

HANDBOOK OF MATHEMATICAL TABLES AND FORMULAS

RICHARD STEVENS BURINGTON



Lb 3871

COMMON MATHEMATICAL SYMBOLS AND ABBREVIATIONS*

+	Plus or positive	sec	Secant
-	Minus or negative	csc	Cosecant
±	{ Plus or minus Positive or negative	vers	Versed sine
±	{ Minus or plus Negative or positive	covers	Covered sine
×	Multipled by	exsec	Exsecant
÷ or :	Divided by	$\sin^{-1} a$ or arc sin a	{ Antisine a Angle whose sine is a Inverse sine a
= or ::	Equals, as	sinh	Hyperbolic sine
≠, ≠	Does not equal	cosh	Hyperbolic cosine
≡	Identical with, identically equal to	tanh	Hyperbolic tangent
≢	Not identical with, is not identically equal to	$\sinh^{-1} a$ or arc sinh a	{ Antihyperbolic sine a Number whose hyper- bolic sine is a
≈	{ Equals approximately Congruent	$P(x,y)$	Rectangular coordinate of point P
>	Greater than	$P(r,\theta)$	Polar coordinate of point P
<	Less than	$f(x), F(x),$ or $\Phi(x)$	Function of x
≥	Greater than or equal to	Δy	Increment of y
≤	Less than or equal to	\doteq or \rightarrow	Approaches as a limit
~	Similar to	Σ	Summation of
∴	Therefore	∞	Infinity
√	Square root	dy	Differential of y
∛	n th root	$\frac{dy}{dx}$ or $f'(x)$	Derivative of $y = f(x)$ with respect to x
a^n	n th power of a	$\frac{d^2y}{dx^2}$ or $f''(x)$	Second derivative of $y = f(x)$ with respect to x
log or log ₁₀	{ Common logarithm Briggsian logarithm	$\frac{d^ny}{dx^n}$ or $f^{(n)}(x)$	n th derivative of $y = f(x)$ with respect to x
ln or log _e or log	{ Natural logarithm Hyperbolic logarithm Napierian logarithm	$\frac{\partial z}{\partial x}$	Partial derivative of z with respect to x
e or ϵ	Base (2.718) of natural system of logarithms	$\frac{\partial^2 z}{\partial x \partial y}$	Second partial deriva- tive of z with respect to y and x
π	Pi (3.1416)	\int	Integral of
∠	Angle	\int_a^b	Integral between the limits a and b
⊥	Perpendicular to	j or i	{ Imaginary quantity ($\sqrt{-1}$), $i^2 = -1$
∥	Parallel to	$x = a + jb$	Symbolic vector notation
a°	a degrees (angle)	$n! = 1 \cdot 2 \cdot 3 \dots n$...
a'	{ a minutes (angle) a prime		
a''	{ a seconds (angle) a second a double prime		
a'''	{ a third a triple prime		
a_n	a sub n		
sin	Sine		
cos	Cosine		
tan	Tangent		
cot or ctn	Cotangent		

* See Glossary for a more complete list of symbols.

GREEK ALPHABET

A	α	Alpha	N	ν	Nu
B	β	Beta	ξ	ξ	Xi
Γ	γ	Gamma	O	o	Omicron
Δ	δ	Delta	Π	π	Pi
E	ϵ	Epsilon	P	ρ	Rho
Z	ζ	Zeta	Σ	σ	Sigma
H	η	Eta	T	τ	Tau
Θ	θ	Theta	Υ	υ	Upsilon
I	ι	Iota	Φ	ϕ	Phi
K	κ	Kappa	X	χ	Chi
Λ	λ	Lambda	Ψ	ψ	Psi
M	μ	Mu	Ω	ω	Omega

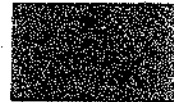
How to Use the Table Locator

Each locator block at the side of this page corresponds in position to an index block on the title page of each group of tables. These readily appear along the side of the book when the pages are flexed as shown.



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LOGARITHMS



**TRIGONOMETRIC
FUNCTIONS**



**EXPONENTIAL AND
HYPERBOLIC
FUNCTIONS**



**POWERS, ROOTS,
RECIPROCAL,
AREAS, AND
CIRCUMFERENCES**



**STATISTICS AND
PROBABILITY**



**INTEREST AND
ACTUARIAL
EXPERIENCE**



**CONSTANTS,
EQUIVALENTS,
AND CONVERSION
FACTORS**



**FOUR-PLACE
TABLES**



THIS BOOK CONTAINS

Part One: Summaries of formulas and theorems of

ALGEBRA	LOGIC
ELEMENTARY GEOMETRY	CALCULUS OF PROPOSITIONS
TRIGONOMETRY	CALCULUS OF RELATIONS
ANALYTIC GEOMETRY	ALGEBRAIC STRUCTURES
DIFFERENTIAL CALCULUS	MATHEMATICAL SYSTEMS
TABLE OF DERIVATIVES	GROUPS
TABLE OF SERIES	POSTULATES FOR COMMON STRUCTURES
INTEGRAL CALCULUS	BOOLEAN ALGEBRAS
TABLE OF INTEGRALS	NUMBER SYSTEMS
VECTOR ANALYSIS	MATRICES
SETS, RELATIONS, AND FUNCTIONS	STATISTICS

Part Two: Tables of

LOGARITHMS	STATISTICS AND PROBABILITY
TRIGONOMETRIC FUNCTIONS	INTEREST AND ACTUARIAL EXPERIENCE
EXPONENTIAL AND HYPERBOLIC FUNCTIONS	CONSTANTS, EQUIVALENTS, AND CONVERSION FACTORS
POWERS, ROOTS, RECIPROCAL, AREAS, AND CIRCUMFERENCES	FOUR-PLACE TABLES

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HANDBOOK OF
MATHEMATICAL TABLES
AND FORMULAS

FOURTH EDITION

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PREFACE

THE WIDE USE and excellent reception of the earlier editions of this book and the valuable suggestions from its many users for additions and improvements have resulted in the publisher's requesting a revision and enlargement of the third edition. As in the cases of the earlier editions, this book has been compiled and arranged to meet the needs of students and workers in mathematics, engineering, physics, chemistry, science, and other fields in which mathematical computations or processes of a mathematical nature are required.

Within the last few years the content of and manner in which mathematics is taught and used have been undergoing many changes. These changes have occurred or are taking place at all levels of instruction and use in the elementary and secondary schools, in colleges, institutes, and universities, and in the many laboratories and industries where mathematics is now so widely used. For example, elementary topics from set theory, symbolic logic, and algebraic structures, which in earlier years were taught to only a few in the latter years of college, are now being taught in some elementary and secondary schools and are now commonly included in the early college courses in mathematics. The rapidly growing and wide use of various types of digital and analog computers has had a marked impact on the teaching and use of mathematics. On the other hand, there are many aspects of the use, tradition, and teaching of mathematics which remain much the same as they have been for many years. The degree to which contemporary changes are accepted and utilized varies widely. The existence of many automatic computers has not diminished the need for compilations of a mathematical character; in fact it has increased the

need for various types of compilations, of which this book covers certain areas. These and related matters have been carefully weighed by the author in preparing this edition. It is felt that the present edition will thus meet the needs of both the contemporary and the more traditional users of mathematics.

The general arrangement of the third edition has been retained. In the first part of the book a summary of the more important formulas and theorems of algebra, trigonometry, analytical geometry, calculus, and vector analysis is given. New sections on sets, logic, relations, functions, algebraic structures, Boolean algebra, matrices, number systems, and elementary statistics have been added. A table of derivatives and a comprehensive table of integrals are included. The second part of the book contains logarithmic and trigonometric tables to five places; tables of natural logarithms, exponential and hyperbolic functions; tables of squares, cubes, square roots, and cube roots; reciprocals and other numerical quantities; probability functions.

This edition is designed as a companion to the *Handbook of Probability and Statistics with Tables* by Richard S. Burington and Donald C. May, McGraw-Hill Book Company.

The entire book has been gone over, and many changes, revisions, and additions have been made within the text as well as in the tables themselves. Every effort has been made to arrange the tables in such a manner that the user will interpret them readily and properly. Considerable effort has been taken to ensure accuracy.

The author wishes to acknowledge valuable suggestions from many of the users of the book and to express his appreciation to Dr. Donald C. May for critical suggestions. He is indebted to the late Sir Ronald A. Fisher, F. R. S., Cambridge, and also to Messrs. Oliver & Boyd, Ltd., Edinburgh, for permission to reprint Table no. III from his book *Statistical Methods for Research Workers* as Table 24 in this book.

The author is especially indebted to his wife, Jennet Mae Burington, for her very considerable assistance in the preparation of the manuscript and in the reading of the proof.

RICHARD STEVENS BURINGTON

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PART ONE

*Formulas, definitions, and
theorems from elementary
mathematics*

I. ALGEBRA

1. FUNDAMENTAL LAWS

- (a) Commutative law: $a + b = b + a$ $ab = ba$
(b) Associative law: $a + (b + c) = (a + b) + c$,
 $a(bc) = (ab)c$
(c) Distributive law: $a(b + c) = ab + ac$

2. LAWS OF EXPONENTS

$$\begin{aligned} a^x a^y &= a^{x+y} & (ab)^x &= a^x b^x & (a^x)^y &= a^{xy} \\ a^0 &= 1 \text{ if } a \neq 0 & a^{-x} &= \frac{1}{a^x} & \frac{a^x}{a^y} &= a^{x-y} \\ a^{x/y} &= \sqrt[y]{a^x} & a^{1/y} &= \sqrt[y]{a} \end{aligned}$$

3. OPERATIONS WITH ZERO

$$a - a = 0 \quad a \times 0 = 0 \times a = 0$$

If $a \neq 0$, $\frac{0}{a} = 0$, $0a = 0$, $a^0 = 1$ (Division by zero undefined)

4. COMPLEX NUMBERS (a number of the form $a + bi$ where a and b are real)

$$i = \sqrt{-1}, i^2 = -1, i^3 = -i, i^4 = 1, i^5 = i, \text{ etc.}$$

$$a + bi = c + di \text{ if and only if } a = c, b = d$$

$$(a + bi) + (c + di) = (a + c) + (b + d)i$$

$$(a + bi)(c + di) = (ac - bd) + (ad + bc)i$$

$$\frac{a + bi}{c + di} = \frac{(a + bi)(c - di)}{(c + di)(c - di)} = \frac{ac + bd}{c^2 + d^2} + \frac{bc - ad}{c^2 + d^2}i$$

5. LAWS OF LOGARITHMS (see explanation of Table 1)

If M, N, b are positive numbers and $b \neq 1$:

$$\log_b MN = \log_b M + \log_b N \quad \log_b \frac{M}{N} = \log_b M - \log_b N$$

4. Algebra

$$\log_b M^p = p \log_b M \quad \log_b \sqrt[q]{M} = \frac{1}{q} \log_b M$$

$$\log_b \frac{1}{M} = -\log_b M \quad \log_b b = 1 \quad \log_b 1 = 0$$

Change of base of logarithms ($c \neq 1$):

$$\log_b M = \log_c M \log_b c = \frac{\log_c M}{\log_c b}$$

6. BINOMIAL THEOREM (n a positive integer)

$$(a + b)^n = a^n + na^{n-1}b + \frac{n(n-1)}{2!}a^{n-2}b^2$$

$$+ \frac{n(n-1)(n-2)}{3!}a^{n-3}b^3 + \dots + nab^{n-1} + b^n$$

where $n! = \underline{n} = 1 \cdot 2 \cdot 3 \dots (n-1)n$

7. EXPANSIONS AND FACTORS

$$(a \pm b)^2 = a^2 \pm 2ab + b^2$$

$$(a \pm b)^3 = a^3 \pm 3a^2b + 3ab^2 \pm b^3$$

$$(a + b + c)^2 = a^2 + b^2 + c^2 + 2ab + 2ac + 2bc$$

$$a^2 - b^2 = (a - b)(a + b)$$

$$a^3 - b^3 = (a - b)(a^2 + ab + b^2)$$

$$a^3 + b^3 = (a + b)(a^2 - ab + b^2)$$

$$a^n - b^n = (a - b)(a^{n-1} + a^{n-2}b + \dots + b^{n-1})$$

$$a^n - b^n = (a + b)(a^{n-1} - a^{n-2}b + \dots - b^{n-1})$$

for n an even integer

$$a^n + b^n = (a + b)(a^{n-1} - a^{n-2}b + \dots + b^{n-1})$$

for n an odd integer

$$a^4 + a^2b^2 + b^4 = (a^2 + ab + b^2)(a^2 - ab + b^2)$$

8. RATIO AND PROPORTION

If $a:b = c:d$ or $\frac{a}{b} = \frac{c}{d}$, then $ad = bc$, $\frac{a}{c} = \frac{b}{d}$

If $\frac{a}{b} = \frac{c}{d} = \frac{e}{f} = \dots = k,$

then $k = \frac{a + c + e + \dots}{b + d + f + \dots} = \frac{pa + qc + re + \dots}{pb + qd + rf + \dots}$

9. CONSTANT FACTOR OF PROPORTIONALITY (OR VARIATION), k

If y varies directly as x , or y is proportional to x ,

$$y = kx$$

If y varies inversely as x , or y is inversely proportional to x ,

$$y = \frac{k}{x}$$

If y varies jointly as x and z ,

$$y = kxz$$

If y varies directly as x and inversely as z ,

$$y = \frac{kx}{z}$$

10. ARITHMETIC PROGRESSION

$$a \quad a + d \quad a + 2d \quad a + 3d \quad \dots$$

If a is the first term, d the common difference, n the number of terms, l the last term, and s the sum of n terms,

$$l = a + (n - 1)d \quad s = \frac{n}{2}(a + l)$$

The *arithmetic mean* of a and b is $(a + b)/2$.

11. GEOMETRIC PROGRESSION

$$a, ar, ar^2, ar^3 \dots$$

If a is the first term, r the common ratio, n the number of terms, l the last term, and S_n the sum of n terms,

$$l = ar^{n-1} \quad S_n = a \frac{r^n - 1}{r - 1} = \frac{rl - a}{r - 1}$$

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If $r^2 < 1$, S_n approaches the limit S_∞ as n increases without limit,

$$S_\infty = \frac{a}{1-r}$$

The *geometric mean* of a and b is \sqrt{ab} .

12. HARMONIC PROGRESSION A sequence of numbers whose reciprocals form an arithmetic progression is called an *harmonic progression*. Thus

$$\frac{1}{a} \quad \frac{1}{a+d} \quad \frac{1}{a+2d} \quad \dots$$

The *harmonic mean* of a and b is $2ab/(a+b)$.

13. PERMUTATION Each different arrangement of all or a part of a set of things is called a *permutation*. The number of permutations of n different things taken r at a time is

$$P(n,r) = {}_n P_r = n(n-1)(n-2) \dots (n-r+1) = \frac{n!}{(n-r)!}$$

where $n! = n(n-1)(n-2) \dots (1)$

14. COMBINATIONS Each of the groups or relations which can be made by taking part or all of a set of things, without regard to the arrangement of the things in a group, is called a *combination*. The number of combinations of n different things taken r at a time is

$$\begin{aligned} C(n,r) = {}_n C_r &= \binom{n}{r} = \frac{{}_n P_r}{r!} \\ &= \frac{n(n-1) \dots (n-r+1)}{r(r-1) \dots 1} = \frac{n!}{r!(n-r)!} \end{aligned}$$

15. PROBABILITY If an event may occur in p ways and may fail in q ways, all ways being equally likely, the *probability* of its occurrence is $p/(p+q)$ and that of its failure to occur is $q/(p+q)$. (For further details see Richard S. Burington and Donald C. May, *Handbook of Probability and Statistics with Tables*, McGraw-Hill Book Company, New York.)

16. REMAINDER THEOREM (see Sec. 30) If the polynomial $f(x)$ is

divided by $x - a$, the remainder is $f(a)$. Hence, if a is a root of the equation $f(x) = 0$, then $f(x)$ is divisible by $x - a$.

17. DETERMINANTS* The determinant D of order n ,

$$D = \begin{vmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \cdots & \vdots \\ a_{n1} & a_{n2} & \cdots & a_{nn} \end{vmatrix}$$

is defined to be the sum

$$\sum (\pm) a_{1i} a_{2j} a_{3k} \cdots a_{nl}$$

of $n!$ terms, the sign in a given term being taken plus or minus according as the number of inversions (of the numbers $1, 2, 3, \dots, n$) in the corresponding sequence i, j, k, \dots, l is even or odd.

The cofactor A_{ij} of the element a_{ij} is defined to be the product of $(-1)^{i+j}$ by the determinant obtained from D by deleting the i th row and the j th column.

The following theorems are true:

(a) If the corresponding rows and columns of D are interchanged, D is unchanged.

(b) If any two rows (or columns) of D are interchanged, D is changed to $-D$.

(c) If any two rows (or columns) are alike, then $D = 0$.

(d) If each element of a row (or column) of D is multiplied by m , the new determinant is equal to mD .

(e) If to each element of a row (or column) is added m times the corresponding element in another row (or column), D is unchanged.

(f) $D = a_{1j}A_{1j} + a_{2j}A_{2j} + \cdots + a_{nj}A_{nj}$, $j = 1, 2, \dots, n$

(g) $0 = a_{1k}A_{1j} + a_{2k}A_{2j} + \cdots + a_{nk}A_{nj}$, if $j \neq k$

(h) The solution of the system of equations

$$a_{i1}x_1 + a_{i2}x_2 + \cdots + a_{in}x_n = c_i \quad i = 1, 2, \dots, n$$

is unique if $D \neq 0$. The solution is given by the equations

$$Dx_1 = C_1 \quad Dx_2 = C_2, \dots \quad Dx_n = C_n$$

where C_k is what D becomes when the elements of its k th column are replaced by c_1, c_2, \dots, c_n , respectively.

* See Chap. 13, Matrices, for further theorems.

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Example 1.

$$\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} = a_{11} \begin{vmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{vmatrix} - a_{12} \begin{vmatrix} a_{21} & a_{23} \\ a_{31} & a_{33} \end{vmatrix} + a_{13} \begin{vmatrix} a_{21} & a_{22} \\ a_{31} & a_{32} \end{vmatrix} \\ = a_{11}(a_{22}a_{33} - a_{32}a_{23}) - a_{12}(a_{21}a_{33} - a_{31}a_{23}) \\ + a_{13}(a_{21}a_{32} - a_{31}a_{22})$$

Example 2. Find the values of x_1, x_2, x_3 which satisfy the system

$$\begin{aligned} 2x_1 + x_2 - 2x_3 &= -6 \\ x_1 + x_2 + x_3 &= 2 \\ -x_1 - 2x_2 + 2x_3 &= 12 \end{aligned}$$

By 17(h), we find

$$x_1 = \frac{\begin{vmatrix} -6 & 1 & -2 \\ 2 & 1 & 1 \\ 2 & 1 & -2 \end{vmatrix}}{\begin{vmatrix} 2 & 1 & -2 \\ 1 & 1 & 1 \\ -1 & -2 & 3 \end{vmatrix}} = \frac{8}{8} = 1; \quad x_2 = \frac{\begin{vmatrix} 2 & -6 & -2 \\ 1 & 2 & 1 \\ -1 & 12 & 3 \end{vmatrix}}{\begin{vmatrix} 2 & 1 & -2 \\ 1 & 1 & 1 \\ -1 & -2 & 3 \end{vmatrix}} = \frac{-16}{8} = -2; \\ x_3 = \frac{\begin{vmatrix} 2 & 1 & -6 \\ 1 & 1 & 2 \\ -1 & -2 & 12 \end{vmatrix}}{\begin{vmatrix} 2 & 1 & -2 \\ 1 & 1 & 1 \\ -1 & -2 & 3 \end{vmatrix}} = 3$$

Interest, Annuities, Sinking Funds

In this section, n is the number of years and r the rate of interest expressed as a decimal.

13. AMOUNT A principal P placed at a rate of interest r for n years accumulates to an amount A_n as follows:

At simple interest: $A_n = P(1 + nr)$

At interest compounded annually:* $A_n = P(1 + r)^n$

At interest compounded q times a year: $A_n = P\left(1 + \frac{r}{q}\right)^{nq}$

* See Table 25.

19. NOMINAL AND EFFECTIVE RATES The rate of interest quoted in describing a given variety of compound interest is called the *nominal rate*. The rate per year at which interest is earned during each year is called the *effective rate*. The effective rate i corresponding to the nominal rate r , compounded q times a year, is:

$$i = \left(1 + \frac{r}{q}\right)^q - 1$$

20. PRESENT OR DISCOUNTED VALUE OF A FUTURE AMOUNT The present quantity P which in n years will accumulate to the amount A_n at the rate of interest r is:

At simple interest:
$$P = \frac{A_n}{1 + nr}$$

At interest compounded annually: *
$$P = \frac{A_n}{(1 + r)^n}$$

At interest compounded q times a year:
$$P = \frac{A_n}{(1 + r/q)^{nq}}$$

P is called the *present value* of A_n due in n years at rate r .

21. TRUE DISCOUNT The true discount is

$$D = A_n - P$$

22. ANNUITY A fixed sum of money paid at regular intervals is called an *annuity*.

23. AMOUNT OF AN ANNUITY † If an annuity P is deposited at the end of each successive year (beginning one year hence), and the interest at rate r , compounded annually, is paid on the accumulated deposit at the end of each year, the total amount N accumulated at the end of n years is

$$N = P \frac{(1 + r)^n - 1}{r}$$

N is called the *amount* of an annuity P .

* See Table 26.

† See Table 27.

24. PRESENT VALUE OF AN ANNUITY* The total present amount P which will supply an annuity N at the end of each year for n years, beginning one year hence (assuming that in successive years the amount not yet paid out earns interest at rate r , compounded annually), is

$$P = N \frac{(1+r)^n - 1}{r(1+r)^n} = N \frac{1 - (1+r)^{-n}}{r}$$

P is called the *present value* of an annuity.

25. AMOUNT OF A SINKING FUND† If a fixed investment N is made at the end of each successive year (beginning at the end of the first year), and interest paid at rate r , compounded annually, is paid on the accumulated amount of the investment at the end of each year, the total amount S accumulated at the end of n years is

$$S = N \frac{(1+r)^n - 1}{r}$$

S is called the *amount of the sinking fund*.

26. FIXED INVESTMENT, OR ANNUAL INSTALLMENT The amount N that must be placed at the end of each year (beginning one year hence), with compound interest paid at rate r on the accumulated deposit, in order to accumulate a sinking fund S in n years is

$$N = S \frac{r}{(1+r)^n - 1}$$

N is called a *fixed investment* or *annual installment*.

Algebraic Equations‡

27. QUADRATIC EQUATIONS If

$$ax^2 + bx + c = 0 \quad a \neq 0$$

then

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

* See Table 28.

† Compare with Sec. 23.

‡ For linear equations, see Sec. 17.

If a, b, c are real and
 if $b^2 - 4ac > 0$ the roots are real and unequal
 if $b^2 - 4ac = 0$ the roots are real and equal
 if $b^2 - 4ac < 0$ the roots are imaginary

28. CUBIC EQUATIONS The cubic equation

$$y^3 + py^2 + qy + r = 0$$

may be reduced by the substitution

$$y = x - \frac{p}{3}$$

to the normal form

$$x^3 + ax + b = 0$$

where $a = \frac{1}{3}(3q - p^2)$ $b = \frac{1}{27}(2p^3 - 9pq + 27r)$

which has the solutions x_1, x_2, x_3 ,

$$x_1 = A + B, \quad x_2, x_3 = -\frac{1}{2}(A + B) \pm \frac{i\sqrt{3}}{2}(A - B)$$

where

$$i^2 = -1 \quad A = \sqrt[3]{\frac{b}{2} + \sqrt{\frac{b^2}{4} + \frac{a^3}{27}}}$$

$$B = \sqrt[3]{-\frac{b}{2} - \sqrt{\frac{b^2}{4} + \frac{a^3}{27}}}$$

If p, q, r are real (and hence if a and b are real) and

if $\frac{b^2}{4} + \frac{a^3}{27} > 0$ there are one real root and two conjugate imaginary roots
 if $\frac{b^2}{4} + \frac{a^3}{27} = 0$ there are three real roots of which at least two are equal
 if $\frac{b^2}{4} + \frac{a^3}{27} < 0$ there are three real and unequal roots

12 *Algebra*

If
$$\frac{b^2}{4} + \frac{a^3}{27} < 0$$

the above formulas are impractical. The real roots are

$$x_k = 2 \sqrt{-\frac{a}{3}} \cos \left(\frac{\phi}{3} + 120^\circ k \right) \quad k = 0, 1, 2$$

where
$$\cos \phi = \mp \sqrt{\frac{b^2/4}{-a^3/27}}$$

and where the upper sign is to be used if b is positive and the lower sign if b is negative.

If
$$\frac{b^2}{4} + \frac{a^3}{27} > 0 \quad \text{and} \quad a > 0$$

the real root is

$$x = 2 \sqrt{\frac{a}{3}} \cot 2\phi$$

where ϕ and ψ are to be computed from

$$\cot 2\psi = \mp \sqrt{\frac{b^2/4}{a^3/27}} \quad \tan \phi = \sqrt[3]{\tan \psi}$$

and where the upper sign is to be used if b is positive and the lower sign if b is negative.

If
$$\frac{b^2}{4} + \frac{a^3}{27} = 0$$

the roots are

$$x = \mp 2 \sqrt{-\frac{a}{3}}, \quad \pm \sqrt{-\frac{a}{3}}, \quad \pm \sqrt{-\frac{a}{3}}$$

where the upper sign is to be used if b is positive and the lower sign if b is negative.

29. BIQUADRATIC (QUARTIC) EQUATION The quartic equation

$$y^4 + py^3 + qy^2 + ry + s = 0$$

may be reduced to the form

$$x^4 + ax^2 + bx + c = 0$$

by the substitution

$$y = x - \frac{P}{4}$$

Let l , m , and n denote the roots of the resolvent cubic

$$t^3 + \frac{a}{2}t^2 + \frac{a^2 - 4c}{16}t - \frac{b^2}{64} = 0$$

The required roots of the reduced quartic are

$$\begin{aligned} x_1 &= +\sqrt{l} + \sqrt{m} + \sqrt{n} & x_2 &= +\sqrt{l} - \sqrt{m} - \sqrt{n} \\ x_3 &= -\sqrt{l} + \sqrt{m} - \sqrt{n} & x_4 &= -\sqrt{l} - \sqrt{m} + \sqrt{n} \end{aligned}$$

where the selection of the square root to be attached to each of the quantities \sqrt{l} , \sqrt{m} , \sqrt{n} must give $\sqrt{l}\sqrt{m}\sqrt{n} = -b/8$.

30. GENERAL EQUATIONS OF THE n th DEGREE

$$P \equiv a_0x^n + a_1x^{n-1} + a_2x^{n-2} + \cdots + a_{n-1}x + a_n = 0$$

If $n > 4$, there is no formula which gives the roots of this general equation. The following methods may be used to advantage:

(a) *Roots by factors.* By trial, find a number r such that $x = r$ satisfies the equation, that is, such that

$$a_0r^n + a_1r^{n-1} + a_2r^{n-2} + \cdots + a_{n-1}r + a_n = 0 \quad a_0 \neq 0$$

Then $x - r$ is a factor of the left-hand member P of the equation. Divide P by $x - r$, leaving an equation of degree one less than that of the original equation. Next, proceed in the same manner with the reduced equation. (All integer roots of $P = 0$ are divisors of a_n/a_0 .)

(b) *Roots by approximation.* Suppose the coefficients a_i in P are real. Let a and b be real numbers. If for $x = a$ and $x = b$ the left member P of the equation has opposite signs, then a root lies between a and b . By repeated application of this principle, real roots to any desired degree of accuracy may be obtained.

(c) *Roots by graphing.* If a graph of P is plotted as a function of x , the real roots are the values of x where the graph crosses the x axis.

By increasing the scale of the portion of the graph near an estimated root, the root may be obtained to any desired degree of accuracy.

(d) *Descartes' rule.* The number of positive real roots of an equation with real coefficients either is equal to the number of its variations of sign or is less than that number by a positive even integer. A root of multiplicity m is here counted as m roots.

31. THE EQUATION $x^n = a$ The n roots of this equation are

$$x = \begin{cases} \sqrt[n]{a} \left(\cos \frac{2k\pi}{n} + \sqrt{-1} \sin \frac{2k\pi}{n} \right) & \text{if } a > 0 \\ \sqrt[n]{-a} \left(\cos \frac{(2k+1)\pi}{n} + \sqrt{-1} \sin \frac{(2k+1)\pi}{n} \right) & \text{if } a < 0 \end{cases}$$

where k takes successively the values $0, 1, 2, 3, \dots, n-1$.

31a. STIRLING'S FORMULA For large values of n ,

$$\sqrt{2n\pi} \left(\frac{n}{e} \right)^n < n! < \sqrt{2n\pi} \left(\frac{n}{e} \right)^n \left(1 + \frac{1}{12n-1} \right)$$

where $\pi = 3.14159 \dots$ and $e = 2.71828 \dots$.

$$\log n! \cong (n + \frac{1}{2}) \log n - n \log e + \log \sqrt{2\pi}$$

2. ELEMENTARY GEOMETRY

Let a, b, c, d , and s denote lengths, A denote areas, and V denote volumes.

32. TRIANGLE (see Sec. 65) $A = bh/2$, where b denotes the base and h the altitude.

33. RECTANGLE $A = ab$, where a and b denote the lengths of the sides.

34. PARALLELOGRAM (OPPOSITE SIDES PARALLEL) $A = ah = ab \sin \theta$, where a and b denote the sides, h the altitude, and θ the angle between the sides.

35. TRAPEZOID (FOUR SIDES, TWO PARALLEL) $A = \frac{1}{2}h(a + b)$, where a and b are the sides and h the altitude.

36. REGULAR POLYGON OF n SIDES (Fig. 1, see Sec. 37)

$$A = \frac{1}{4} na^2 \cot \frac{180^\circ}{n} \quad \text{where } a \text{ is length of side}$$

$$R = \frac{a}{2} \csc \frac{180^\circ}{n} \quad \text{where } R \text{ is radius of circumscribed circle}$$

$$r = \frac{a}{2} \cot \frac{180^\circ}{n} \quad \text{where } r \text{ is radius of inscribed circle}$$

$$\alpha = \frac{360^\circ}{n} = \frac{2\pi}{n} \quad \text{radians}$$

$$\beta = \frac{n-2}{n} \times 180^\circ = \frac{n-2}{n} \pi$$

$$a = 2r \tan \frac{\alpha}{2} = 2R \sin \frac{\alpha}{2}$$

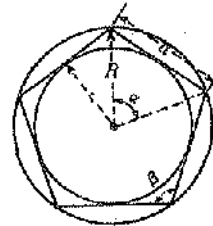


Fig. 1

radians where α and β are the angles indicated in Fig. 1

37. CIRCLE (Fig. 2)

Let C = circumference S = length of arc subtended by θ
 R = radius l = chord subtended by arc S
 D = diameter h = rise
 A = area θ = central angle, radians

$$C = 2\pi R = \pi D \quad \pi = 3.14159 \dots$$

$$S = R\theta = \frac{1}{2}D\theta = D \cos^{-1} \frac{d}{R}$$

$$l = 2\sqrt{R^2 - d^2} = 2R \sin \frac{\theta}{2} = 2d \tan \frac{\theta}{2}$$

$$d = \frac{1}{2}\sqrt{4R^2 - l^2} = R \cos \frac{\theta}{2} = \frac{1}{2}l \cot \frac{\theta}{2}$$

$$h = R - d$$

$$\theta = \frac{S}{R} = \frac{2S}{D} = 2 \cos^{-1} \frac{d}{R} = 2 \tan^{-1} \frac{l}{2d} = 2 \sin^{-1} \frac{l}{D}$$

$$A \text{ (circle)} = \pi R^2 = \frac{1}{4}\pi D^2$$

$$A \text{ (sector)} = \frac{1}{2}Rs = \frac{1}{2}R^2\theta$$

$$A \text{ (segment)} = A \text{ (sector)} - A \text{ (triangle)} = \frac{1}{2}R^2(\theta - \sin \theta)$$

$$= R^2 \cos^{-1} \frac{R-h}{R} - (R-h)\sqrt{2Rh-h^2}$$

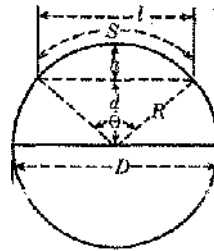


Fig. 2

Perimeter of an n -side regular polygon inscribed in a circle

$$= 2nR \sin \frac{\pi}{n}$$

$$\text{Area of inscribed polygon} = \frac{1}{2}nR^2 \sin \frac{2\pi}{n}$$

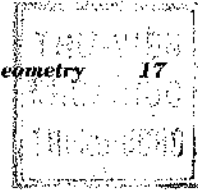
Perimeter of an n -side regular polygon circumscribed about a circle

$$= 2nR \tan \frac{\pi}{n}$$

$$\text{Area of circumscribed polygon} = nR^2 \tan \frac{\pi}{n}$$

Radius of circle inscribed in a triangle of sides a , b , and c is

$$r = \frac{\sqrt{(s-a)(s-b)(s-c)}}{s} \quad s = \frac{1}{2}(a+b+c)$$



Radius of circle circumscribed about a triangle is

$$R = \frac{abc}{4\sqrt{s(s-a)(s-b)(s-c)}}$$

38. ELLIPSE (see Sec. 84) Perimeter = $4aE(k)$, where $k^2 = 1 - (b^2/a^2)$ and $E(k)$ is the complete elliptic integral E given in Table 34. $A = \pi ab$, where a and b are lengths of semimajor and semiminor axes respectively.



Fig. 3

39. PARABOLA (see Sec. 83)

$$A = \frac{2ld}{3}$$

$$\text{Height of } d_1 = \frac{d}{l^2} (l^2 - l_1^2)$$

$$\text{Width of } l_1 = l \sqrt{\frac{d - d_1}{d}}$$

$$\text{Length of arc} = l \left[1 + \frac{2}{3} \left(\frac{2d}{l} \right)^2 - \frac{2}{5} \left(\frac{2d}{l} \right)^4 + \dots \right]$$



Fig. 4

40. CATENARY, CYCLOID, ETC. (see Secs. 91 to 101)

41. AREA BY APPROXIMATION If $y_0, y_1, y_2, \dots, y_n$ are the lengths of a series of equally spaced parallel chords and if h is their distance apart, the area enclosed by boundary is given approximately by any one of the following formulas:

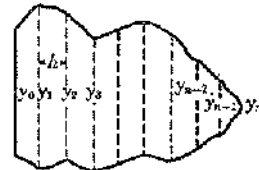


Fig. 5

$$A_T = h \left[\frac{1}{2}(y_0 + y_n) + y_1 + y_2 + \dots + y_{n-1} \right] \quad (\text{trapezoidal rule})$$

$$A_D = h [0.4(y_0 + y_n) + 1.1(y_1 + y_{n-1}) + y_2 + y_3 + \dots + y_{n-2}] \quad (\text{Durand's rule})$$

$$A_S = \frac{1}{3}h [(y_0 + y_n) + 4(y_1 + y_3 + \dots + y_{n-1}) + 2(y_2 + y_4 + \dots + y_{n-2})] \quad (n \text{ even, Simpson's rule})$$

In general, A_S gives the more accurate approximation. The greater the value of n , the greater the accuracy of approximation.

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42. **CUBE** $V = a^3$; $d = a\sqrt{3}$; total surface area = $6a^2$, where a is length of side and d is length of diagonal.

43. **RECTANGULAR PARALLELOPIPED**

$$V = abc \quad d = \sqrt{a^2 + b^2 + c^2}$$

Total surface area = $2(ab + bc + ca)$, where a , b , and c are lengths of sides and d is length of diagonal.

44. **PRISM OR CYLINDER** $V = (\text{area of base}) \times (\text{altitude})$

Lateral area = (perimeter of right section) \times (lateral edge)

45. **PYRAMID OR CONE** $V = \frac{1}{3}(\text{area of base}) \times (\text{altitude})$

Lateral area of regular pyramid
= $\frac{1}{2}(\text{perimeter of base}) \times (\text{slant height})$

46. **FRUSTUM OF PYRAMID OR CONE** $V = \frac{1}{3}(A_1 + A_2 + \sqrt{A_1A_2})h$, where h is the altitude and A_1 and A_2 are the areas of the bases.

Lateral area of a regular figure
= $\frac{1}{2}(\text{sum of perimeters of base}) \times (\text{slant height})$

46a. **PRISMOID** $V = (h/6)(A_1 + A_2 + 4A_3)$, where h = altitude, A_1 and A_2 are the areas of the bases, and A_3 is the area of the midsection parallel to bases.

47. **AREA OF SURFACE AND VOLUME OF REGULAR POLYHEDRA OF EDGE l**

Name	Type of surface	Area of surface	Volume
Tetrahedron	4 equilateral triangles	$1.73205l^2$	$0.11785l^3$
Hexahedron (cube)	6 squares	$6.00000l^2$	$1.00000l^3$
Octahedron	8 equilateral triangles	$3.46410l^2$	$0.47140l^3$
Dodecahedron	12 pentagons	$20.64578l^2$	$7.66312l^3$
Icosahedron	20 equilateral triangles	$8.66025l^2$	$2.18169l^3$

48. SPHERE

$$\begin{aligned}
 A \text{ (sphere)} &= 4\pi R^2 = \pi D^2 \\
 A \text{ (zone)} &= 2\pi R h_1 = \pi D h_1 \\
 V \text{ (sphere)} &= \frac{4}{3}\pi R^3 = \frac{1}{6}\pi D^3 \\
 V \text{ (spherical sector)} &= \frac{2}{3}\pi R^2 h_1 \\
 &= \frac{1}{8}\pi D^2 h_1 \\
 V \text{ (spherical segment of one base)} & \\
 &= \frac{1}{6}\pi h_3(3r_3^2 + h_3^2) \\
 V \text{ (spherical segment of two bases)} & \\
 &= \frac{1}{6}\pi h_2(3r_3^2 + 3r_2^2 + h_2^2) \\
 A \text{ (lune)} &= 2R^2\theta, \text{ where } \theta \text{ is angle, radians of lune}
 \end{aligned}$$

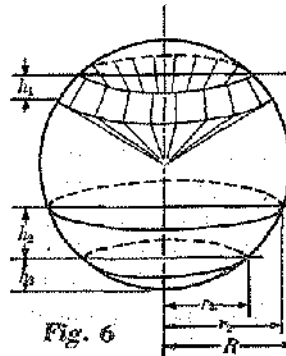


Fig. 6

49. SOLID ANGLE ψ The solid angle ψ at any point P subtended by a surface S is equal to the area A of the portion of the surface of a sphere of unit radius, center at P , which is cut out by a conical surface, with vertex at P , passing through the perimeter of S .



Fig. 7

The unit solid angle ψ is called the *steradian*.

The total solid angle about a point is 4π steradians.

50. ELLIPSOID $V = \frac{4}{3}\pi abc$, where a , b , and c are the lengths of the semiaxes.

51. TORUS

$$\begin{aligned}
 V &= 2\pi^2 R r^2 \\
 \text{Area of surface} = S &= 4\pi^2 R r
 \end{aligned}$$

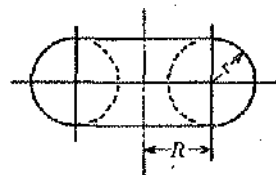


Fig. 8

51a. THEOREMS OF PAPPUS

(a) If a plane area A is rotated about a line l in the plane of A and not cutting A , the volume of the solid generated is equal to the product of A and the distance traveled by the center of gravity of A .

(b) If a plane curve C is rotated about a line l in the plane of C and not cutting C , the area of the surface generated is equal to the product of the length of C and the distance traveled by the center of gravity of C .

3. TRIGONOMETRY

52. ANGLE If two lines intersect, one line may be rotated about their point of intersection through the *angle* which they form until it coincides with the other line.

The angle is said to be *positive* if the rotation is counterclockwise, and *negative* if clockwise.

A complete revolution of a line is a rotation through an angle of 360° . Thus

A *degree* is $\frac{1}{360}$ of the plane angle about a point.

A *radian* is the angle subtended at the center of a circle by an arc whose length is equal to that of the radius.

$$180^\circ = \pi \text{ radians; } 1^\circ = \frac{\pi}{180} \text{ radians; } 1 \text{ radian} = \frac{180}{\pi} \text{ degrees}$$

53. TRIGONOMETRIC FUNCTIONS OF AN ANGLE α Let α be any angle whose initial side lies on the positive x axis and whose vertex is at the origin and (x, y) be any point on the terminal side of the angle. (x is positive if measured along OX to the right from the y axis and negative if measured along OX' to the left from the y axis. Likewise, y is positive if measured parallel to OY and negative if measured parallel to OY' .) Let r be the positive distance from the origin to the point. The trigonometric functions of an angle are defined as follows:

$$\text{sine } \alpha = \sin \alpha = \frac{y}{r}$$

$$\text{cosine } \alpha = \cos \alpha = \frac{x}{r}$$

$$\text{tangent } \alpha = \tan \alpha = \frac{y}{x}$$

$$\text{cotangent } \alpha = \cot \alpha = \text{ctn } \alpha = \frac{x}{y}$$

$$\text{secant } \alpha = \sec \alpha = \frac{r}{x}$$

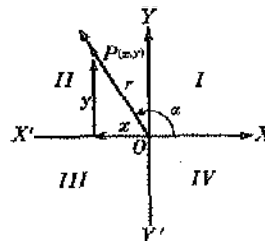


Fig. 9

$$\text{cosecant } \alpha = \text{csc } \alpha = \frac{r}{y}$$

$$\text{exsecant } \alpha = \text{exsec } \alpha = \sec \alpha - 1$$

$$\text{versine } \alpha = \text{vers } \alpha = 1 - \cos \alpha$$

$$\text{coversine } \alpha = \text{covers } \alpha = 1 - \sin \alpha$$

$$\text{haversine } \alpha = \text{hav } \alpha = \frac{1}{2} \text{vers } \alpha$$

54. SIGNS OF THE FUNCTIONS

Quadrant	sin	cos	tan	cot	sec	csc
I	+	+	+	+	+	+
II	+	-	-	-	-	+
III	-	-	+	+	-	-
IV	-	+	-	-	+	-

55. FUNCTIONS OF 0°, 30°, 45°, 60°, 90°, 180°, 270°, 360°

	0°	30°	45°	60°	90°	180°	270°	360°
sin	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1	0	-1	0
cos	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0	-1	0	1
tan	0	$\frac{\sqrt{3}}{3}$	1	$\sqrt{3}$		0		0
cot		$\sqrt{3}$	1	$\frac{\sqrt{3}}{3}$	0		0	
sec	1	$\frac{2\sqrt{3}}{3}$	$\sqrt{2}$	2		-1		1
csc		2	$\sqrt{2}$	$\frac{2\sqrt{3}}{3}$	1		-1	

56. VARIATIONS OF THE FUNCTIONS

Quadrant	sin	cos	tan	cot	sec	csc
I	0 → +1	+1 → 0	0 → +∞	+∞ → 0	+1 → +∞	+∞ → +1
II	+1 → 0	0 → -1	-∞ → 0	0 → -∞	-∞ → -1	+1 → +∞
III	0 → -1	-1 → 0	0 → +∞	+∞ → 0	-1 → -∞	-∞ → -1
IV	-1 → 0	0 → +1	-∞ → 0	0 → -∞	+∞ → +1	-1 → -∞

57. FUNCTIONS OF ANGLES IN ANY QUADRANT IN TERMS OF ANGLES IN FIRST QUADRANT

	$-\alpha$	$90^\circ \pm \alpha$	$180^\circ \pm \alpha$	$270^\circ \pm \alpha$	$n(360)^\circ \pm \alpha$
sin	$-\sin \alpha$	$+\cos \alpha$	$\mp \sin \alpha$	$-\cos \alpha$	$\pm \sin \alpha$
cos	$+\cos \alpha$	$\mp \sin \alpha$	$-\cos \alpha$	$\pm \sin \alpha$	$+\cos \alpha$
tan	$-\tan \alpha$	$\mp \cot \alpha$	$\pm \tan \alpha$	$\mp \cot \alpha$	$\pm \tan \alpha$
cot	$-\cot \alpha$	$\mp \tan \alpha$	$\pm \cot \alpha$	$\mp \tan \alpha$	$\pm \cot \alpha$
sec	$+\sec \alpha$	$\mp \csc \alpha$	$-\sec \alpha$	$\pm \csc \alpha$	$+\sec \alpha$
csc	$-\csc \alpha$	$+\sec \alpha$	$\mp \csc \alpha$	$-\sec \alpha$	$\pm \csc \alpha$

n denotes any integer.

For example:

$$\tan (270^\circ + \alpha) = -\cot \alpha$$

58. FUNDAMENTAL IDENTITIES

$$\sin \alpha = \frac{1}{\csc \alpha}; \cos \alpha = \frac{1}{\sec \alpha}; \tan \alpha = \frac{1}{\cot \alpha} = \frac{\sin \alpha}{\cos \alpha}$$

$$\csc \alpha = \frac{1}{\sin \alpha}; \sec \alpha = \frac{1}{\cos \alpha}; \cot \alpha = \frac{1}{\tan \alpha} = \frac{\cos \alpha}{\sin \alpha}$$

$$\sin^2 \alpha + \cos^2 \alpha = 1; 1 + \tan^2 \alpha = \sec^2 \alpha; 1 + \cot^2 \alpha = \csc^2 \alpha$$

$$\sin 2\alpha = 2 \sin \alpha \cos \alpha$$

$$\cos 2\alpha = 2 \cos^2 \alpha - 1 = 1 - 2 \sin^2 \alpha = \cos^2 \alpha - \sin^2 \alpha$$

$$\tan 2\alpha = \frac{2 \tan \alpha}{1 - \tan^2 \alpha}$$

$$\sin 3\alpha = 3 \sin \alpha - 4 \sin^3 \alpha$$

$$\cos 3\alpha = 4 \cos^3 \alpha - 3 \cos \alpha$$

$$\sin n\alpha = 2 \sin (n-1)\alpha \cos \alpha - \sin (n-2)\alpha$$

$$\cos n\alpha = 2 \cos (n-1)\alpha \cos \alpha - \cos (n-2)\alpha$$

$$\sin (\alpha \pm \beta) = \sin \alpha \cos \beta \pm \cos \alpha \sin \beta$$

$$\cos (\alpha \pm \beta) = \cos \alpha \cos \beta \mp \sin \alpha \sin \beta$$

$$\tan (\alpha \pm \beta) = \frac{\tan \alpha \pm \tan \beta}{1 \mp \tan \alpha \tan \beta}$$

$$\sin \alpha + \sin \beta = 2 \sin \frac{1}{2}(\alpha + \beta) \cos \frac{1}{2}(\alpha - \beta)$$

$$\sin \alpha - \sin \beta = 2 \cos \frac{1}{2}(\alpha + \beta) \sin \frac{1}{2}(\alpha - \beta)$$

$$\cos \alpha + \cos \beta = 2 \cos \frac{1}{2}(\alpha + \beta) \cos \frac{1}{2}(\alpha - \beta)$$

$$\cos \alpha - \cos \beta = -2 \sin \frac{1}{2}(\alpha + \beta) \sin \frac{1}{2}(\alpha - \beta)$$

$$\sin \frac{\alpha}{2} = \pm \sqrt{\frac{1 - \cos \alpha}{2}}$$

positive if $\alpha/2$ in I or II quadrants, negative otherwise

$$\cos \frac{\alpha}{2} = \pm \sqrt{\frac{1 + \cos \alpha}{2}}$$

positive if $\alpha/2$ in I or IV, negative otherwise

$$\tan \frac{\alpha}{2} = \frac{1 - \cos \alpha}{\sin \alpha} = \frac{\sin \alpha}{1 + \cos \alpha} = \pm \sqrt{\frac{1 - \cos \alpha}{1 + \cos \alpha}}$$

positive if $\alpha/2$ in I or III, negative otherwise

$$\sin^2 \alpha = \frac{1}{2}(1 - \cos 2\alpha); \cos^2 \alpha = \frac{1}{2}(1 + \cos 2\alpha)$$

$$\sin^3 \alpha = \frac{3}{4}(3 \sin \alpha - \sin 3\alpha); \cos^3 \alpha = \frac{1}{4}(\cos 3\alpha + 3 \cos \alpha)$$

$$\sin \alpha \sin \beta = \frac{1}{2} \cos (\alpha - \beta) - \frac{1}{2} \cos (\alpha + \beta)$$

$$\cos \alpha \cos \beta = \frac{1}{2} \cos (\alpha - \beta) + \frac{1}{2} \cos (\alpha + \beta)$$

$$\sin \alpha \cos \beta = \frac{1}{2} \sin (\alpha + \beta) + \frac{1}{2} \sin (\alpha - \beta)$$

59. EQUIVALENT EXPRESSIONS FOR $\sin \alpha$, $\cos \alpha$, $\tan \alpha$, etc.

Function	$\sin \alpha$	$\cos \alpha$	$\tan \alpha$	$\cot \alpha$	$\sec \alpha$	$\csc \alpha$
$\sin \alpha$	$\sin \alpha$	$\pm \sqrt{1 - \cos^2 \alpha}$	$\frac{\tan \alpha}{\pm \sqrt{1 + \tan^2 \alpha}}$	$\frac{1}{\pm \sqrt{1 + \cot^2 \alpha}}$	$\frac{\pm \sqrt{\sec^2 \alpha - 1}}{\sec \alpha}$	$\frac{1}{\csc \alpha}$
$\cos \alpha$	$\pm \sqrt{1 - \sin^2 \alpha}$	$\cos \alpha$	$\frac{1}{\pm \sqrt{1 + \tan^2 \alpha}}$	$\frac{\cot \alpha}{\pm \sqrt{1 + \cot^2 \alpha}}$	$\frac{1}{\sec \alpha}$	$\frac{\pm \sqrt{\csc^2 \alpha - 1}}{\csc \alpha}$
$\tan \alpha$	$\frac{\sin \alpha}{\pm \sqrt{1 - \sin^2 \alpha}}$	$\pm \sqrt{1 - \cos^2 \alpha}$	$\tan \alpha$	$\frac{1}{\cot \alpha}$	$\pm \sqrt{\sec^2 \alpha - 1}$	$\frac{1}{\pm \sqrt{\csc^2 \alpha - 1}}$
$\cot \alpha$	$\frac{\pm \sqrt{1 - \sin^2 \alpha}}{\sin \alpha}$	$\frac{\cos \alpha}{\pm \sqrt{1 - \cos^2 \alpha}}$	$\frac{1}{\tan \alpha}$	$\cot \alpha$	$\frac{1}{\pm \sqrt{\sec^2 \alpha - 1}}$	$\pm \sqrt{\csc^2 \alpha - 1}$
$\sec \alpha$	$\frac{1}{\pm \sqrt{1 - \sin^2 \alpha}}$	$\frac{1}{\cos \alpha}$	$\pm \sqrt{1 + \tan^2 \alpha}$	$\frac{\pm \sqrt{1 + \cot^2 \alpha}}{\cot \alpha}$	$\sec \alpha$	$\frac{\csc \alpha}{\pm \sqrt{\csc^2 \alpha - 1}}$
$\csc \alpha$	$\frac{1}{\sin \alpha}$	$\frac{1}{\pm \sqrt{1 - \cos^2 \alpha}}$	$\frac{\pm \sqrt{1 + \tan^2 \alpha}}{\tan \alpha}$	$\pm \sqrt{1 + \cot^2 \alpha}$	$\frac{\sec \alpha}{\pm \sqrt{\sec^2 \alpha - 1}}$	$\csc \alpha$

Note: The quadrant in which α terminates determines the sign to be used. See table of signs in Sec. 54.

60. INVERSE TRIGONOMETRIC FUNCTIONS The complete solution of $x = \sin y$ is (in radians)

$$y = (-1)^n \text{Sin}^{-1} x + n\pi \quad -\frac{\pi}{2} \leq \text{Sin}^{-1} x \leq \frac{\pi}{2}$$

where $\text{Sin}^{-1} x$ is the *principal value* of the angle whose sine is x , n an integer. Similarly, if $x = \tan y$,

$$y = \text{Tan}^{-1} x + n\pi \quad -\frac{\pi}{2} < \text{Tan}^{-1} x < \frac{\pi}{2}$$

The symbol $\sin^{-1} x$ (or arc $\sin x$) is used to denote any angle y whose sine is x ; and $\text{Sin}^{-1} x$ (or $\text{sin}^{-1} x$) is commonly used to designate the *principal value*. Similarly for other inverse relations, as defined below:

Relation	Principal value
$\sin^{-1} x$	$-\frac{\pi}{2} \leq \text{Sin}^{-1} x \leq \frac{\pi}{2}$
$\tan^{-1} x$	$-\frac{\pi}{2} < \text{Tan}^{-1} x < \frac{\pi}{2}$
$\sec^{-1} x$	$0 \leq \text{Sec}^{-1} x < \frac{\pi}{2}$ if $x > 0$ $-\pi \leq \text{Sec}^{-1} x < -\frac{\pi}{2}$ if $x < 0$
$\cosh^{-1} x$	$0 \leq \text{Cosh}^{-1} x$
$\cos^{-1} x$	$0 \leq \text{Cos}^{-1} x \leq \pi$
$\cot^{-1} x$	$0 < \text{Cot}^{-1} x < \pi$
$\csc^{-1} x$	$0 < \text{Csc}^{-1} x \leq \frac{\pi}{2}$ if $x > 0$ $-\pi < \text{Csc}^{-1} x \leq -\frac{\pi}{2}$ if $x < 0$
$\text{sech}^{-1} x$	$0 \leq \text{Sech}^{-1} x$

61. CERTAIN RELATIONS For principal values as in Sec. 60; $a > 0$:

$$\begin{aligned} \sin^{-1} a &= \cos^{-1} \sqrt{1 - a^2} = \tan^{-1} \frac{a}{\sqrt{1 - a^2}} = \cot^{-1} \frac{\sqrt{1 - a^2}}{a} \\ &= \sec^{-1} \frac{1}{\sqrt{1 - a^2}} = \csc^{-1} \frac{1}{a} \end{aligned}$$

$$\begin{aligned} \cos^{-1} a &= \sin^{-1} \sqrt{1 - a^2} = \tan^{-1} \frac{\sqrt{1 - a^2}}{a} = \cot^{-1} \frac{a}{\sqrt{1 - a^2}} \\ &= \sec^{-1} \frac{1}{a} = \csc^{-1} \frac{1}{\sqrt{1 - a^2}} \end{aligned}$$

$$\begin{aligned} \tan^{-1} a &= \sin^{-1} \frac{a}{\sqrt{1 + a^2}} = \cos^{-1} \frac{1}{\sqrt{1 + a^2}} = \cot^{-1} \frac{1}{a} \\ &= \sec^{-1} \sqrt{1 + a^2} = \csc^{-1} \frac{\sqrt{1 + a^2}}{a} \end{aligned}$$

$$\cot^{-1} a = \tan^{-1} \frac{1}{a}; \sec^{-1} a = \cos^{-1} \frac{1}{a}; \csc^{-1} a = \sin^{-1} \frac{1}{a}$$

62. SOLUTION OF TRIGONOMETRIC EQUATIONS To solve a trigonometric equation, reduce the given equation by means of the relations expressed in Secs. 58 to 60 to an equation containing only a single function of a single angle. Solve the resulting equation by algebraic methods (Sec. 30) for the remaining function, and from this find the values of the angle, by Sec. 57 and Table 7. All these values should then be tested in the original equation, discarding those which do not satisfy it.

63. RELATIONS BETWEEN SIDES AND ANGLES OF PLANE TRIANGLES

Let a, b, c denote the sides and α, β, γ the corresponding opposite angles with

- $2s = a + b + c$
- $A = \text{area}$
- $h_b = \text{altitude on side } b$
- $r = \text{radius of inscribed circle}$
- $R = \text{radius of circumscribed circle}$

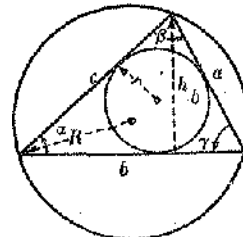


Fig. 10

$$\alpha + \beta + \gamma = 180^\circ$$

$$\frac{a}{\sin \alpha} = \frac{b}{\sin \beta} = \frac{c}{\sin \gamma} \quad (\text{law of sines})$$

$$\frac{a+b}{a-b} = \frac{\tan \frac{1}{2}(\alpha + \beta)}{\tan \frac{1}{2}(\alpha - \beta)} \quad (\text{law of tangents})^*$$

$$a^2 = b^2 + c^2 - 2bc \cos \alpha \quad (\text{law of cosines})^*$$

$$a = b \cos \gamma + c \cos \beta^*$$

$$\cos \alpha = \frac{b^2 + c^2 - a^2}{2bc} \quad \sin \alpha = \frac{2}{bc} \sqrt{s(s-a)(s-b)(s-c)}^*$$

$$\sin \frac{\alpha}{2} = \sqrt{\frac{(s-b)(s-c)}{bc}}^* \quad \cos \frac{\alpha}{2} = \sqrt{\frac{s(s-a)}{bc}}^*$$

$$\tan \frac{\alpha}{2} = \sqrt{\frac{(s-b)(s-c)}{s(s-a)}} = \frac{r}{s-a}^*$$

where $r = \sqrt{\frac{(s-a)(s-b)(s-c)}{s}}$

$$A = \frac{1}{2}bh_b^* = \frac{1}{2}ab \sin \gamma^* = \frac{a^2 \sin \beta \sin \gamma}{2 \sin \alpha} = \sqrt{s(s-a)(s-b)(s-c)} = rs$$

$$R = \frac{a^*}{2 \sin \alpha} = \frac{abc}{4A} \quad h_b = c \sin \alpha^* = a \sin \gamma^* = \frac{2rs}{b}$$

64. SOLUTION OF A RIGHT TRIANGLE Given one side and any acute angle α , or any two sides, to find the remaining parts.

$$a = \sqrt{(c+b)(c-b)} = c \sin \alpha = b \tan \alpha$$

$$b = \sqrt{(c+a)(c-a)} = c \cos \alpha = \frac{a}{\tan \alpha}$$

$$\sin \alpha = \frac{a}{c} \quad \cos \alpha = \frac{b}{c} \quad \tan \alpha = \frac{a}{b} \quad \beta = 90^\circ - \alpha$$

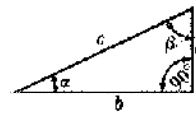


Fig. 11

* Two more formulas may be obtained by replacing a by b , b by c , c by a , α by β , β by γ , γ by α .

$$c = \frac{a}{\sin \alpha} = \frac{b}{\cos \alpha} = \sqrt{a^2 + b^2}$$

$$A = \frac{1}{2}ab = \frac{a^2}{2 \tan \alpha} = \frac{b^2 \tan \alpha}{2} = \frac{c^2 \sin 2\alpha}{4}$$

65. SOLUTION OF OBLIQUE TRIANGLES (Fig. 12) The formulas of the preceding Sec. 63 will suffice to solve any oblique triangle. Use the trigonometric tables for numerical work. We give one method. Solutions should be checked by some other method.

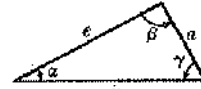


Fig. 12

(a) Given any two sides b and c and included angle α ,

$$\frac{1}{2}(\beta + \gamma) = 90^\circ - \frac{1}{2}\alpha \quad \tan \frac{1}{2}(\beta - \gamma) = \frac{b - c}{b + c} \tan \frac{1}{2}(\beta + \gamma)$$

$$\beta = \frac{1}{2}(\beta + \gamma) + \frac{1}{2}(\beta - \gamma) \quad \gamma = \frac{1}{2}(\beta + \gamma) - \frac{1}{2}(\beta - \gamma)$$

$$a = \frac{b \sin \alpha}{\sin \beta}$$

(b) Given any two angles α and β and any side c ,

$$\gamma = 180^\circ - (\alpha + \beta) \quad a = \frac{c \sin \alpha}{\sin \gamma} \quad b = \frac{c \sin \beta}{\sin \gamma}$$

(c) Given any two sides a and c and an angle opposite one of these, say, α ,

$$\sin \gamma = \frac{c \sin \alpha}{a} \quad \beta = 180^\circ - (\alpha + \gamma) \quad b = \frac{a \sin \beta}{\sin \alpha}$$

This case may have two sets of solutions, for γ may have two values, $\gamma_1 < 90^\circ$ and $\gamma_2 = 180^\circ - \gamma_1 > 90^\circ$. If $\alpha + \gamma_2 > 180^\circ$, use only γ_1 .

(d) Given the three sides a , b , and c ,

$$s = \frac{1}{2}(a + b + c) \quad r = \sqrt{\frac{(s - a)(s - b)(s - c)}{s}}$$

$$\tan \frac{1}{2}\alpha = \frac{r}{s - a} \quad \tan \frac{1}{2}\beta = \frac{r}{s - b} \quad \tan \frac{1}{2}\gamma = \frac{r}{s - c}$$

Spherical Trigonometry

66. THE RIGHT SPHERICAL TRIANGLE

Let O be the center of a sphere and a, b, c the sides of a right triangle with opposite angles α, β , and $\gamma = 90^\circ$, respectively, the sides being measured by the angle subtended at the center of the sphere.

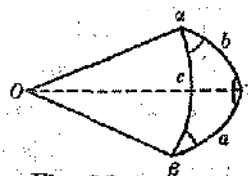


Fig. 13

$\sin a = \sin \alpha \sin c$	$\sin b = \sin \beta \sin c$
$\sin a = \tan b \cot \beta$	$\sin b = \tan a \cot \alpha$
$\cos \alpha = \cos a \sin \beta$	$\cos \beta = \cos b \sin \alpha$
$\cos \alpha = \tan b \cot c$	$\cos \beta = \tan a \cot c$
$\cos c = \cot \alpha \cot \beta$	$\cos c = \cos a \cos b$

67. NAPIER'S RULES OF CIRCULAR PARTS

(see Fig. 14) Let the five quantities $a, b, \text{co-}\alpha$ (complement of α), $\text{co-}c$, $\text{co-}\beta$ be arranged in order as indicated in the figure. If we denote any one of these quantities a *middle* part, then two of the other parts are *adjacent* to it, and the other two parts are *opposite* to it. The above formulas may be remembered by means of the following rules:

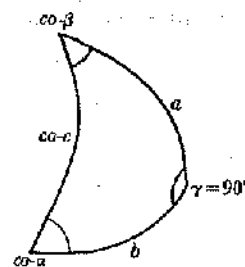


Fig. 14

- (a) The sine of a middle part is equal to the product of the tangents of the adjacent parts.
- (b) The sine of a middle part is equal to the product of the cosines of the opposite parts.

68. THE OBLIQUE SPHERICAL TRIANGLE

Let a, b, c denote the sides and α, β, γ the corresponding opposite angles of the spherical triangle, Δ its area, E its spherical excess, R the radius of the sphere upon which the triangle lies, and $\alpha', \beta', \gamma', a', b', c'$ the corresponding parts of the polar triangle.

$$0^\circ < a + b + c < 360^\circ \qquad 180^\circ < \alpha + \beta + \gamma < 540^\circ$$

$$\alpha = 180^\circ - a' \qquad \beta = 180^\circ - b' \qquad \gamma = 180^\circ - c'$$

$$a = 180^\circ - \alpha' \qquad b = 180^\circ - \beta' \qquad c = 180^\circ - \gamma'$$

$$\frac{\sin \alpha}{\sin a} = \frac{\sin \beta}{\sin b} = \frac{\sin \gamma}{\sin c} \quad (\text{law of sines})$$

$$\cos a = \cos b \cos c + \sin b \sin c \cos \alpha$$

$$\cos \alpha = -\cos \beta \cos \gamma + \sin \beta \sin \gamma \cos a \quad (\text{law of cosines})$$

$$\tan \frac{\alpha}{2} = \sqrt{\frac{\sin(s-b)\sin(s-c)}{\sin s \sin(s-a)}}, \text{ where } s = \frac{1}{2}(a+b+c)$$

$$\tan \frac{a}{2} = \sqrt{\frac{-\cos \sigma \cos(\sigma-\alpha)}{\cos(\sigma-\beta)\cos(\sigma-\gamma)}}, \text{ where } \sigma = \frac{1}{2}(\alpha+\beta+\gamma)$$

$$\frac{\sin \frac{1}{2}(\alpha+\beta)}{\sin \frac{1}{2}(\alpha-\beta)} = \frac{\tan \frac{1}{2}c}{\tan \frac{1}{2}(a-b)} \quad \frac{\cos \frac{1}{2}(\alpha+\beta)}{\cos \frac{1}{2}(\alpha-\beta)} = \frac{\tan \frac{1}{2}c}{\tan \frac{1}{2}(a+b)}$$

$$\frac{\sin \frac{1}{2}(a+b)}{\sin \frac{1}{2}(a-b)} = \frac{\cot \frac{1}{2}\gamma}{\tan \frac{1}{2}(\alpha-\beta)} \quad \frac{\cos \frac{1}{2}(a+b)}{\cos \frac{1}{2}(a-b)} = \frac{\cot \frac{1}{2}\gamma}{\tan \frac{1}{2}(\alpha+\beta)}$$

$$\sin \frac{1}{2}(\alpha+\beta) \cos \frac{1}{2}c = \cos \frac{1}{2}(a-b) \cos \frac{1}{2}\gamma$$

$$\cos \frac{1}{2}(\alpha+\beta) \cos \frac{1}{2}c = \cos \frac{1}{2}(a+b) \sin \frac{1}{2}\gamma$$

$$\sin \frac{1}{2}(\alpha-\beta) \sin \frac{1}{2}c = \sin \frac{1}{2}(a-b) \cos \frac{1}{2}\gamma$$

$$\cos \frac{1}{2}(\alpha-\beta) \sin \frac{1}{2}c = \sin \frac{1}{2}(a+b) \sin \frac{1}{2}\gamma$$

$$\tan \frac{E}{4} = \sqrt{\tan \frac{1}{2}s \tan \frac{1}{2}(s-a) \tan \frac{1}{2}(s-b) \tan \frac{1}{2}(s-c)}$$

$$\Delta = \frac{\pi R^2 E}{180} \quad \alpha + \beta + \gamma - 180^\circ = E$$

*Hyperbolic Functions**

69. DEFINITIONS (for definition of e see Sec. 108)

$$\text{Hyperbolic sine of } x = \sinh x = \frac{1}{2}(e^x - e^{-x})$$

$$\text{Hyperbolic cosine of } x = \cosh x = \frac{1}{2}(e^x + e^{-x})$$

$$\text{Hyperbolic tangent of } x = \tanh x = \frac{e^x - e^{-x}}{e^x + e^{-x}} = \frac{\sinh x}{\cosh x}$$

$$\operatorname{csch} x = \frac{1}{\sinh x} \quad \operatorname{sech} x = \frac{1}{\cosh x} \quad \operatorname{coth} x = \frac{1}{\tanh x}$$

* For derivatives and integrals of hyperbolic functions see Secs. 108 and 121.

70. INVERSE OR ANTIHYPERBOLIC FUNCTIONS (see Sec. 60).

If $x = \sinh y$, then y is the *inverse hyperbolic sine* of x , written $y = \sinh^{-1} x$ or $\text{arc sinh } x$.

$$\begin{aligned} \sinh^{-1} x &= \log_e (x + \sqrt{x^2 + 1})^* \\ \cosh^{-1} x &= \log_e (x + \sqrt{x^2 - 1})^\dagger \\ \tanh^{-1} x &= \frac{1}{2} \log_e \left(\frac{1+x}{1-x} \right) & \coth^{-1} x &= \frac{1}{2} \log_e \left(\frac{x+1}{x-1} \right) \\ \operatorname{sech}^{-1} x &= \log_e \left(\frac{1 + \sqrt{1-x^2}}{x} \right)^\ddagger \\ \operatorname{csch}^{-1} x &= \log_e \left(\frac{1 + \sqrt{1+x^2}}{x} \right)^\ddagger \end{aligned}$$

71. FUNDAMENTAL IDENTITIES

$$\begin{aligned} \sinh(-x) &= -\sinh x & \cosh^2 x - \sinh^2 x &= 1 \\ \cosh(-x) &= \cosh x & \operatorname{sech}^2 x + \tanh^2 x &= 1 \\ \tanh(-x) &= -\tanh x & \operatorname{csch}^2 x - \coth^2 x &= -1 \\ \sinh(x \pm y) &= \sinh x \cosh y \pm \cosh x \sinh y \\ \cosh(x \pm y) &= \cosh x \cosh y \pm \sinh x \sinh y \\ \tanh(x \pm y) &= \frac{\tanh x \pm \tanh y}{1 \pm \tanh x \tanh y} \\ \sinh 2x &= 2 \sinh x \cosh x & \cosh 2x &= \cosh^2 x + \sinh^2 x \\ 2 \sinh^2 \frac{x}{2} &= \cosh x - 1 & 2 \cosh^2 \frac{x}{2} &= \cosh x + 1 \end{aligned}$$

72. CONNECTION WITH CIRCULAR FUNCTIONS

$$\begin{aligned} \sin x &= \frac{e^{ix} - e^{-ix}}{2i} & \cos x &= \frac{e^{ix} + e^{-ix}}{2} & i^2 &= -1 \\ \sin x &= -i \sinh ix & \cos x &= \cosh ix & \tan x &= -i \tanh ix \end{aligned}$$

* If $x \geq 0$. † For principal values. ‡ If $x > 0$.

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$$\begin{array}{lll} \sin ix = i \sinh x & \cos ix = \cosh x & \tan ix = i \tanh x \\ \sinh ix = i \sin x & \cosh ix = \cos x & \tanh ix = i \tan x \end{array}$$

$$\begin{aligned} \sinh(x \pm iy) &= \sinh x \cos y \pm i \cosh x \sin y \\ \cosh(x \pm iy) &= \cosh x \cos y \pm i \sinh x \sin y \end{aligned}$$

$$\sinh(x + 2i\pi) = \sinh x \qquad \cosh(x + 2i\pi) = \cosh x$$

$$\sinh(x + i\pi) = -\sinh x \qquad \cosh(x + i\pi) = -\cosh x$$

$$\sinh(x + \frac{1}{2}i\pi) = i \cosh x \qquad \cosh(x + \frac{1}{2}i\pi) = i \sinh x$$

$$e^\theta = \cosh \theta + \sinh \theta \qquad e^{-\theta} = \cosh \theta - \sinh \theta$$

$$e^{i\theta} = \cos \theta + i \sin \theta \qquad e^{-i\theta} = \cos \theta - i \sin \theta$$

$$x + iy = re^{i\theta}$$

$$\text{where } r = +\sqrt{x^2 + y^2}, \theta = \arccos \frac{x}{r} = \arcsin \frac{y}{r}$$

$$\log_e(x \pm iy) = \frac{1}{2} \log_e(x^2 + y^2) \pm i \arctan \frac{y}{x}$$

$$(\cos \theta + i \sin \theta)^n = \cos n\theta + i \sin n\theta$$

$$\left(\cos \frac{2k\pi}{n} + i \sin \frac{2k\pi}{n} \right)^n = 1 \qquad k = 0, 1, \dots, n-1$$

$$\sinh x + \sinh y = 2 \sinh \frac{x+y}{2} \cosh \frac{x-y}{2}$$

$$\sinh x - \sinh y = 2 \cosh \frac{x+y}{2} \sinh \frac{x-y}{2}$$

$$\cosh x + \cosh y = 2 \cosh \frac{x+y}{2} \cosh \frac{x-y}{2}$$

$$\cosh x - \cosh y = 2 \sinh \frac{x+y}{2} \sinh \frac{x-y}{2}$$

4. ANALYTIC GEOMETRY

73. RECTANGULAR COORDINATES Let $X'X$ (x axis) and $Y'Y$ (y axis) be two perpendicular lines meeting in the point O called the origin. The point $P(x, y)$ in the plane of the x and y axes is fixed by the distances x (abscissa) and y (ordinate) from $Y'Y$ and $X'X$, respectively, to P . x is positive to the right and negative to the left of the y axis, and y is positive above and negative below the x axis.

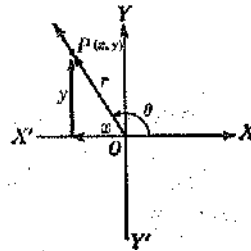


Fig. 15

74. POLAR COORDINATES Let OX (initial line) be a fixed line in the plane and O (pole or origin) a point on this line. The position of any point $P(r, \theta)$ in the plane is determined by the distance r (radius vector) from O to the point together with the angle θ (vectorial angle) measured from OX to OP . θ is positive if measured counterclockwise, negative if measured clockwise; r is positive if measured along the terminal side of θ and negative if measured along the terminal side of θ produced through the pole.

75. RELATIONS BETWEEN RECTANGULAR AND POLAR COORDINATES
(see Sec. 113)

$$\begin{cases} x = r \cos \theta \\ y = r \sin \theta \end{cases} \quad \begin{cases} r = \sqrt{x^2 + y^2} \\ \theta = \tan^{-1} \frac{y}{x} \end{cases} \quad \begin{cases} \sin \theta = \frac{y}{\sqrt{x^2 + y^2}} \\ \cos \theta = \frac{x}{\sqrt{x^2 + y^2}} \end{cases}$$

76. POINTS AND SLOPES Let $P_1(x_1, y_1)$ and $P_2(x_2, y_2)$ be any two points and α_1 be the angle measured counterclockwise from OX to P_1P_2 :

$$\begin{aligned} \text{Distance between } P_1 \text{ and } P_2 &= P_1P_2 \\ &= d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \end{aligned}$$

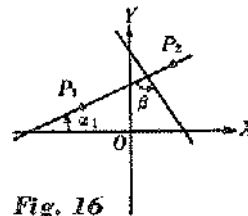


Fig. 16

$$\text{Slope of } P_1P_2 = \tan \alpha_1 = m = \frac{y_2 - y_1}{x_2 - x_1}$$

Point dividing P_1P_2 in ratio $m_1:m_2$

$$= \left(\frac{m_1x_2 + m_2x_1}{m_1 + m_2}, \frac{m_1y_2 + m_2y_1}{m_1 + m_2} \right)$$

$$\text{Midpoint of } P_1P_2 = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$$

The angle β between lines of slopes m_1 and m_2 , respectively, is given by

$$\tan \beta = \frac{m_2 - m_1}{1 + m_1m_2}$$

Two lines of slopes m_1 and m_2 are perpendicular if $m_2 = -(1/m_1)$ and parallel if $m_1 = m_2$.

77. AREA OF TRIANGLE If the vertices are the points (x_1, y_1) , (x_2, y_2) , (x_3, y_3) , then the area is equal to the numerical value of

$$\frac{1}{2} \begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} = \frac{1}{2}(x_1y_2 - x_1y_3 + x_2y_3 - x_2y_1 + x_3y_1 - x_3y_2)$$

78. LOCUS AND EQUATION The set of all points which satisfy a given condition is called the *locus* of that condition. An *equation* is called the equation of the locus if it is satisfied by the coordinates of every point on the locus and by no other points. There are three common representations of the locus by means of equations:

(a) *Rectangular equation*, which involves the rectangular coordinates (x, y)

(b) *Polar equation*, which involves the polar coordinates (r, θ)

(c) *Parametric equations*, which express x and y or r and θ in terms of a third independent variable called a parameter

79. TRANSFORMATION OF COORDINATES To transform an equation or a curve from one system of coordinates in x, y to another such system in x', y' , substitute for each variable its value in terms of variables of the new system.

(a) *Rectangular system: old axes parallel to new axes.*
 The coordinates of new origin in terms of old system are (h, k) .

$$\begin{cases} x = x' + h \\ y = y' + k \end{cases}$$

(b) *Rectangular system: old origin coincident with new origin and the x' axis making an angle θ with the x axis*

$$\begin{cases} x = x' \cos \theta - y' \sin \theta \\ y = x' \sin \theta + y' \cos \theta \end{cases}$$

(c) *Rectangular system: old axes not parallel with new. New origin at (h, k) in old system*

$$\begin{cases} x = x' \cos \theta - y' \sin \theta + h \\ y = x' \sin \theta + y' \cos \theta + k \end{cases}$$

30. STRAIGHT LINE The equations of the straight line may assume the following forms:

(a) $y = mx + b$ ($m =$ slope, $b =$ intercept on y axis)

(b) $y - y_1 = m(x - x_1)$ [$m =$ slope, line passes through point (x_1, y_1)]

(c) $\frac{y - y_1}{x - x_1} = \frac{y_2 - y_1}{x_2 - x_1}$ [line passes through points (x_1, y_1) and (x_2, y_2)]

(d) $\frac{x}{a} + \frac{y}{b} = 1$ (a and b are the intercepts on x and y axes, respectively)

(e) $x \cos \alpha + y \sin \alpha = p$ (*normal form*, p is distance from origin to the line, α is angle which normal to the line makes with x axis)

(f) $Ax + By + C = 0$ [*general form*, slope = $-(A/B)$]

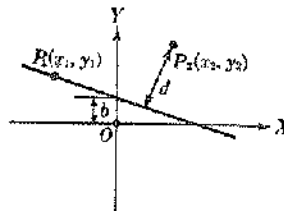


Fig. 17

To reduce $Ax + By + C = 0$ to normal form (e), divide by $\pm\sqrt{A^2 + B^2}$, where the sign of the radical is taken opposite to that of C when $C \neq 0$.

The distance from the line $Ax + By + C = 0$ to the point $P_2(x_2, y_2)$ is

$$d = \frac{Ax_2 + By_2 + C}{\pm\sqrt{A^2 + B^2}}$$

81. CIRCLE (see Sec. 37) General equation of circle with radius R and center at (h, k) is

$$(x - h)^2 + (y - k)^2 = R^2$$

82. CONIC The locus of a point P which moves so that its distance from a fixed point F (focus) bears a constant ratio e (eccentricity) to its distance from a fixed straight line (directrix) is a conic.

If d is the distance from focus to directrix and F is at the origin,

$$x^2 + y^2 = e^2(d + x)^2$$

$$r = \frac{de}{1 - e \cos \theta}$$

If $e = 1$, the conic is a *parabola*; if $e > 1$, a *hyperbola*; and if $e < 1$, an *ellipse*.

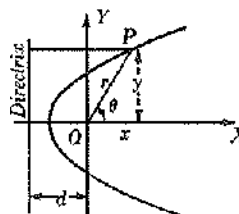


Fig. 18

83. PARABOLA (see Sec. 39) $e = 1$

(a) $(y - k)^2 = 4a(x - h)$. Vertex at (h, k) , axis $\parallel OX$ (Fig. 19). Figure 19 is drawn for the case when a is positive.

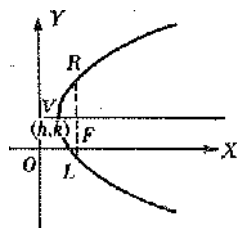


Fig. 19

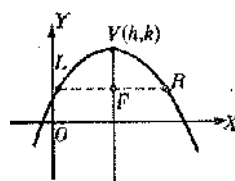


Fig. 20

(b) $(x - h)^2 = 4a(y - k)$. Vertex at (h, k) , axis $\parallel OY$ (Fig. 20).
 Figure 20 is drawn for the case when a is negative.

Distance from vertex to focus $= VF = a$
 Distance from vertex to directrix $= a$
 Latus rectum $= LR = 4a$

84. ELLIPSE (see Sec. 38) $e < 1$

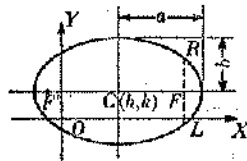


Fig. 21

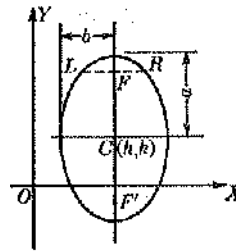


Fig. 22

(a) $\frac{(x - h)^2}{a^2} + \frac{(y - k)^2}{b^2} = 1$. Center at (h, k) , major axis $\parallel OX$ (Fig. 21).

(b) $\frac{(y - k)^2}{a^2} + \frac{(x - h)^2}{b^2} = 1$. Center at (h, k) , major axis $\parallel OY$ (Fig. 22).

Major axis $= 2a$ Minor axis $= 2b$
 Distance from center to either focus $= \sqrt{a^2 - b^2}$
 Distance from center to either directrix $= a/e$
 Eccentricity $= e = \sqrt{a^2 - b^2}/a$
 Latus rectum $= 2b^2/a$
 Sum of distances from any point P on ellipse to foci, $PF' + PF = 2a$

85. HYPERBOLA $e > 1$

(a) $\frac{(x - h)^2}{a^2} - \frac{(y - k)^2}{b^2} = 1$.

Center at (h, k) , transverse axis $\parallel OX$.
 Slopes of asymptotes $= \pm b/a$ (Fig. 23).

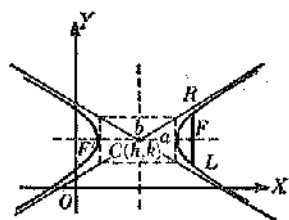


Fig. 23

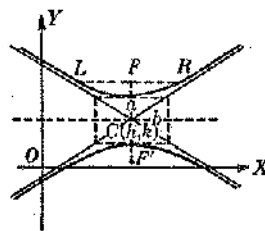


Fig. 24

$$(b) \frac{(y - k)^2}{a^2} - \frac{(x - h)^2}{b^2} = 1.$$

Center at (h, k) , transverse axis $\parallel OY$.
Slopes of asymptotes $= \pm a/b$ (Fig. 24).

Transverse axis $= 2a$

Conjugate axis $= 2b$

Distance from center to either focus $= \sqrt{a^2 + b^2}$

Distance from center to either directrix $= a/e$

Difference of distances of any point on hyperbola from the foci $= 2a$

Eccentricity $= e = \frac{\sqrt{a^2 + b^2}}{a}$

Latus rectum $= 2b^2/a$

86. SINE CURVE

$$y = a \sin (bx + c)$$

$$y = a \cos (bx + c')$$

$$= a \sin (bx + c), \text{ where } c = c' + \frac{\pi}{2}$$

$$y = p \sin bx + q \cos bx = a \sin (bx + c)$$

$$\text{where } c = \tan^{-1} \frac{q}{p}, a = \sqrt{p^2 + q^2}$$

a = amplitude = maximum height of wave

$\frac{2\pi}{b}$ = wavelength = distance from any point on wave to the corresponding point on the next wave

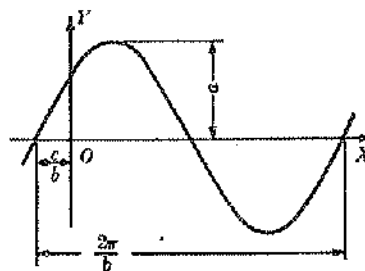


Fig. 25

$x = -\frac{c}{b}$ = phase, indicates a point on x axis from which the positive half of the wave starts

87. TRIGONOMETRIC CURVES

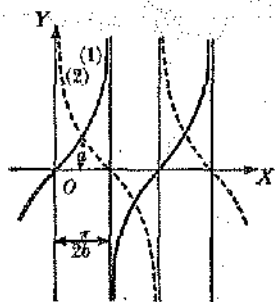


Fig. 26

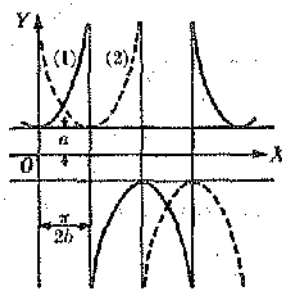


Fig. 27

In Fig. 26,

(1) $y = a \tan bx$ or $x = \frac{1}{b} \tan^{-1} \frac{y}{a}$

(2) $y = a \cot bx$ or $x = \frac{1}{b} \cot^{-1} \frac{y}{a}$

In Fig. 27,

(1) $y = a \sec bx$ or $x = \frac{1}{b} \sec^{-1} \frac{y}{a}$

(2) $y = a \csc bx$ or $x = \frac{1}{b} \csc^{-1} \frac{y}{a}$

88. LOGARITHMIC AND EXPONENTIAL CURVES

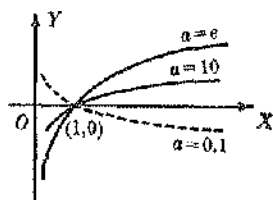


Fig. 28. Logarithmic curve.
 $y = \log_a x$ or $x = a^y$

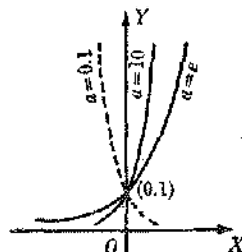


Fig. 29. Exponential curve.
 $y = a^x$ or $x = \log_a y$

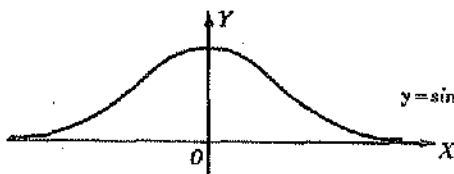


Fig. 30

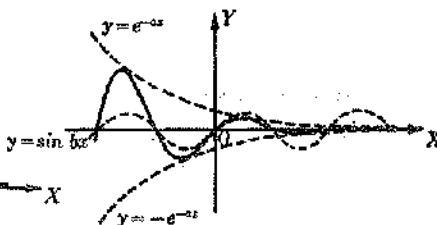


Fig. 31

89. PROBABILITY CURVE (Fig. 30) $y = e^{-x^2}$

90. OSCILLATORY WAVE OF DECREASING AMPLITUDE (Fig. 31)

$$y = e^{-ax} \sin bx$$

91. CATENARY (Fig. 32)

$$y = \frac{a}{2} (e^{x/a} + e^{-x/a})$$

A curve made by a cord of uniform weight suspended freely between two points. (See Sec. 69.)

$$\text{Length of arc} = s = l \left[1 + \frac{2}{3} \left(\frac{2d}{l} \right)^2 \right]$$

approximately, if d is small in comparison with l .

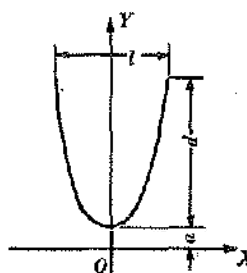


Fig. 32

92. CYCLOID (Fig. 33)

$$\begin{cases} x = a(\phi - \sin \phi) \\ y = a(1 - \cos \phi) \end{cases}$$

A curve described by a point on the circumference of circle which rolls along a fixed straight line.

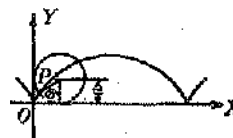


Fig. 33

$$\begin{aligned} \text{Area one arch} &= 3\pi a^2 \\ \text{Length of arc of one arch} &= 8a \end{aligned}$$

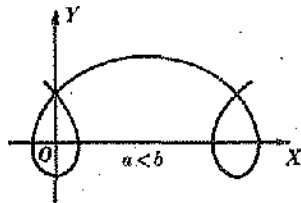


Fig. 34. Prolate cycloid.

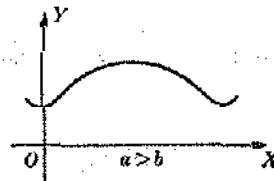


Fig. 35. Curtate cycloid.

93. PROLATE AND CURTATE CYCLOID

$$\begin{cases} x = a\phi - b \sin \phi \\ y = a - b \cos \phi \end{cases}$$

A curve described by a point on a circle at a distance b from the center of the circle of radius a which rolls along a fixed straight line.

94. EPICYCLOID (Fig. 36)

$$\begin{cases} x = (a + b) \cos \phi - a \cos \left(\frac{a + b}{a} \phi \right) \\ y = (a + b) \sin \phi - a \sin \left(\frac{a + b}{a} \phi \right) \end{cases}$$

A curve described by a point on the circumference of a circle which rolls along the outside of a fixed circle.

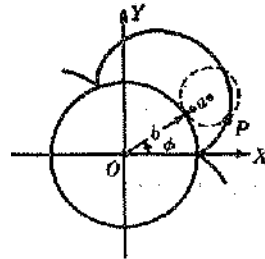


Fig. 36

95. CARDIOID (Fig. 37)

$$r = a(1 + \cos \theta)$$

An epicycloid in which both circles have the same radii is called a cardioid.

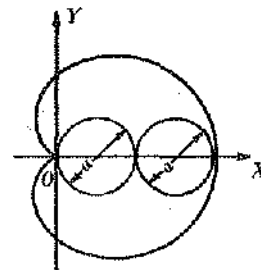


Fig. 37

96. HYPOCYCLOID

$$\begin{cases} x = (a - b) \cos \phi + b \cos \left(\frac{a - b}{b} \phi \right) \\ y = (a - b) \sin \phi - b \sin \left(\frac{a - b}{b} \phi \right) \end{cases}$$

A curve described by a point on the circumference of a circle which rolls along the inside of a fixed circle.

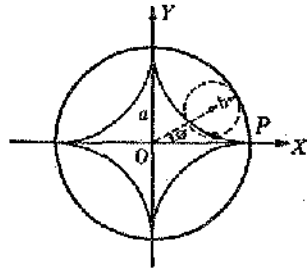


Fig. 38

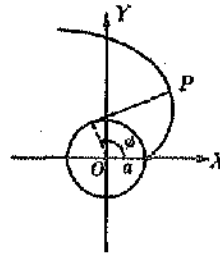


Fig. 39

97. HYPOCYCLOID OF FOUR CUSPS (Fig. 38)

$$x^{2/3} + y^{2/3} = a^{2/3}$$

$$x = a \cos^3 \phi \qquad y = a \sin^3 \phi$$

The radius of fixed circle is four times the radius of rolling circle.

98. INVOLUTE OF THE CIRCLE (Fig. 39)

$$\begin{cases} x = a \cos \phi + a \phi \sin \phi \\ y = a \sin \phi - a \phi \cos \phi \end{cases}$$

A curve generated by the end of a string which is kept taut while being unwound from a circle.

99. LEMNISCATE (Fig. 40)

$$r^2 = 2a^2 \cos 2\theta$$

100. n-LEAVED ROSE (Fig. 41)

- (1) $r = a \sin n\theta$
- (2) $r = a \cos n\theta$

If n is an odd integer, there are n leaves; if n is even, $2n$ leaves, of which the figure shows one.

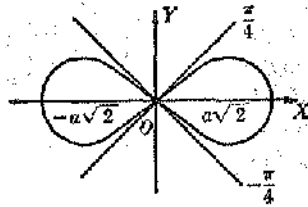


Fig. 40

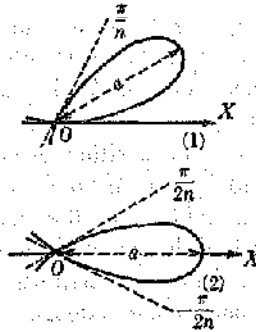


Fig. 41

101. SPIRALS

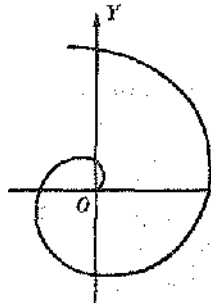


Fig. 42.
Archimedian.
 $r = a\theta$

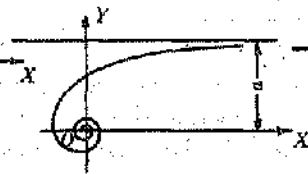


Fig. 43.
Hyperbolic.
 $r\theta = a$

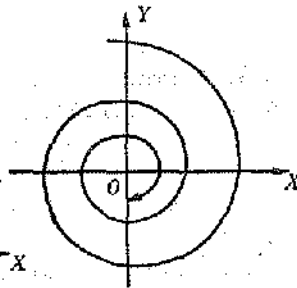


Fig. 44.
Logarithmic.
 $r = e^{a\theta}$ or $a\theta = \log_e r$

Solid Analytic Geometry

102. COORDINATES (Fig. 45)

(a) *Rectangular system.* The position of a point $P(x, y, z)$ in space is fixed by its three distances x , y , and z from three coordinate planes XOY , XOZ , ZOY , which are mutually perpendicular and meet in a point O (origin).

(b) *Cylindrical system.* The position of any point $P(r, \theta, z)$ is fixed

by (r, θ) , the polar coordinates of the projection of P in the XOY plane, and by z , its distance from the XOY plane.

(c) *Spherical (or polar or geographical) system.* The position of any point $P(\rho, \theta, \phi)$ is fixed by the distance $\rho = \overline{OP}$, the angle $\theta = \angle XOM$, and the angle $\phi = \angle ZOP$. θ is called the *longitude* and ϕ the *colatitude*.

The following relations exist between the three coordinate systems:

$$\begin{cases} x = \rho \sin \phi \cos \theta \\ y = \rho \sin \phi \sin \theta \\ z = \rho \cos \phi \end{cases} \quad \begin{cases} r = \rho \sin \phi \\ z = \rho \cos \phi \end{cases} \quad \begin{cases} \rho = \sqrt{r^2 + z^2} \\ \cos \phi = \frac{z}{\sqrt{r^2 + z^2}} \end{cases}$$

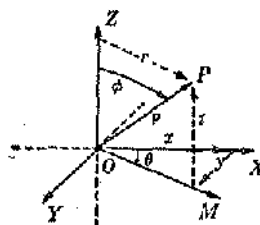


Fig. 45

103. POINTS, LINES, AND PLANES Distance between two points $P_1(x_1, y_1, z_1)$ and $P_2(x_2, y_2, z_2)$ is

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$$

Direction cosines of a line are the cosines of the angles α, β, γ which the line or any parallel line makes with the coordinate axes.

The direction cosines of the line segment $P_1(x_1, y_1, z_1)$ to $P_2(x_2, y_2, z_2)$ are

$$\cos \alpha = \frac{x_2 - x_1}{d}, \quad \cos \beta = \frac{y_2 - y_1}{d}, \quad \cos \gamma = \frac{z_2 - z_1}{d}$$

If $\cos \alpha : \cos \beta : \cos \gamma = a : b : c$, then

$$\cos \alpha = \frac{a}{\sqrt{a^2 + b^2 + c^2}}, \quad \cos \beta = \frac{b}{\sqrt{a^2 + b^2 + c^2}}, \quad \cos \gamma = \frac{c}{\sqrt{a^2 + b^2 + c^2}}$$

$$\cos^2 \alpha + \cos^2 \beta + \cos^2 \gamma = 1$$

Angle θ between two lines, whose direction angles are $\alpha_1, \beta_1, \gamma_1$ and $\alpha_2, \beta_2, \gamma_2$, is given by

$$\cos \theta = \cos \alpha_1 \cos \alpha_2 + \cos \beta_1 \cos \beta_2 + \cos \gamma_1 \cos \gamma_2$$

Equation of a plane is

$$Ax + By + Cz + D = 0$$

where A, B, C are proportional to the direction cosines of a normal (a line perpendicular to the plane) to the plane.

Angle between two planes is the angle between their normals.

Equation of a straight line through the point $P_1(x_1, y_1, z_1)$ is

$$\frac{x - x_1}{a} = \frac{y - y_1}{b} = \frac{z - z_1}{c}$$

where a, b, c are proportional to the direction cosines of the line. a, b, c are called *direction numbers* of the line.

104. FIGURES IN THREE DIMENSIONS

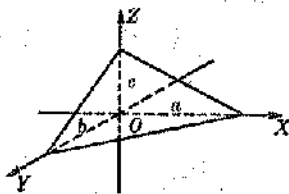


Fig. 46. Plane.*

$$\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$$

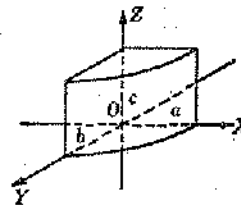


Fig. 47. Elliptic cylinder.*

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

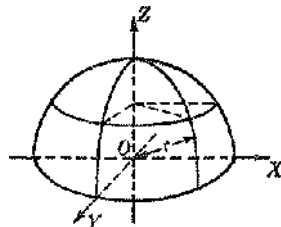


Fig. 48. Sphere.*

$$x^2 + y^2 + z^2 = r^2$$

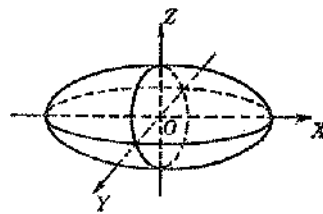


Fig. 49. Ellipsoid.*

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$$

* Only a portion of the figure is shown.

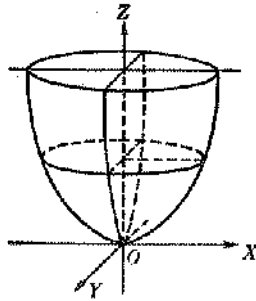


Fig. 50.
Elliptic paraboloid.

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = cz$$

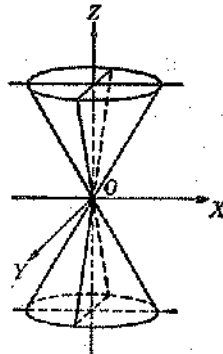


Fig. 51.
Portion of cone.

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{z^2}{c^2} = 0$$

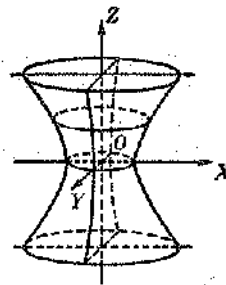


Fig. 52.
Hyperboloid of one sheet.

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{z^2}{c^2} = 1$$

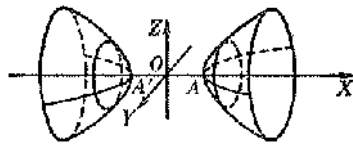


Fig. 53.
Hyperboloid of two sheets.

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} - \frac{z^2}{c^2} = 1$$

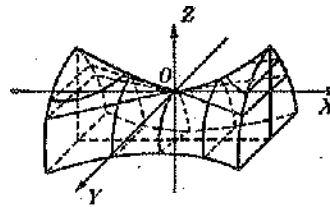


Fig. 54.
Hyperbolic paraboloid.

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = cz$$

5. DIFFERENTIAL CALCULUS

105. DEFINITIONS The symbols $f(x)$, $F(x)$, $g(x)$, $\phi(x)$, ... are used to denote the values of the (real) functions f , F , g , ϕ , ..., respectively, at x . $f(a)$ and $f(a + h)$ represent the values of $f(x)$ at $x = a$ and $x = a + h$, respectively.

For definitions of function, limit, and continuous function, see Chap. 9.

106. DERIVATIVE Let $f(x)$ be the value of f at x . Consider the relation $f'(a)$ defined by

$$f'(a) = \lim_{h \rightarrow 0} \frac{f(a + h) - f(a)}{h}$$

The value of $f'(a)$, if it exists, may depend on how $h \rightarrow 0$. If $f'(a)$ exists as a unique finite limit, independent of how $h \rightarrow 0$, then $f(x)$ is said to be *differentiable* at $x = a$, and $f'(a)$ is called the *derivative of f with respect to x at $x = a$* . The phrase *the derivative of f* is also used to mean the function f' whose value at x is the number $f'(x)$.

The derivative of $y = f(x)$ at x is written in various ways, e.g.,

$$\frac{dy}{dx} = \lim_{\Delta x \rightarrow 0} \frac{\Delta y}{\Delta x} = \lim_{\Delta x \rightarrow 0} \frac{f(x + \Delta x) - f(x)}{\Delta x} = f'(x) = D_x y = y'$$

where $\Delta x = h$ is called an *increment* in x and $\Delta y = k$ an *increment* in y .

Higher derivatives are defined as follows:

$$\frac{d^2 y}{dx^2} = \frac{d}{dx} \left(\frac{dy}{dx} \right) = \frac{d}{dx} f'(x) = f''(x) \quad (2\text{d derivative}) \quad \text{e)}$$

$$\frac{d^3 y}{dx^3} = \frac{d}{dx} \left(\frac{d^2 y}{dx^2} \right) = \frac{d}{dx} f''(x) = f'''(x) \quad (3\text{d derivative}) \quad \text{e)}$$

$$\frac{d^n y}{dx^n} = \frac{d}{dx} \left(\frac{d^{n-1} y}{dx^{n-1}} \right) = \frac{d}{dx} f^{(n-1)}(x) = f^{(n)}(x) \quad (n\text{th derivative})$$

The symbol $f^{(n)}(a)$ represents the value of $f^{(n)}(x)$ when $x = a$.

107. CERTAIN RELATIONS AMONG DERIVATIVES

If $x = f(y)$, then

$$\frac{dy}{dx} = \frac{1}{dx/dy}$$

If $y = f(u)$ and $u = F(x)$, then

$$\frac{dy}{dx} = \frac{dy}{du} \frac{du}{dx}$$

If $x = f(\alpha)$ and $y = \phi(\alpha)$, then

$$\frac{dy}{dx} = \frac{\phi'(\alpha)}{f'(\alpha)} \quad \frac{d^2y}{dx^2} = \frac{f'(\alpha)\phi''(\alpha) - \phi'(\alpha)f''(\alpha)}{[f'(\alpha)]^3}$$

108. TABLE OF DERIVATIVES In this table, u and v represent functions of x ; a , n , e represent constants ($e = 2.7183\dots$); and all angles are measured in radians.

$$\frac{d}{dx} x = 1 \quad \frac{d}{dx} a = 0$$

$$\frac{d}{dx} (u \pm v \pm \dots) = \frac{du}{dx} \pm \frac{dv}{dx} \pm \dots$$

$$\frac{d}{dx} (au) = a \frac{du}{dx}$$

$$\frac{d}{dx} (uv) = u \frac{dv}{dx} + v \frac{du}{dx}$$

$$\frac{d}{dx} \left(\frac{u}{v} \right) = \frac{v \frac{du}{dx} - u \frac{dv}{dx}}{v^2}$$

$$\frac{d}{dx} \sin u = \cos u \frac{du}{dx}$$

$$\frac{d}{dx} u^n = nu^{n-1} \frac{du}{dx}$$

$$\frac{d}{dx} \cos u = -\sin u \frac{du}{dx}$$

$$\frac{d}{dx} \log_a u = \frac{\log_a e}{u} \frac{du}{dx}$$

$$\frac{d}{dx} \tan u = \sec^2 u \frac{du}{dx}$$

$$\frac{d}{dx} \log_e u = \frac{1}{u} \frac{du}{dx}$$

$$\frac{d}{dx} \cot u = -\csc^2 u \frac{du}{dx}$$

$$\frac{d}{dx} a^u = a^u \log_e a \frac{du}{dx}$$

$$\frac{d}{dx} \sec u = \sec u \tan u \frac{du}{dx}$$

$$\frac{d}{dx} e^u = e^u \frac{du}{dx} \qquad \frac{d}{dx} \csc u = -\csc u \cot u \frac{du}{dx}$$

$$\frac{d}{dx} u^v = vu^{v-1} \frac{du}{dx} + u^v \log_e u \frac{dv}{dx} \qquad \frac{d}{dx} \text{vers } u = \sin u \frac{du}{dx}$$

$$\lim_{x \rightarrow 0} \frac{\sin x}{x} = 1, \quad \lim_{x \rightarrow 0} (1+x)^{1/x} = e = 2.71828 \dots$$

$$= 1 + 1 + \frac{1}{2!} + \frac{1}{3!} + \dots$$

$$\frac{d}{dx} \sin^{-1} u = \frac{1}{\sqrt{1-u^2}} \frac{du}{dx} \qquad -\frac{\pi}{2} \leq \sin^{-1} u \leq \frac{\pi}{2}$$

$$\frac{d}{dx} \cos^{-1} u = -\frac{1}{\sqrt{1-u^2}} \frac{du}{dx} \qquad 0 \leq \cos^{-1} u \leq \pi$$

$$\frac{d}{dx} \tan^{-1} u = \frac{1}{1+u^2} \frac{du}{dx} \qquad \frac{d}{dx} \cot^{-1} u = -\frac{1}{1+u^2} \frac{du}{dx}$$

$$\frac{d}{dx} \sec^{-1} u = \frac{1}{u\sqrt{u^2-1}} \frac{du}{dx}, \quad -\pi \leq \sec^{-1} u < \frac{\pi}{2},$$

$$0 \leq \sec^{-1} u < \frac{\pi}{2}$$

$$\frac{d}{dx} \csc^{-1} u = -\frac{1}{u\sqrt{u^2-1}} \frac{du}{dx}, \quad -\pi < \csc^{-1} u \leq -\frac{\pi}{2},$$

$$0 < \csc^{-1} u \leq \frac{\pi}{2}$$

$$\frac{d}{dx} \text{vers}^{-1} u = \frac{1}{\sqrt{2u-u^2}} \frac{du}{dx} \qquad 0 \leq \text{vers}^{-1} u \leq \pi$$

$$\frac{d}{dx} \sinh u = \cosh u \frac{du}{dx} \qquad \frac{d}{dx} \cosh u = \sinh u \frac{du}{dx}$$

$$\frac{d}{dx} \tanh u = \text{sech}^2 u \frac{du}{dx} \qquad \frac{d}{dx} \coth u = -\text{csch}^2 u \frac{du}{dx}$$

$$\frac{d}{dx} \text{sech } u = -\text{sech } u \tanh u \frac{du}{dx} \qquad \frac{d}{dx} \text{csch } u$$

$$= -\text{csch } u \coth u \frac{du}{dx}$$

$$\begin{aligned} \frac{d}{dx} \sinh^{-1} u &= \frac{1}{\sqrt{u^2 + 1}} \frac{du}{dx} & \frac{d}{dx} \cosh^{-1} u &= \frac{1}{\sqrt{u^2 - 1}} \frac{du}{dx}, \quad u > 1 \\ \frac{d}{dx} \tanh^{-1} u &= \frac{1}{1 - u^2} \frac{du}{dx} & \frac{d}{dx} \coth^{-1} u &= -\frac{1}{u^2 - 1} \frac{du}{dx} \\ \frac{d}{dx} \operatorname{sech}^{-1} u &= -\frac{1}{u\sqrt{1 - u^2}} \frac{du}{dx} \dagger & \frac{d}{dx} \operatorname{csch}^{-1} u &= -\frac{1}{u\sqrt{u^2 + 1}} \frac{du}{dx}, \quad u > 0 \end{aligned}$$

109. SLOPE OF A CURVE. EQUATION OF TANGENT AND NORMAL (RECTANGULAR COORDINATES)

The slope of the curve $y = f(x)$ at the point $P(x, y)$ is defined as the slope of the tangent line to the curve at P .

$$\text{Slope} = m = \tan \alpha = \frac{dy}{dx} = f'(x)$$

$$\text{Slope at } x = x_1 \text{ is } m_1 = f'(x_1)$$

Equation of *tangent line* to curve at $P_1(x_1, y_1)$ is

$$y - y_1 = m_1(x - x_1)$$

Equation of *normal line* to curve at $P_1(x_1, y_1)$ is

$$y - y_1 = -\frac{1}{m_1}(x - x_1)$$

Angle (θ) of intersection of two curves whose slopes are m_1 and m_2 at the common point is given by

$$\tan \theta = \frac{m_2 - m_1}{1 + m_1 m_2}$$

The sign of $\tan \theta$ determines whether the acute or obtuse angle is meant.

$$* \cosh^{-1} u > 0. \quad \dagger \operatorname{sech}^{-1} u > 0.$$

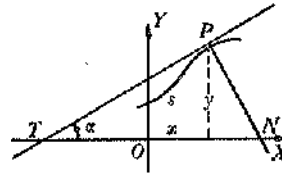


Fig. 55

110. DIFFERENTIAL Let $y = f(x)$. $f'(x_0)$ is the derivative of f with respect to x at x_0 . The *differential of x* , written dx , is the increment of x , that is, $dx = \Delta x$; the *differential of f at x_0* , written df , is the product $df = f'(x_0) \Delta x$. The differential of f at x is written in various ways, e.g.,

$$dy = df = f'(x) dx = \frac{df(x)}{dx} dx = \frac{dy}{dx} dx \quad \text{where } \frac{dy}{dx} = f'(x)$$

If $x = f(t)$, $y = \phi(t)$, then $dx = f'(t) dt$ and $dy = \phi'(t) dt$.

Every derivative formula has a corresponding differential formula. For example, from the table in Sec. 108,

$$d(\sin u) = \cos u du \quad d(uv) = u dv + v du$$

111. MAXIMUM AND MINIMUM VALUES OF A FUNCTION A *maximum (minimum)* value of a function $f(x)$ in the interval (a, b) is a value of the function which is greater (less) than the values of the function in the immediate vicinity.

The values of x which give a maximum or minimum value to $y = f(x)$ are found by solving the equations $f'(x) = 0$ or ∞ . If a is a root of $f'(x) = 0$ and if $f''(a) < 0$, $f(a)$ is a maximum; if $f''(a) > 0$, $f(a)$ is a minimum. If $f''(a) = 0$, $f'''(a) \neq 0$, $f(a)$ is neither a maximum nor a minimum, but if $f''(a) = f'''(a) = 0$, $f(a)$ is maximum or minimum according as $f^4(a) \lessgtr 0$.*

In general, if the first derivative which does not vanish for $x = a$ is of an odd order, $f(a)$ is neither a maximum nor a minimum; but if it is of an even order, the $2n$ th, say, then $f(a)$ is a maximum or minimum according as $f^{(2n)}(a) \lessgtr 0$.*

To find the largest or smallest values of a function in an interval (a, b) , find $f(a)$, $f(b)$ and compare with the maximum and minimum values as found in the interval.

112. POINTS OF INFLECTION OF A CURVE.* The curve is said to have a *point of inflection* at $x = a$ if $f''(a) = 0$ and $f''(x) < 0$ on one side of $x = a$ and $f''(x) > 0$ on the other side of $x = a$. Wherever $f''(x) < 0$, the curve is *concave downward*, and wherever $f''(x) > 0$, the curve is *concave upward*.

* Here it is assumed that $f'(x)$, $f''(x)$, . . . exist.

113. DERIVATIVE OF ARC LENGTH. RADIUS OF CURVATURE (see Secs. 75, 107, 126) Let s be the length of arc measured along the curve $y = f(x)$ [or in polar coordinates $r = \phi(\theta)$] from some fixed point to any point $P(x, y)$ and α be the angle of inclination of the tangent line at P with OX . Then

$$\frac{dx}{ds} = \cos \alpha = \frac{1}{\sqrt{1 + \left(\frac{dy}{dx}\right)^2}}, \quad \frac{dy}{ds} = \sin \alpha = \frac{1}{\sqrt{1 + \left(\frac{dx}{dy}\right)^2}}$$

$$ds = \sqrt{dx^2 + dy^2} = \sqrt{1 + \left(\frac{dy}{dx}\right)^2} dx = \sqrt{1 + \left(\frac{dx}{dy}\right)^2} dy$$

$$ds = \sqrt{dr^2 + r^2 d\theta^2} = \sqrt{r^2 + \left(\frac{dr}{d\theta}\right)^2} d\theta = \sqrt{1 + r^2 \left(\frac{d\theta}{dr}\right)^2} dr$$

$$\text{If } \quad x = r \cos \theta \quad \text{and} \quad y = r \sin \theta \\ dx = \cos \theta dr - r \sin \theta d\theta, \quad dy = \sin \theta dr + r \cos \theta d\theta$$

The *radius of curvature* R at any point $P(x, y)$ of the curve $y = f(x)$ is

$$R = \frac{ds}{d\alpha} = \frac{\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{3/2}}{\frac{d^2y}{dx^2}} = \frac{\{1 + [f'(x)]^2\}^{3/2}}{f''(x)} \\ = \frac{\left[r^2 + \left(\frac{dr}{d\theta}\right)^2\right]^{3/2}}{r^2 + 2\left(\frac{dr}{d\theta}\right)^2 - r \frac{d^2r}{d\theta^2}}$$

The *curvature* (K) at (x, y) is $K = 1/R$.

The *center of curvature* corresponding to the point (x_1, y_1) on $y = f(x)$ is (h, k) where

$$h = x_1 - \frac{f'(x_1)\{1 + [f'(x_1)]^2\}}{f''(x_1)} \\ k = y_1 + \frac{1 + [f'(x_1)]^2}{f''(x_1)}$$

114. THEOREM OF THE MEAN. ROLLE'S THEOREM If $y = f(x)$ and its derivative $f'(x)$ are continuous on the interval (a, b) , there exists a value of x somewhere between a and b such that

$$f(b) = f(a) + (b - a)f'(x_1) \quad a < x_1 < b$$

Rolle's theorem is a special case of the theorem of the mean with $f(a) = f(b) = 0$; that is, there exists at least one value of x between a and b for which $f'(x_1) = 0$.

115. EVALUATION OF INDETERMINATE FORMS If $f(x)$ and $F(x)$ are two continuous functions of x having continuous derivatives, $f'(x)$ and $F'(x)$, then

(a) If $\lim_{x \rightarrow a} f(x) = 0$, $\lim_{x \rightarrow a} F(x) = 0$, and $\lim_{x \rightarrow a} F'(x) \neq 0$ [or if $\lim_{x \rightarrow a} f(x) = \lim_{x \rightarrow a} F(x) = \infty$], then

$$\lim_{x \rightarrow a} \frac{f(x)}{F(x)} = \lim_{x \rightarrow a} \frac{f'(x)}{F'(x)}$$

(b) If $\lim_{x \rightarrow a} f(x) = 0$ and $\lim_{x \rightarrow a} F(x) = \infty$ [that is, $f(x)$ becomes infinite as x approaches a as a limit], then $\lim_{x \rightarrow a} [f(x)F(x)]$ may often be determined by writing

$$f(x)F(x) = \frac{f(x)}{1/F(x)}$$

thus expressing the functions in the form of (a).

(c) If $\lim_{x \rightarrow a} f(x) = \infty$ and $\lim_{x \rightarrow a} F(x) = \infty$, then $\lim_{x \rightarrow a} [f(x) - F(x)]$ may often be determined by writing

$$f(x) - F(x) = \frac{[1/F(x)] - [1/f(x)]}{1/[f(x)F(x)]}$$

and using (a).

(d) The $\lim_{x \rightarrow a} [f(x)^{F(x)}]$ may frequently be evaluated upon writing

$$f(x)^{F(x)} = e^{F(x) \log_e f(x)}$$

When one factor of the last exponent approaches zero and the other becomes infinite, the exponent is of the type considered in (b).

Thus we are led to the indeterminate forms which are symbolized by $0/0$, ∞/∞ , 0^0 , 1^∞ , ∞^0 .

116. TAYLOR'S AND MACLAUREN'S THEOREMS Any function (continuous and having derivatives) may, in general, be expanded into a *Taylor's series*:

$$f(x) = f(a) + f'(a) \frac{x-a}{1!} + f''(a) \frac{(x-a)^2}{2!} \\ + f'''(a) \frac{(x-a)^3}{3!} + \dots + f^{(n-1)}(a) \frac{(x-a)^{n-1}}{(n-1)!} + R_n$$

where a is any quantity for which $f(a), f'(a), f''(a), \dots$ are finite.

If the series is to be used for approximating $f(x)$ for some value of x , then a should be picked so that the difference $(x-a)$ is numerically very small, and thus only a few terms of the series need be used. The remainder, after n terms, is $R_n = f^{(n)}(x_1)(x-a)^n/n!$, where x_1 lies between a and x . R_n gives the limits of error in using n terms of the series for the approximation of the function.

$$n! = \underline{n} = 1 \cdot 2 \cdot 3 \cdot 4 \cdot \dots \cdot n$$

If $a = 0$, the above series is called *Maclaurin's series*.

$$f(x) = f(0) + f'(0) \frac{x}{1!} + f''(0) \frac{x^2}{2!} + f'''(0) \frac{x^3}{3!} + \dots \\ + f^{(n-1)}(0) \frac{x^{n-1}}{(n-1)!} + R_n$$

117. SERIES The following series may be obtained through the expansion of the functions by Taylor's or Maclaurin's theorem. The expressions following a series indicate the region of convergence of the series, that is, the values of x for which R_n approaches zero as n becomes infinite, so that an approximation of the function may be obtained by using a number of terms of the series. If the region of convergence is not indicated, the series converges for all finite values of x . ($n! = 1 \cdot 2 \cdot 3 \cdot \dots \cdot n$). $\log_e u \equiv \log u$.

(a) *Binomial series*

$$(a+x)^n = a^n + na^{n-1}x + \frac{n(n-1)}{2!} a^{n-2}x^2 \\ + \frac{n(n-1)(n-2)}{3!} a^{n-3}x^3 + \dots \quad x^2 < a^2$$

If n is a positive integer, the series consists of $(n + 1)$ terms; otherwise, the number of terms is infinite.

$$(a - bx)^{-1} = \frac{1}{a} \left(1 + \frac{bx}{a} + \frac{b^2x^2}{a^2} + \frac{b^3x^3}{a^3} \dots \right) \quad b^2x^2 < a^2$$

(b) Exponential, logarithmic, and trigonometric series*

$$e = 1 + \frac{1}{1!} + \frac{1}{2!} + \frac{1}{3!} + \frac{1}{4!} + \dots$$

$$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \dots$$

$$a^x = 1 + x \log a + \frac{(x \log a)^2}{2!} + \frac{(x \log a)^3}{3!} + \dots \quad a > 0$$

$$e^{-x^2} = 1 - x^2 + \frac{x^4}{2!} - \frac{x^6}{3!} + \frac{x^8}{4!} - \dots$$

$$\log x = (x - 1) - \frac{1}{2}(x - 1)^2 + \frac{1}{3}(x - 1)^3 - \dots \quad 0 < x \leq 2$$

$$\log x = \frac{x-1}{x} + \frac{1}{2} \left(\frac{x-1}{x} \right)^2 + \frac{1}{3} \left(\frac{x-1}{x} \right)^3 + \dots \quad x > \frac{1}{2}$$

$$\log x = 2 \left[\frac{x-1}{x+1} + \frac{1}{3} \left(\frac{x-1}{x+1} \right)^3 + \frac{1}{5} \left(\frac{x-1}{x+1} \right)^5 + \dots \right] \quad x > 0$$

$$\log(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \dots \quad -1 < x \leq 1$$

$$\begin{aligned} \log(a+x) = \log a + 2 \left[\frac{x}{2a+x} + \frac{1}{3} \left(\frac{x}{2a+x} \right)^3 \right. \\ \left. + \frac{1}{5} \left(\frac{x}{2a+x} \right)^5 + \dots \right] \quad a > 0, -a < x < +\infty \end{aligned}$$

$$\log \left(\frac{1+x}{1-x} \right) = 2 \left(x + \frac{x^3}{3} + \frac{x^5}{5} + \frac{x^7}{7} + \dots \right) \quad x^2 < 1$$

$$\begin{aligned} \log \left(\frac{x+1}{x-1} \right) = 2 \left[\frac{1}{x} + \frac{1}{3} \left(\frac{1}{x} \right)^3 + \frac{1}{5} \left(\frac{1}{x} \right)^5 \right. \\ \left. + \frac{1}{7} \left(\frac{1}{x} \right)^7 + \dots \right] \quad x^2 > 1 \end{aligned}$$

* $\log x \equiv \log_e x$. For principal values of inverse functions, see Sec. 60.

$$\log\left(\frac{x+1}{x}\right) = 2\left[\frac{1}{2x+1} + \frac{1}{3(2x+1)^3} + \frac{1}{5(2x+1)^5} + \dots\right] \quad (2x+1)^2 > 1$$

$$\log(x + \sqrt{1+x^2}) = x - \frac{1}{2} \frac{x^3}{3} + \frac{1 \cdot 3}{2 \cdot 4} \frac{x^5}{5} - \frac{1 \cdot 3 \cdot 5}{2 \cdot 4 \cdot 6} \frac{x^7}{7} + \dots \quad x^2 < 1$$

$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \dots$$

$$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \dots$$

$$\tan x = x + \frac{x^3}{3} + \frac{2x^5}{15} + \frac{17x^7}{315} + \frac{62x^9}{2835} + \dots \quad x^2 < \frac{\pi^2}{4}$$

$$\sin^{-1} x = x + \frac{x^3}{6} + \frac{1}{2} \frac{3}{4} \frac{x^5}{5} + \frac{1}{2} \frac{3}{4} \frac{5}{6} \frac{x^7}{7} + \dots \quad x^2 < 1$$

$$\tan^{-1} x = \begin{cases} x - \frac{1}{3}x^3 + \frac{1}{5}x^5 - \frac{1}{7}x^7 + \dots & x^2 < 1 \\ \frac{\pi}{2} - \frac{1}{x} + \frac{1}{3x^3} - \frac{1}{5x^5} + \dots & x > 1 \end{cases}$$

$$\log|\sin x| = \log|x| - \frac{x^2}{6} - \frac{x^4}{180} - \frac{x^6}{2835} - \dots \quad x^2 < \frac{\pi^2}{2}$$

$$\log \cos x = -\frac{x^2}{2} - \frac{x^4}{12} - \frac{x^6}{45} - \frac{17x^8}{2520} - \dots \quad x^2 < \frac{\pi^2}{4}$$

$$\log|\tan x| = \log|x| + \frac{x^2}{3} + \frac{7x^4}{90} + \frac{62x^6}{2835} + \dots \quad x^2 < \frac{\pi^2}{4}$$

$$e^{\sin x} = 1 + x + \frac{x^2}{2!} - \frac{3x^4}{4!} - \frac{8x^5}{5!} - \frac{3x^6}{6!} + \dots$$

$$e^{\cos x} = e\left(1 - \frac{x^2}{2!} + \frac{4x^4}{4!} - \frac{31x^6}{6!} + \dots\right)$$

$$e^{\tan x} = 1 + x + \frac{x^2}{2!} + \frac{3x^3}{3!} + \frac{9x^4}{4!} + \frac{37x^5}{5!} + \dots \quad x^2 < \frac{\pi^2}{4}$$

$$\sinh x = x + \frac{x^3}{3!} + \frac{x^5}{5!} + \frac{x^7}{7!} + \dots$$

$$\begin{aligned} \cosh x &= 1 + \frac{x^2}{2!} + \frac{x^4}{4!} + \frac{x^6}{6!} + \dots \\ \tanh x &= x - \frac{x^3}{3} + \frac{2x^5}{15} - \frac{17x^7}{315} + \dots \quad x^2 < \frac{\pi^2}{4} \\ \sinh^{-1} x &= x - \frac{1}{2} \frac{x^3}{3} + \frac{1 \cdot 3}{2 \cdot 4} \frac{x^5}{5} - \frac{1 \cdot 3 \cdot 5}{2 \cdot 4 \cdot 6} \frac{x^7}{7} + \dots \quad x^2 < 1 \\ \sinh^{-1} x &= \log 2x + \frac{1}{2} \frac{1}{2x^2} - \frac{1 \cdot 3}{2 \cdot 4} \frac{1}{4x^4} \\ &\quad + \frac{1 \cdot 3 \cdot 5}{2 \cdot 4 \cdot 6} \frac{1}{6x^6} \dots \quad x > 1 \\ \cosh^{-1} x &= \log 2x - \frac{1}{2} \frac{1}{2x^2} - \frac{1 \cdot 3}{2 \cdot 4} \frac{1}{4x^4} \\ &\quad - \frac{1 \cdot 3 \cdot 5}{2 \cdot 4 \cdot 6} \frac{1}{6x^6} - \dots \quad x > 1 \\ \tanh^{-1} x &= x + \frac{x^3}{3} + \frac{x^5}{5} + \frac{x^7}{7} + \dots \quad x^2 < 1 \end{aligned}$$

118. PARTIAL DERIVATIVES. DIFFERENTIALS If $z = f(x, y)$ is a function of two variables, then the derivative of z with respect to x , treating y as a constant, is called the *first partial derivative of z with respect to x* and is denoted by $\partial z / \partial x$. Similarly, the derivative of z with respect to y , treating x as a constant, is called the *first partial derivative of z with respect to y* and is denoted by $\partial z / \partial y$.

Similarly, if $z = f(x, y, u, \dots)$, then the first derivative of z with respect to x , treating y, u, \dots as constants, is called the first partial derivative of z with respect to x and is denoted by $\partial z / \partial x$. Likewise, the second partial derivatives are defined as indicated below:

$$\frac{\partial^2 z}{\partial x^2} = \frac{\partial}{\partial x} \frac{\partial z}{\partial x}; \frac{\partial^2 z}{\partial y^2} = \frac{\partial}{\partial y} \frac{\partial z}{\partial y}; \frac{\partial^2 z}{\partial x \partial y} = \frac{\partial}{\partial x} \frac{\partial z}{\partial y} = \frac{\partial}{\partial y} \frac{\partial z}{\partial x} = \frac{\partial^2 z}{\partial y \partial x}$$

If $z = f(x, y, \dots, u)$ and x, y, \dots, u are functions of a single variable t , then

$$\begin{aligned} \frac{dz}{dt} &= \frac{\partial z}{\partial x} \frac{dx}{dt} + \frac{\partial z}{\partial y} \frac{dy}{dt} + \dots + \frac{\partial z}{\partial u} \frac{du}{dt} \\ dz &= \frac{\partial z}{\partial x} dx + \frac{\partial z}{\partial y} dy + \dots + \frac{\partial z}{\partial u} du \end{aligned}$$

If $F(x, y, z, \dots, u) = 0$, then $\frac{\partial F}{\partial x} dx + \frac{\partial F}{\partial y} dy + \dots + \frac{\partial F}{\partial u} du = 0$.

119. SURFACES. SPACE CURVES (see Analytic Geometry, Secs. 97 and 98) The *tangent plane* to the surface $F(x, y, z) = 0$ at the point (x_1, y_1, z_1) on the surface is

$$(x - x_1) \left(\frac{\partial F}{\partial x} \right)_1 + (y - y_1) \left(\frac{\partial F}{\partial y} \right)_1 + (z - z_1) \left(\frac{\partial F}{\partial z} \right)_1 = 0$$

where $(\partial F/\partial x)_1$ is the value of $\partial F/\partial x$ at (x_1, y_1, z_1) , etc.

The equations of the *normal* to the surface at (x_1, y_1, z_1) are

$$\frac{x - x_1}{(\partial F/\partial x)_1} = \frac{y - y_1}{(\partial F/\partial y)_1} = \frac{z - z_1}{(\partial F/\partial z)_1}$$

The *direction cosines* of the normal to the surface at the point (x_1, y_1, z_1) are proportional to

$$\left(\frac{\partial F}{\partial x} \right)_1, \left(\frac{\partial F}{\partial y} \right)_1, \left(\frac{\partial F}{\partial z} \right)_1$$

Given the *space curve* $x = x(t)$, $y = y(t)$, $z = z(t)$. The direction cosines of the tangent line to the curve at any point are proportional to

$$\frac{dx}{dt}, \frac{dy}{dt}, \frac{dz}{dt} \quad \text{or to} \quad dx, dy, dz$$

The equations of the *tangent line* to the curve at (x_1, y_1, z_1) on the curve are

$$\frac{x - x_1}{(dx/dt)_1} = \frac{y - y_1}{(dy/dt)_1} = \frac{z - z_1}{(dz/dt)_1}$$

where $(dx/dt)_1$ is the value of dx/dt at (x_1, y_1, z_1) , etc.

6. INTEGRAL CALCULUS

120. DEFINITION OF INDEFINITE INTEGRAL $F(x)$ is said to be an *indefinite integral* of $f(x)$ if the derivative of $F(x)$ is $f(x)$ or if the differential of $F(x)$ is $f(x) dx$; symbolically:

$$F(x) = \int f(x) dx \quad \text{if} \quad \frac{dF(x)}{dx} = f(x) \quad \text{or} \quad dF(x) = f(x) dx$$

In general, $\int f(x) dx = F(x) + C$, where C is an arbitrary constant.

121. FUNDAMENTAL THEOREMS ON INTEGRALS. SHORT TABLE OF INTEGRALS* (u and v denote functions of x ; a , b , and C denote constants.) Interpret symbols for inverse relations (Sec. 60) to be principal values.

1. $\int df(x) = f(x) + C$

2. $d \int f(x) dx = f(x) dx$

3. $\int 0 dx = C$

4. $\int a f(x) dx = a \int f(x) dx$

5. $\int (u \pm v) dx = \int u dx \pm \int v dx$

6. $\int u dv = uv - \int v du$

7. $\int \frac{u dv}{dx} dx = uv - \int v \frac{du}{dx} dx$

8. $\int f(y) dx = \int \frac{f(y) dy}{dy/dx}$

* See Sec. 148. Where an integration gives a term of the form $A \log N$, where A is a constant and $N < 0$, use $A \log |N|$; if $N > 0$, use $A \log N$.

60 *Integral Calculus*

$$9. \int u^n du = \frac{u^{n+1}}{n+1} + C \quad n \neq -1$$

$$10. \int \frac{du}{u} = \log_e u + C, u > 0 \quad \text{or, } \log_e |u| + C \quad u \neq 0$$

$$11. \int e^u du = e^u + C$$

$$12. \int b^u du = \frac{b^u}{\log_e b} + C \quad b > 0, b \neq 1$$

$$13. \int \sin u du = -\cos u + C$$

$$14. \int \cos u du = \sin u + C$$

$$15. \int \tan u du = \log_e \sec u + C = -\log_e \cos u + C$$

$$16. \int \cot u du = \log_e \sin u + C = -\log_e \csc u + C$$

$$17. \int \sec u du = \log_e (\sec u + \tan u) + C \\ = \log_e \tan \left(\frac{u}{2} + \frac{\pi}{4} \right) + C$$

$$18. \int \csc u du = \log_e (\csc u - \cot u) + C = \log_e \tan \frac{u}{2} + C$$

$$19. \int \sin^2 u du = \frac{1}{2} u - \frac{1}{2} \sin u \cos u + C$$

$$20. \int \cos^2 u du = \frac{1}{2} u + \frac{1}{2} \sin u \cos u + C$$

$$21. \int \sec^2 u du = \tan u + C$$

$$22. \int \csc^2 u du = -\cot u + C$$

$$23. \int \tan^2 u du = \tan u - u + C$$

$$24. \int \cot^2 u du = -\cot u - u + C$$

$$25.* \int \frac{du}{u^2 + a^2} = \frac{1}{a} \tan^{-1} \frac{u}{a} + C$$

$$26. \int \frac{du}{u^2 - a^2} = \begin{cases} \frac{1}{2a} \log_e \frac{u-a}{u+a} + C = -\frac{1}{a} \coth^{-1} \frac{u}{a} + C & u^2 > a^2 \\ \frac{1}{2a} \log_e \frac{a-u}{a+u} + C = -\frac{1}{a} \tanh^{-1} \frac{u}{a} + C & u^2 < a^2 \end{cases}$$

$$27. \int \frac{du}{\sqrt{a^2 - u^2}} = \begin{cases} \sin^{-1} \frac{u}{a} + C & a > 0 \\ \sin^{-1} \frac{u}{|a|} + C & |a| \neq 0 \end{cases}$$

$$28. \int \frac{du}{\sqrt{u^2 \pm a^2}} = \log_e (u + \sqrt{u^2 \pm a^2})^\dagger + C$$

$$29.* \int \frac{du}{\sqrt{2au - u^2}} = \cos^{-1} \frac{a-u}{a} + C$$

$$30. \int \frac{du}{u\sqrt{u^2 - a^2}} = \frac{1}{a} \sec^{-1} \frac{u}{a} + C = \frac{1}{a} \cos^{-1} \frac{a}{u} + C$$

$$31. \int \frac{du}{u\sqrt{a^2 \pm u^2}} = -\frac{1}{a} \log_e \frac{a + \sqrt{a^2 \pm u^2}}{u} + C$$

$$32. \int \sqrt{a^2 - u^2} du = \frac{1}{2} \left(u\sqrt{a^2 - u^2} + a^2 \sin^{-1} \frac{u}{a} \right) + C$$

$$33. \int \sqrt{u^2 \pm a^2} du = \frac{1}{2} \left[u\sqrt{u^2 \pm a^2} \pm a^2 \log_e (u + \sqrt{u^2 \pm a^2}) \right]^\dagger + C$$

$$34. \int \sinh u du = \cosh u + C$$

* In formulas 25 to 32 and footnote †, use $a > 0$.

$$\dagger \log_e \frac{u + \sqrt{u^2 + a^2}}{a} = \sinh^{-1} \frac{u}{a} \qquad \log_e \frac{a + \sqrt{a^2 - u^2}}{u} = \operatorname{sech}^{-1} \frac{u}{a}$$

$$\log_e \frac{u + \sqrt{u^2 - a^2}}{a} = \cosh^{-1} \frac{u}{a} \qquad \log_e \frac{a + \sqrt{a^2 + u^2}}{u} = \operatorname{csch}^{-1} \frac{u}{a}$$

35. $\int \cosh u \, du = \sinh u + C$

36. $\int \tanh u \, du = \log_e (\cosh u) + C$

37. $\int \coth u \, du = \log_e (\sinh u) + C$

38. $\int \operatorname{sech} u \, du = \sin^{-1} (\tanh u) + C$

39. $\int \operatorname{csch} u \, du = \log_e \left(\tanh \frac{u}{2} \right) + C$

40. $\int \operatorname{sech} u \tanh u \, du = -\operatorname{sech} u + C$

41. $\int \operatorname{csch} u \coth u \, du = -\operatorname{csch} u + C$

Definite Integrals

122. DEFINITION OF DEFINITE INTEGRAL Let $f(x)$ be continuous for the interval from $x = a$ to $x = b$ inclusive. Divide this interval into n equal parts by the points $a, x_1, x_2, \dots, x_{n-1}, b$ such that $\Delta x = (b - a)/n$. The definite integral of $f(x)$ with respect to x between the limits $x = a$ to $x = b$ is

$$\int_a^b f(x) \, dx = \lim_{n \rightarrow \infty} [f(a) \Delta x + f(x_1) \Delta x + f(x_2) \Delta x + \dots + f(x_{n-1}) \Delta x]$$

$$= \left[\int f(x) \, dx \right]_a^b = \left[F(x) \right]_a^b = F(b) - F(a)$$

where $F(x)$ is a function whose derivative with respect to x is $f(x)$.

123. APPROXIMATE VALUES OF DEFINITE INTEGRAL Approximate values of the above definite integral are given by the rules of Sec. 41, where $y_0, y_1, y_2, \dots, y_{n-1}, y_n$ are the values of $f(x)$ for $x = a, x_1, x_2, \dots, x_{n-1}, b$, respectively, and $h = (b - a)/n$.

124. SOME FUNDAMENTAL THEOREMS

$$\int_a^b [f_1(x) + f_2(x) + \dots + f_n(x)] dx = \int_a^b f_1(x) dx + \int_a^b f_2(x) dx + \dots + \int_a^b f_n(x) dx$$

$$\int_a^b k f(x) dx = k \int_a^b f(x) dx \quad \text{if } k \text{ is a constant}$$

$$\int_a^b f(x) dx = - \int_b^a f(x) dx$$

$$\int_a^b f(x) dx = \int_a^c f(x) dx + \int_c^b f(x) dx$$

$$\int_a^b f(x) dx = (b - a) f(x_1) \quad \text{where } x_1 \text{ lies between } a \text{ and } b$$

$$\int_a^\infty f(x) dx = \lim_{t \rightarrow \infty} \int_a^t f(x) dx$$

$$\int_{-\infty}^b f(x) dx = \lim_{t \rightarrow -\infty} \int_t^b f(x) dx$$

$$\int_{-\infty}^{+\infty} f(x) dx = \int_{-\infty}^c f(x) dx + \int_c^{+\infty} f(x) dx$$

If $f(x)$ has a singular point* at $x = b$, $b \neq a$,

$$\int_a^b f(x) dx = \lim_{e \rightarrow 0} \int_a^{b-e} f(x) dx$$

The mean value of the function $f(x)$ on the interval (a,b) is

$$\frac{1}{b-a} \int_a^b f(x) dx$$

Some Applications of the Definite Integral

125. PLANE AREA The area bounded by $y = f(x)$, $y = 0$, $x = a$, $x = b$, where y has the same sign for all values of x between a and b , is

$$A = \int_a^b f(x) dx \quad dA = f(x) dx$$

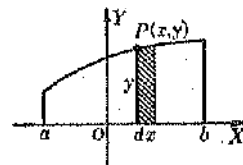


Fig. 56

* For example, when $\lim_{x \rightarrow b} f(x) = \infty$.

The area bounded by the curve $r = f(\theta)$ and the two radii $\theta = \alpha$ and $\theta = \beta$ is

$$A = \frac{1}{2} \int_{\alpha}^{\beta} [f(\theta)]^2 d\theta \quad dA = \frac{1}{2} r^2 d\theta$$

126. LENGTH OF ARC (see Sec. 113) The length (s) of arc of a plane curve $f(x, y) = 0$ from the point (a, c) to the point (b, d) is

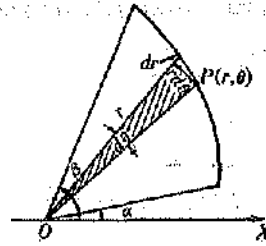


Fig. 57

$$s = \int_a^b \sqrt{1 + \left(\frac{dy}{dx}\right)^2} dx = \int_c^d \sqrt{1 + \left(\frac{dx}{dy}\right)^2} dy$$

If the equation of the curve is $x = f(t)$, $y = f(t)$, the length of arc from $t = a$ to $t = b$ is

$$s = \int_a^b \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2} dt$$

If the equation of the curve is $r = f(\theta)$, then

$$s = \int_{\theta_1}^{\theta_2} \sqrt{r^2 + \left(\frac{dr}{d\theta}\right)^2} d\theta = \int_{r_1}^{r_2} \sqrt{r^2 \left(\frac{d\theta}{dr}\right)^2 + 1} dr$$

127. VOLUME BY PARALLEL SECTIONS If the plane perpendicular to the x axis at $(x, 0, 0)$ cuts from a given solid a section whose area is $A(x)$, then the volume of that part of the solid between $x = a$ and $x = b$ is

$$\int_a^b A(x) dx$$

128. VOLUME OF REVOLUTION The volume of a solid of revolution generated by revolving that portion of the curve $y = f(x)$ between $x = a$ and $x = b$

(a) about the x axis is $\pi \int_a^b y^2 dx$

(b) about the y axis is $\pi \int_c^d x^2 dy$, where c and d are the values of y corresponding to the values a and b of x

129. AREA OF SURFACE OF REVOLUTION The area of the surface of a solid of revolution generated by revolving the curve $y = f(x)$ between $x = a$ and $x = b$

(a) about the x axis is $2\pi \int_a^b y \sqrt{1 + \left(\frac{dy}{dx}\right)^2} dx$

(b) about the y axis is $2\pi \int_a^d x \sqrt{1 + \left(\frac{dx}{dy}\right)^2} dy$

130. PLANE AREAS BY DOUBLE INTEGRATION

(a) Rectangular coordinates

$$A = \int_a^b \int_{\varphi(x)}^{f(x)} dy dx \quad \text{or} \quad \int_c^d \int_{g(y)}^{\psi(y)} dx dy$$

(b) Polar coordinates

$$A = \int_{\theta_1}^{\theta_2} \int_{f_1(\theta)}^{f_2(\theta)} r dr d\theta \quad \text{or} \quad \int_{r_1}^{r_2} \int_{\varphi_1(r)}^{\varphi_2(r)} r d\theta dr$$

131. VOLUMES BY DOUBLE INTEGRATION If $z = f(x, y)$,

$$V = \int_a^b \int_{\phi(x)}^{\psi(x)} f(x, y) dy dx \quad \text{or} \quad \int_c^d \int_{a(y)}^{\beta(y)} f(x, y) dx dy$$

132. VOLUMES BY TRIPLE INTEGRATION (see Sec. 102)

(a) Rectangular coordinates

$$V = \iiint dx dy dz$$

(b) Cylindrical coordinates

$$V = \iiint r dr d\theta dz$$

(c) Spherical coordinates

$$V = \iiint \rho^2 \sin \phi d\theta d\phi d\rho$$

where the limits of integration must be supplied. Other formulas may be obtained by changing the order of integration.

$$\begin{aligned} & \text{Area of surface } z = f(x, y) \\ A &= \iint \sqrt{\left(\frac{\partial z}{\partial x}\right)^2 + \left(\frac{\partial z}{\partial y}\right)^2 + 1} \, dy \, dx \end{aligned}$$

where the limits of integration must be supplied.

133. MASS* The mass of a body of density δ is

$$m = \int dm, \quad dm = \delta \, dA, \text{ or } \delta \, ds, \text{ or } \delta \, dV, \text{ or } \delta \, dS$$

where dA , ds , dV , dS are, respectively, the elements of area, length, volume, and surface of Secs. 125 to 132.

134. DENSITY* If δ is a variable (or constant) density (mass per unit of element) and $\bar{\delta}$ is the mean density of a solid of volume V , then

$$\bar{\delta} = \frac{\int \delta \, dV}{\int dV}$$

135. MOMENT* The moments M_{yz} , M_{zx} , M_{xy} of a mass m with respect to the coordinate planes (as indicated by the subscripts) are

$$M_{yz} = \int x \, dm, \quad M_{zx} = \int y \, dm, \quad M_{xy} = \int z \, dm$$

136. CENTROID OF MASS OR CENTER OF GRAVITY* The coordinates $(\bar{x}, \bar{y}, \bar{z})$ of the centroid of a mass m are

$$\bar{x} = \frac{\int x \, dm}{\int dm}, \quad \bar{y} = \frac{\int y \, dm}{\int dm}, \quad \bar{z} = \frac{\int z \, dm}{\int dm}$$

Note: In the above equations x , y , z are the coordinates of the center of gravity of the element dm .

* The limits of integration are to be supplied.

137. CENTROID OF SEVERAL MASSES The x coordinate (\bar{x}) of the centroid of several masses m_1, m_2, \dots, m_n , having $\bar{x}_1, \bar{x}_2, \dots, \bar{x}_n$, respectively, as the x coordinates of their centroids, is

$$\bar{x} = \frac{m_1\bar{x}_1 + m_2\bar{x}_2 + \dots + m_n\bar{x}_n}{m_1 + m_2 + \dots + m_n}$$

Similar formulas hold for the other coordinates \bar{y}, \bar{z} .

138. MOMENT OF INERTIA (SECOND MOMENT)* The moments of inertia (I):

(a) for a plane curve about the x axis, y axis, and origin, respectively, are

$$I_x = \int y^2 ds, \quad I_y = \int x^2 ds, \quad I_0 = \int (x^2 + y^2) ds$$

(b) for a plane area about the x axis, y axis, and origin, respectively, are

$$I_x = \int y^2 dA, \quad I_y = \int x^2 dA, \quad I_0 = \int (x^2 + y^2) dA$$

(c) for a solid of mass m about the yz, xz, xy planes, x axis, etc., respectively, are

$$I_{yz} = \int x^2 dm, \quad I_{xz} = \int y^2 dm, \quad I_{xy} = \int z^2 dm, \quad I_x = I_{xz} + I_{xy}, \text{ etc.}$$

138a. THEOREM OF PARALLEL AXES Let L be any line in space and L_0 a line parallel to L , passing through the centroid of the body of mass m . If d is the distance between the lines L and L_0 , then

$$I_L = I_{L_0} + d^2m$$

where I_L and I_{L_0} are the moments of inertia of the body about the lines L and L_0 , respectively.

139. RADIUS OF GYRATION If I is the moment of inertia of a mass m and K is the radius of gyration, $I = mK^2$. Similarly for areas, lengths, volumes, etc.

* The limits of integration are to be supplied.

If masses (or areas, etc.) m_1, m_2, \dots, m_n have, respectively, the radii of gyration k_1, k_2, \dots, k_n , with respect to a line or plane, then with respect to this line or plane, the several masses taken together have the radius of gyration K , where

$$K^2 = \frac{m_1 k_1^2 + m_2 k_2^2 + \dots + m_n k_n^2}{m_1 + m_2 + \dots + m_n}$$

140. WORK The work W done in moving a particle from $s = a$ to $s = b$ by a force whose component expressed as a function of s in the direction of motion is F_s is

$$W = \int_{s=a}^{s=b} F_s ds \quad dW = F_s ds$$

141. PRESSURE The pressure (p) against an area vertical to the surface of a liquid and between the depths a and b is

$$P = \int_{y=a}^{y=b} wly dy \quad dp = wly dy$$

where w is the weight of liquid per unit volume, y is the depth beneath the surface of liquid of a horizontal element of area, and l is the length of the horizontal element of area expressed in terms of y .

142. CENTER OF PRESSURE The depth \bar{y} of the center of pressure against an area vertical to the surface of the liquid and between the depths a and b is

$$\bar{y} = \frac{\int_{y=a}^{y=b} y dp}{\int_{y=a}^{y=b} dp} \quad (\text{for } dp \text{ see Sec. 141})$$

7. TABLE OF INTEGRALS

Certain Elementary Processes

143. To integrate $\int R(x) dx$, where $R(x)$ is rational function of x . Write $R(x)$ in the form of a fraction whose terms are polynomials. If the fraction is improper (i.e., the degree of the denominator is less than or equal to the degree of the numerator), divide the numerator by the denominator and thus write $R(x)$ as the sum of a quotient $Q(x)$ and a proper fraction $P(x)$. (In a proper fraction the degree of the numerator is less than the degree of the denominator.) The polynomial $Q(x)$ is readily integrated. To integrate $P(x)$, separate it into a sum of partial fractions (as indicated below) and integrate each term of the sum separately.

To separate the proper fraction $P(x)$ into partial fractions, write $P(x) = f(x)/\psi(x)$, where $f(x)$ and $\psi(x)$ are polynomials,

$$\psi(x) = (x - a)^p(x - b)^q(x - c)^r \dots$$

and the constants a, b, c are all different. By algebra, there exist constants $A_1, A_2, \dots, B_1, B_2, \dots$ such that

$$\begin{aligned} \frac{f(x)}{\psi(x)} = & \frac{A_1}{x - a} + \frac{A_2}{(x - a)^2} + \dots + \frac{A_p}{(x - a)^p} \\ & + \frac{B_1}{x - b} + \frac{B_2}{(x - b)^2} + \dots + \frac{B_q}{(x - b)^q} + \dots \end{aligned}$$

The separate terms of this sum may be integrated by the formulas

$$\int \frac{dx}{(x - \alpha)^t} = \frac{-1}{(t - 1)(x - \alpha)^{t-1}}, \quad t > 1, \quad \int \frac{dx}{x - \alpha} = \log(x - \alpha)$$

If $f(x)$ and $\psi(x)$ have real coefficients and $\psi(x) = 0$ has imaginary roots, the above method leads to imaginary quantities. To avoid this, separate $P(x)$ in a different way. As before, corresponding to each p -fold real root a of $\psi(x)$, use the sum

$$\frac{A_1}{x - a} + \frac{A_2}{(x - a)^2} + \dots + \frac{A_p}{(x - a)^p}$$

To each λ -fold real quadratic factor $x^2 + \alpha x + \beta$ of $\psi(x)$ which does not factor into real linear factors, use, instead of the two sets of terms occurring in the first expansion of $R(x)$ and dependent on the conjugate complex roots of $x^2 + \alpha x + \beta = 0$, sums of the forms

$$\frac{D_1x + E_1}{x^2 + \alpha x + \beta} + \frac{D_2x + E_2}{(x^2 + \alpha x + \beta)^2} + \cdots + \frac{D_\lambda x + E_\lambda}{(x^2 + \alpha x + \beta)^\lambda}$$

where the quantities $D_1, D_2, \dots, E_1, E_2, \dots$ are real constants.

These new sums may be separately integrated by means of integral formulas 149 to 156, etc.

144. TO INTEGRATE AN IRRATIONAL ALGEBRAIC FUNCTION If no convenient method of integration is apparent, the integration may frequently be performed by means of a change of variable. For example, if R is a rational function of two arguments and n is an integer, then to integrate

(a) $\int R[x, (ax + b)^{1/n}] dx$, let $(ax + b) = y^n$, whence the integral reduces to $\int P(y) dy$, where $P(y)$ is a rational function of y and may be integrated as in Sec. 143;

(b) $\int R[x, (x^2 + bx + c)^{1/2}] dx$, let $(x^2 + bx + c)^{1/2} = z - x$, reduce R to a rational function of z , and proceed as in Sec. 143;

(c) $\int R(\sin x, \cos x) dx$, let $\tan \frac{x}{2} = t$, whence

$$\sin x = \frac{2t}{1+t^2} \quad \cos x = \frac{1-t^2}{1+t^2} \quad dx = \frac{2 dt}{1+t^2}$$

reduce R to a rational function of t and proceed as in Sec. 143.

145. TO INTEGRATE EXPRESSIONS CONTAINING

$$\sqrt{a^2 - x^2} \quad \sqrt{x^2 \pm a^2}$$

Expressions containing these radicals can frequently be integrated after making the following transformations:

- (a) if $\sqrt{a^2 - x^2}$ occurs, let $x = a \sin t$
- (b) if $\sqrt{x^2 - a^2}$ occurs, let $x = a \sec t$
- (c) if $\sqrt{x^2 + a^2}$ occurs, let $x = a \tan t$

146. TRIGONOMETRIC EXPRESSIONS The identities of Sec. 58 frequently facilitate the integration of such functions.

147. INTEGRATION BY PARTS The formula $\int u dv = uv - \int v du$ is often effective in reducing an integral to simpler integrals.

148. TABLE OF INDEFINITE INTEGRALS (see Sec. 121) In this table the constant C of integration is omitted, but C should be added to the result of every integration. The letter x represents any variable; u , any function of x ; a, b, \dots arbitrary constants; and $\log_e u \equiv \log u$, unless otherwise stated. All angles are in radians. In general, interpret symbols for inverse relations (Sec. 60) as principal values (for example, $\sin^{-1} x$ means $\text{Sin}^{-1} x$); interpret $\sqrt{\quad}$ as the positive value (for example, $t = \sqrt{x^2 - a^2}$ means the positive value of t). Where an integration gives a constant A times the logarithm of a negative number N , say, $A \log N$, use $A \log |N|$. [For example, if $X = ax + b$, $\int (1/X) dx = (1/a) \log_e X + C$, which is used if $X > 0$. If $X < 0$, use $\int (1/X) dx = (1/a) \log_e |X| + C$, since $\log(-1) \equiv (2k + 1)\pi i$, k an integer, may be considered a part of the constant of integration.]

Expressions Containing $(ax + b)$

$$42. \int (ax + b)^n dx = \frac{1}{a(n + 1)} (ax + b)^{n+1} \quad n \neq -1$$

$$43. \int \frac{dx}{ax + b} = \frac{1}{a} \log_e (ax + b)$$

$$44. \int \frac{dx}{(ax + b)^2} = -\frac{1}{a(ax + b)}$$

$$45. \int \frac{dx}{(ax + b)^3} = -\frac{1}{2a(ax + b)^2}$$

$$46. \int x(ax + b)^n dx = \frac{1}{a^2(n+2)} (ax + b)^{n+2} - \frac{b}{a^2(n+1)} (ax + b)^{n+1} \quad n \neq -1, -2$$

$$47. \int \frac{x dx}{ax + b} = \frac{x}{a} - \frac{b}{a^2} \log(ax + b)$$

$$48. \int \frac{x dx}{(ax + b)^2} = \frac{b}{a^2(ax + b)} + \frac{1}{a^2} \log(ax + b)$$

$$49. \int \frac{x dx}{(ax + b)^3} = \frac{b}{2a^2(ax + b)^2} - \frac{1}{a^2(ax + b)}$$

$$50. \int x^2(ax + b)^n dx = \frac{1}{a^3} \left[\frac{(ax + b)^{n+3}}{n+3} - 2b \frac{(ax + b)^{n+2}}{n+2} + b^2 \frac{(ax + b)^{n+1}}{n+1} \right] \quad n \neq -1, -2, -3$$

$$51. \int \frac{x^2 dx}{ax + b} = \frac{1}{a^3} \left[\frac{1}{2}(ax + b)^2 - 2b(ax + b) + b^2 \log(ax + b) \right]$$

$$52. \int \frac{x^2 dx}{(ax + b)^2} = \frac{1}{a^3} \left[(ax + b) - 2b \log(ax + b) - \frac{b^2}{ax + b} \right]$$

$$53. \int \frac{x^2 dx}{(ax + b)^3} = \frac{1}{a^3} \left[\log(ax + b) + \frac{2b}{ax + b} - \frac{b^2}{2(ax + b)^2} \right]$$

$$54. \int x^m(ax + b)^n dx = \frac{1}{a(m+n+1)} \left[x^m(ax + b)^{n+1} - mb \int x^{m-1}(ax + b)^n dx \right] \\ = \frac{1}{m+n+1} \left[x^{m+1}(ax + b)^n + nb \int x^m(ax + b)^{n-1} dx \right] \quad m > 0, m+n+1 \neq 0$$

$$55. \int \frac{dx}{x(ax+b)} = \frac{1}{b} \log \frac{x}{ax+b}$$

$$56. \int \frac{dx}{x^2(ax+b)} = -\frac{1}{bx} + \frac{a}{b^2} \log \frac{ax+b}{x}$$

$$57. \int \frac{dx}{x^3(ax+b)} = \frac{2ax-b}{2b^2x^2} + \frac{a^2}{b^3} \log \frac{x}{ax+b}$$

$$58. \int \frac{dx}{x(ax+b)^2} = \frac{1}{b(ax+b)} - \frac{1}{b^2} \log \frac{ax+b}{x}$$

$$59. \int \frac{dx}{x(ax+b)^3} = \frac{1}{b^3} \left[\frac{1}{2} \left(\frac{ax+2b}{ax+b} \right)^2 + \log \frac{x}{ax+b} \right]$$

$$60. \int \frac{dx}{x^2(ax+b)^2} = \frac{b+2ax}{b^2x(ax+b)} + \frac{2a}{b^3} \log \frac{ax+b}{x}$$

$$61. \int \sqrt{ax+b} dx = \frac{2}{3a} \sqrt{(ax+b)^3}$$

$$62. \int x \sqrt{ax+b} dx = \frac{2(3ax-2b)}{15a^2} \sqrt{(ax+b)^3}$$

$$63. \int x^2 \sqrt{ax+b} dx = \frac{2(15a^2x^2 - 12abx + 8b^2) \sqrt{(ax+b)^3}}{105a^3}$$

$$64. \int x^3 \sqrt{ax+b} dx = \frac{2(35a^3x^3 - 30a^2bx^2 + 24ab^2x - 16b^3) \sqrt{(ax+b)^3}}{315a^4}$$

$$65. \int x^n \sqrt{ax+b} dx = \frac{2}{a^{n+1}} \int u^2(u^2-b)^n du \quad u = \sqrt{ax+b}$$

$$66. \int \frac{\sqrt{ax+b}}{x} dx = 2\sqrt{ax+b} + b \int \frac{dx}{x\sqrt{ax+b}}$$

$$67. \int \frac{dx}{\sqrt{ax+b}} = \frac{2\sqrt{ax+b}}{a}$$

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$$68. \int \frac{x \, dx}{\sqrt{ax + b}} = \frac{2(ax - 2b)}{3a^2} \sqrt{ax + b}$$

$$69. \int \frac{x^2 \, dx}{\sqrt{ax + b}} = \frac{2(3a^2x^2 - 4abx + 8b^2)}{15a^3} \sqrt{ax + b}$$

$$70. \int \frac{x^3 \, dx}{\sqrt{ax + b}} = \frac{2(5a^3x^3 - 6a^2bx^2 + 8ab^2x - 16b^3)}{35a^4} \sqrt{ax + b}$$

$$71. \int \frac{x^n \, dx}{\sqrt{ax + b}} = \frac{2}{a^{n+1}} \int (a^2 - b)^n \, du \quad u = \sqrt{ax + b}$$

$$72. \int \frac{dx}{x\sqrt{ax + b}} = \frac{1}{\sqrt{b}} \log \frac{\sqrt{ax + b} - \sqrt{b}}{\sqrt{ax + b} + \sqrt{b}} \quad b > 0$$

$$73. \int \frac{dx}{x\sqrt{ax + b}} = \begin{cases} \frac{2}{\sqrt{-b}} \tan^{-1} \sqrt{\frac{ax + b}{-b}} & b < 0 \\ \frac{-2}{\sqrt{b}} \tanh^{-1} \sqrt{\frac{ax + b}{b}} & b > 0 \end{cases}$$

$$74. \int \frac{dx}{x^2\sqrt{ax + b}} = -\frac{\sqrt{ax + b}}{bx} - \frac{a}{2b} \int \frac{dx}{x\sqrt{ax + b}}$$

$$75. \int \frac{dx}{x^3\sqrt{ax + b}} = -\frac{\sqrt{ax + b}}{2bx^2} + \frac{3a\sqrt{ax + b}}{4b^2x} + \frac{3a^2}{8b^2} \int \frac{dx}{x\sqrt{ax + b}}$$

$$76. \int \frac{dx}{x^n(ax + b)^m} = -\frac{1}{b^{m+n-1}} \int \frac{(u - a)^{m+n-2} \, du}{u^m} \quad u = \frac{ax + b}{x}$$

$$77. \int (ax + b)^{\pm(n/2)} \, dx = \frac{2(ax + b)^{(2 \pm n)/2}}{a(2 \pm n)}$$

$$78. \int x(ax + b)^{\pm(n/2)} dx = \frac{2}{a^2} \left[\frac{(ax + b)^{(4 \pm n)/2}}{4 \pm n} - \frac{b(ax + b)^{(2 \pm n)/2}}{2 \pm n} \right]$$

$$79. \int \frac{dx}{x(ax + b)^{n/2}} = \frac{1}{b} \int \frac{dx}{x(ax + b)^{(n-2)/2}} - \frac{a}{b} \int \frac{dx}{(ax + b)^{n/2}}$$

$$80. \int \frac{x^m dx}{\sqrt{ax + b}} = \frac{2x^m \sqrt{ax + b}}{(2m + 1)a} - \frac{2mb}{(2m + 1)a} \int \frac{x^{m-1} dx}{\sqrt{ax + b}}$$

$$81. \int \frac{dx}{x^n \sqrt{ax + b}} = \frac{-\sqrt{ax + b}}{(n-1)bx^{n-1}} - \frac{(2n-3)a}{(2n-2)b} \int \frac{dx}{x^{n-1} \sqrt{ax + b}}$$

$$82. \int \frac{(ax + b)^{n/2}}{x} dx = a \int (ax + b)^{(n-2)/2} dx + b \int \frac{(ax + b)^{(n-2)/2}}{x} dx$$

$$83. \int \frac{dx}{(ax + b)(cx + d)} = \frac{1}{bc - ad} \log \frac{cx + d}{ax + b}, \quad bc - ad \neq 0$$

$$84. \int \frac{dx}{(ax + b)^2(cx + d)} = \frac{1}{bc - ad} \left(\frac{1}{ax + b} + \frac{c}{bc - ad} \log \frac{cx + d}{ax + b} \right), \quad bc - ad \neq 0$$

$$85. \int (ax + b)^n (cx + d)^m dx = \frac{1}{(m + n + 1)a} \left[(ax + b)^{n+1} (cx + d)^m - m(bc - ad) \int (ax + b)^n (cx + d)^{m-1} dx \right]$$

$$86. \int \frac{dx}{(ax + b)^n (cx + d)^m} = \frac{-1}{(m-1)(bc - ad)} \left[\frac{1}{(ax + b)^{n-1} (cx + d)^{m-1}} + a(m+n-2) \int \frac{dx}{(ax + b)^n (cx + d)^{m-1}} \right], \quad m > 1, n > 0, bc - ad \neq 0$$

$$87. \int \frac{(ax+b)^n}{(cx+d)^m} dx = \frac{1}{(m-1)(bc-ad)} \left[\frac{(ax+b)^{n+1}}{(cx+d)^{m-1}} \right. \\ \left. + (m-n-2)a \int \frac{(ax+b)^n dx}{(cx+d)^{m-1}} \right] = \frac{-1}{(m-n-1)c} \left[\frac{(ax+b)^n}{(cx+d)^{m-1}} \right. \\ \left. + n(bc-ad) \int \frac{(ax+b)^{n-1}}{(cx+d)^m} dx \right]$$

$$88. \int \frac{x dx}{(ax+b)(cx+d)} = \frac{1}{bc-ad} \left[\frac{b}{a} \log(ax+b) \right. \\ \left. - \frac{d}{c} \log(cx+d) \right] \quad bc-ad \neq 0$$

$$89. \int \frac{x dx}{(ax+b)^2(cx+d)} = \frac{1}{bc-ad} \left[\frac{b}{a(ax+b)} \right. \\ \left. - \frac{d}{bc-ad} \log \frac{cx+d}{ax+b} \right] \quad bc-ad \neq 0$$

$$90. \int \frac{cx+d}{\sqrt{ax+b}} dx = \frac{2}{3a^2} (3ad - 2bc + acx) \sqrt{ax+b}$$

$$91. \int \frac{\sqrt{ax+b}}{cx+d} dx = \frac{2\sqrt{ax+b}}{c} \\ - \frac{2}{c} \sqrt{\frac{ad-bc}{c}} \tan^{-1} \sqrt{\frac{c(ax+b)}{ad-bc}} \quad c > 0, ad > bc$$

$$92. \int \frac{\sqrt{ax+b}}{cx+d} dx = \frac{2\sqrt{ax+b}}{c} \\ + \frac{1}{c} \sqrt{\frac{bc-ad}{c}} \log \frac{\sqrt{c(ax+b)} - \sqrt{bc-ad}}{\sqrt{c(ax+b)} + \sqrt{bc-ad}} \\ c > 0, bc > ad$$

$$93. \int \frac{dx}{(cx+d)\sqrt{ax+b}} = \frac{2}{\sqrt{c}\sqrt{ad-bc}} \tan^{-1} \sqrt{\frac{c(ax+b)}{ad-bc}} \\ c > 0, ad > bc$$

$$94. \int \frac{dx}{(cx + d)\sqrt{ax + b}} = \frac{1}{\sqrt{c}\sqrt{bc - ad}} \log \frac{\sqrt{c(ax + b)} - \sqrt{bc - ad}}{\sqrt{c(ax + b)} + \sqrt{bc - ad}}, \quad c > 0, bc > ad$$

Expressions Containing $ax^2 + c$, $ax^2 + c$, $x^2 \pm p^2$, and $p^2 - x^2$

$$95. \int \frac{dx}{p^2 + x^2} = \frac{1}{p} \tan^{-1} \frac{x}{p}, \text{ or } -\frac{1}{p} \cot^{-1} \frac{x}{p}$$

$$96. \int \frac{dx}{p^2 - x^2} = \frac{1}{2p} \log \frac{p + x}{p - x}, \text{ or } \frac{1}{p} \tanh^{-1} \frac{x}{p}$$

$$97. \int \frac{dx}{ax^2 + c} = \frac{1}{\sqrt{ac}} \tan^{-1} \left(x \sqrt{\frac{a}{c}} \right) \quad a \text{ and } c > 0$$

$$98. \int \frac{dx}{ax^2 + c} = \begin{cases} \frac{1}{2\sqrt{-ac}} \log \frac{x\sqrt{a} - \sqrt{-c}}{x\sqrt{a} + \sqrt{-c}} & a > 0, c < 0 \\ \frac{1}{2\sqrt{-ac}} \log \frac{\sqrt{c} + x\sqrt{-a}}{\sqrt{c} - x\sqrt{-a}} & a < 0, c > 0 \end{cases}$$

$$99. \int \frac{dx}{(ax^2 + c)^n} = \frac{1}{2(n-1)c} \frac{x}{(ax^2 + c)^{n-1}} + \frac{2n-3}{2(n-1)c} \int \frac{dx}{(ax^2 + c)^{n-1}} \quad n > 1$$

$$100. \int x(ax^2 + c)^n dx = \frac{1}{2a} \frac{(ax^2 + c)^{n+1}}{n+1} \quad n \neq -1$$

$$101. \int \frac{x}{ax^2 + c} dx = \frac{1}{2a} \log(ax^2 + c)$$

$$102. \int \frac{dx}{x(ax^2 + c)} = \frac{1}{2c} \log \frac{x^2}{ax^2 + c}$$

$$103. \int \frac{dx}{x^2(ax^2 + c)} = -\frac{1}{cx} - \frac{a}{c} \int \frac{dx}{ax^2 + c}$$

$$104. \int \frac{x^2 dx}{ax^2 + c} = \frac{x}{a} - \frac{c}{a} \int \frac{dx}{ax^2 + c}$$

$$105. \int \frac{x^n dx}{ax^2 + c} = \frac{x^{n-1}}{a(n-1)} - \frac{c}{a} \int \frac{x^{n-2} dx}{ax^2 + c} \quad n \neq 1$$

$$106. \int \frac{x^2 dx}{(ax^2 + c)^n} = -\frac{1}{2(n-1)a} \frac{x}{(ax^2 + c)^{n-1}} + \frac{1}{2(n-1)a} \int \frac{dx}{(ax^2 + c)^{n-1}}$$

$$107. \int \frac{dx}{x^2(ax^2 + c)^n} = \frac{1}{c} \int \frac{dx}{x^2(ax^2 + c)^{n-1}} - \frac{a}{c} \int \frac{dx}{(ax^2 + c)^n}$$

$$108. \int \sqrt{x^2 \pm p^2} dx = \frac{1}{2} [x \sqrt{x^2 \pm p^2} \pm p^2 \log(x + \sqrt{x^2 \pm p^2})]$$

$$109. \int \sqrt{p^2 - x^2} dx = \frac{1}{2} \left(x \sqrt{p^2 - x^2} + p^2 \sin^{-1} \frac{x}{p} \right)$$

$$110. \int \frac{dx}{\sqrt{x^2 \pm p^2}} = \log(x + \sqrt{x^2 \pm p^2})$$

$$111. \int \frac{dx}{\sqrt{p^2 - x^2}} = \sin^{-1} \frac{x}{p} \quad \text{or} \quad -\cos^{-1} \frac{x}{p}$$

$$112. \int \sqrt{ax^2 + c} dx = \frac{x}{2} \sqrt{ax^2 + c} + \frac{c}{2\sqrt{a}} \log(x\sqrt{a} + \sqrt{ax^2 + c}) \quad a > 0$$

$$113. \int \sqrt{ax^2 + c} dx = \frac{x}{2} \sqrt{ax^2 + c} + \frac{c}{2\sqrt{-a}} \sin^{-1} \left(x \sqrt{\frac{-a}{c}} \right) \quad a < 0$$

$$114. \int \frac{dx}{\sqrt{ax^2 + c}} = \frac{1}{\sqrt{a}} \log(x\sqrt{a} + \sqrt{ax^2 + c}) \quad a > 0$$

$$115. \int \frac{dx}{\sqrt{ax^2 + c}} = \frac{1}{\sqrt{-a}} \sin^{-1} \left(x \sqrt{\frac{-a}{c}} \right) \quad a < 0$$

$$116. \int x \sqrt{ax^2 + c} dx = \frac{1}{3a} (ax^2 + c)^{3/2}$$

$$117. \int x^2 \sqrt{ax^2 + c} dx = \frac{x}{4a} \sqrt{(ax^2 + c)^3} - \frac{cx}{8a} \sqrt{ax^2 + c} - \frac{c^2}{8\sqrt{a^3}} \log(x\sqrt{a} + \sqrt{ax^2 + c}) \quad a > 0$$

$$118. \int x^2 \sqrt{ax^2 + c} dx = \frac{x}{4a} \sqrt{(ax^2 + c)^3} - \frac{cx}{8a} \sqrt{ax^2 + c} - \frac{c^2}{8a\sqrt{-a}} \sin^{-1}\left(x\sqrt{\frac{-a}{c}}\right) \quad a < 0$$

$$119. \int \frac{x dx}{\sqrt{ax^2 + c}} = \frac{1}{a} \sqrt{ax^2 + c}$$

$$120. \int \frac{x^2 dx}{\sqrt{ax^2 + c}} = \frac{x}{a} \sqrt{ax^2 + c} - \frac{1}{a} \int \sqrt{ax^2 + c} dx$$

$$121. \int \frac{\sqrt{ax^2 + c}}{x} dx = \sqrt{ax^2 + c} + \sqrt{c} \log \frac{\sqrt{ax^2 + c} - \sqrt{c}}{x} \quad c > 0$$

$$122. \int \frac{\sqrt{ax^2 + c}}{x} dx = \sqrt{ax^2 + c} - \sqrt{-c} \tan^{-1} \frac{\sqrt{ax^2 + c}}{\sqrt{-c}} \quad c < 0$$

$$123. \int \frac{dx}{x\sqrt{p^2 \pm x^2}} = -\frac{1}{p} \log \frac{p + \sqrt{p^2 \pm x^2}}{x}$$

$$124. \int \frac{dx}{x\sqrt{x^2 - p^2}} = \frac{1}{p} \cos^{-1} \frac{p}{x} \quad \text{or} \quad -\frac{1}{p} \sin^{-1} \frac{p}{x}$$

$$125. \int \frac{dx}{x\sqrt{ax^2 + c}} = \frac{1}{\sqrt{c}} \log \frac{\sqrt{ax^2 + c} - \sqrt{c}}{x} \quad c > 0$$

$$126. \int \frac{dx}{x\sqrt{ax^2 + c}} = \frac{1}{\sqrt{-c}} \sec^{-1}\left(x\sqrt{\frac{-a}{c}}\right) \quad c < 0$$

$$127. \int \frac{dx}{x^2 \sqrt{ax^2 + c}} = -\frac{\sqrt{ax^2 + c}}{cx}$$

$$128. \int \frac{x^n dx}{\sqrt{ax^2 + c}} = \frac{x^{n-1} \sqrt{ax^2 + c}}{na} - \frac{(n-1)c}{na} \int \frac{x^{n-2} dx}{\sqrt{ax^2 + c}} \quad n > 0$$

$$129. \int x^n \sqrt{ax^2 + c} dx = \frac{x^{n-1} (ax^2 + c)^{3/2}}{(n+2)a} - \frac{(n-1)c}{(n+2)a} \int x^{n-2} \sqrt{ax^2 + c} dx \quad n > 0$$

$$130. \int \frac{\sqrt{ax^2 + c}}{x^n} dx = -\frac{(ax^2 + c)^{3/2}}{c(n-1)x^{n-1}} - \frac{(n-4)a}{(n-1)c} \int \frac{\sqrt{ax^2 + c}}{x^{n-2}} dx \quad n > 1$$

$$131. \int \frac{dx}{x^n \sqrt{ax^2 + c}} = -\frac{\sqrt{ax^2 + c}}{c(n-1)x^{n-1}} - \frac{(n-2)a}{(n-1)c} \int \frac{dx}{x^{n-2} \sqrt{ax^2 + c}} \quad n > 1$$

$$132. \int (ax^2 + c)^{3/2} dx = \frac{x}{8} (2ax^2 + 5c) \sqrt{ax^2 + c} + \frac{3c^2}{8\sqrt{a}} \log(x\sqrt{a} + \sqrt{ax^2 + c}) \quad a > 0$$

$$133. \int (ax^2 + c)^{3/2} dx = \frac{x}{8} (2ax^2 + 5c) \sqrt{ax^2 + c} + \frac{3c^2}{8\sqrt{-a}} \sin^{-1} \left(x \sqrt{\frac{-a}{c}} \right) \quad a < 0$$

$$134. \int \frac{dx}{(ax^2 + c)^{3/2}} = \frac{x}{c\sqrt{ax^2 + c}}$$

$$135. \int x(ax^2 + c)^{3/2} dx = \frac{1}{5a} (ax^2 + c)^{5/2}$$

$$136. \int x^2(ax^2 + c)^{3/2} dx = \frac{x^3}{6} (ax^2 + c)^{3/2} + \frac{c}{2} \int x^2 \sqrt{ax^2 + c} dx$$

$$137. \int x^n(ax^2 + c)^{3/2} dx \\ = \frac{x^{n+1}(ax^2 + c)^{3/2}}{n+4} + \frac{3c}{n+4} \int x^n \sqrt{ax^2 + c} dx$$

$$138. \int \frac{x dx}{(ax^2 + c)^{3/2}} = -\frac{1}{a\sqrt{ax^2 + c}}$$

$$139. \int \frac{x^2 dx}{(ax^2 + c)^{3/2}} = -\frac{x}{a\sqrt{ax^2 + c}} \\ + \frac{1}{a\sqrt{a}} \log(x\sqrt{a} + \sqrt{ax^2 + c}) \quad a > 0$$

$$140. \int \frac{x^2 dx}{(ax^2 + c)^{3/2}} = -\frac{x}{a\sqrt{ax^2 + c}} \\ + \frac{1}{a\sqrt{-a}} \sin^{-1}\left(x\sqrt{\frac{-a}{c}}\right) \quad a < 0$$

$$141. \int \frac{x^3 dx}{(ax^2 + c)^{3/2}} = -\frac{x^2}{a\sqrt{ax^2 + c}} + \frac{2}{a^2} \sqrt{ax^2 + c}$$

$$142. \int \frac{dx}{x(ax^n + c)} = \frac{1}{cn} \log \frac{x^n}{ax^n + c}$$

$$143. \int \frac{dx}{(ax^n + c)^m} = \frac{1}{c} \int \frac{dx}{(ax^n + c)^{m-1}} - \frac{a}{c} \int \frac{x^n dx}{(ax^n + c)^m}$$

$$144. \int \frac{dx}{x\sqrt{ax^n + c}} = \frac{1}{n\sqrt{c}} \log \frac{\sqrt{ax^n + c} - \sqrt{c}}{\sqrt{ax^n + c} + \sqrt{c}} \quad c > 0$$

$$145. \int \frac{dx}{x\sqrt{ax^n + c}} = \frac{2}{n\sqrt{-c}} \sec^{-1} \sqrt{\frac{-ax^n}{c}} \quad c < 0$$

$$146. \int x^{m-1}(ax^n + c)^p dx \\ = \frac{1}{m+np} \left[x^m(ax^n + c)^p + npc \int x^{m-1}(ax^n + c)^{p-1} dx \right] \\ = \frac{1}{cn(p+1)} \left[-x^m(ax^n + c)^{p+1} \right. \\ \left. + (m+np+n) \int x^{m-1}(ax^n + c)^{p+1} dx \right]$$

$$= \frac{1}{a(m+np)} \left[x^{m-n}(ax^n+c)^{p+1} - (m-n)c \int x^{m-n-1}(ax^n+c)^p dx \right]$$

$$= \frac{1}{mc} \left[x^m(ax^n+c)^{p+1} - (m+np+n)a \int x^{m+n-1}(ax^n+c)^p dx \right]$$

$$147. \int \frac{x^m dx}{(ax^n+c)^p} = \frac{1}{a} \int \frac{x^{m-n} dx}{(ax^n+c)^{p-1}} - \frac{c}{a} \int \frac{x^{m-n} dx}{(ax^n+c)^p}$$

$$148. \int \frac{dx}{x^m(ax^n+c)^p} = \frac{1}{c} \int \frac{dx}{x^m(ax^n+c)^{p-1}} - \frac{a}{c} \int \frac{dx}{x^{m-n}(ax^n+c)^p}$$

Expressions Containing (ax^2+bx+c)

$$149. \int \frac{dx}{ax^2+bx+c} = \frac{1}{\sqrt{b^2-4ac}} \log \frac{2ax+b-\sqrt{b^2-4ac}}{2ax+b+\sqrt{b^2-4ac}} \quad b^2 > 4ac$$

$$150. \int \frac{dx}{ax^2+bx+c} = \frac{2}{\sqrt{4ac-b^2}} \tan^{-1} \frac{2ax+b}{\sqrt{4ac-b^2}} \quad b^2 < 4ac$$

$$151. \int \frac{dx}{ax^2+bx+c} = \frac{2}{2ax+b} \quad b^2 = 4ac$$

$$152. \int \frac{dx}{(ax^2+bx+c)^{n+1}} = \frac{2ax+b}{n(4ac-b^2)(ax^2+bx+c)^n} + \frac{2(2n-1)a}{n(4ac-b^2)} \int \frac{dx}{(ax^2+bx+c)^n}$$

$$153. \int \frac{x dx}{ax^2+bx+c} = \frac{1}{2a} \log(ax^2+bx+c) - \frac{b}{2a} \int \frac{dx}{ax^2+bx+c}$$

$$154. \int \frac{x^2 dx}{ax^2+bx+c} = \frac{x}{a} - \frac{b}{2a^2} \log(ax^2+bx+c) + \frac{b^2-2ac}{2a^2} \int \frac{dx}{ax^2+bx+c}$$

$$155. \int \frac{x^n dx}{ax^2 + bx + c} = \frac{x^{n-1}}{(n-1)a} - \frac{c}{a} \int \frac{x^{n-2} dx}{ax^2 + bx + c} - \frac{b}{a} \int \frac{x^{n-1} dx}{ax^2 + bx + c}$$

$$156. \int \frac{x dx}{(ax^2 + bx + c)^{n+1}} = \frac{-(2c + bx)}{n(4ac - b^2)(ax^2 + bx + c)^n} - \frac{b(2n-1)}{n(4ac - b^2)} \int \frac{dx}{(ax^2 + bx + c)^n}$$

$$157. \int \frac{x^m dx}{(ax^2 + bx + c)^{n+1}} = -\frac{x^{m-1}}{a(2n-m+1)(ax^2 + bx + c)^n} - \frac{n-m+1}{2n-m+1} \frac{b}{a} \int \frac{x^{m-1} dx}{(ax^2 + bx + c)^{n+1}} + \frac{m-1}{2n-m+1} \frac{c}{a} \int \frac{x^{m-2} dx}{(ax^2 + bx + c)^{n+1}}$$

$$158. \int \frac{dx}{x(ax^2 + bx + c)} = \frac{1}{2c} \log \left(\frac{x^2}{ax^2 + bx + c} \right) - \frac{b}{2c} \int \frac{dx}{ax^2 + bx + c}$$

$$159. \int \frac{dx}{x^2(ax^2 + bx + c)} = \frac{b}{2c^2} \log \left(\frac{ax^2 + bx + c}{x^2} \right) - \frac{1}{cx} + \left(\frac{b^2}{2c^2} - \frac{a}{c} \right) \int \frac{dx}{ax^2 + bx + c}$$

$$160. \int \frac{dx}{x^m(ax^2 + bx + c)^{n+1}} = -\frac{1}{(m-1)cx^{m-1}(ax^2 + bx + c)^n} - \frac{n+m-1}{m-1} \frac{b}{c} \int \frac{dx}{x^{m-1}(ax^2 + bx + c)^{n+1}} - \frac{2n+m-1}{m-1} \frac{a}{c} \int \frac{dx}{x^{m-2}(ax^2 + bx + c)^{n+1}}$$

$$161. \int \frac{dx}{x(ax^2 + bx + c)^n} = \frac{1}{2c(n-1)(ax^2 + bx + c)^{n-1}} - \frac{b}{2c} \int \frac{dx}{(ax^2 + bx + c)^n} + \frac{1}{c} \int \frac{dx}{x(ax^2 + bx + c)^{n-1}}$$

$$162. \int \frac{dx}{\sqrt{ax^2 + bx + c}} = \frac{1}{\sqrt{a}} \log (2ax + b + 2\sqrt{a} \sqrt{ax^2 + bx + c}) \quad a > 0$$

$$163. \int \frac{dx}{\sqrt{ax^2 + bx + c}} = \frac{1}{\sqrt{-a}} \sin^{-1} \frac{-2ax - b}{\sqrt{b^2 - 4ac}} \quad a < 0$$

$$164. \int \frac{x dx}{\sqrt{ax^2 + bx + c}} = \frac{\sqrt{ax^2 + bx + c}}{a} - \frac{b}{2a} \int \frac{dx}{\sqrt{ax^2 + bx + c}}$$

$$165. \int \frac{x^n dx}{\sqrt{ax^2 + bx + c}} = \frac{x^{n-1}}{an} \sqrt{ax^2 + bx + c} - \frac{b(2n-1)}{2an} \int \frac{x^{n-1} dx}{\sqrt{ax^2 + bx + c}} - \frac{c(n-1)}{an} \int \frac{x^{n-2} dx}{\sqrt{ax^2 + bx + c}}$$

$$166. \int \sqrt{ax^2 + bx + c} dx = \frac{2ax + b}{4a} \sqrt{ax^2 + bx + c} + \frac{4ac - b^2}{8a} \int \frac{dx}{\sqrt{ax^2 + bx + c}}$$

$$167. \int x \sqrt{ax^2 + bx + c} dx = \frac{(ax^2 + bx + c)^{3/2}}{3a} - \frac{b}{2a} \int \sqrt{ax^2 + bx + c} dx$$

$$168. \int x^2 \sqrt{ax^2 + bx + c} dx = \left(x - \frac{5b}{6a}\right) \frac{(ax^2 + bx + c)^{3/2}}{4a} + \frac{5b^2 - 4ac}{16a^2} \int \sqrt{ax^2 + bx + c} dx$$

$$169. \int \frac{dx}{x \sqrt{ax^2 + bx + c}} = -\frac{1}{\sqrt{c}} \log \left(\frac{\sqrt{ax^2 + bx + c} + \sqrt{c}}{x} + \frac{b}{2\sqrt{c}} \right) \quad c > 0$$

$$170. \int \frac{dx}{x\sqrt{ax^2 + bx + c}} = \frac{1}{\sqrt{-c}} \sin^{-1} \frac{bx + 2c}{x\sqrt{b^2 - 4ac}} \quad c < 0$$

$$171. \int \frac{dx}{x\sqrt{ax^2 + bx}} = -\frac{2}{bx} \sqrt{ax^2 + bx} \quad c = 0$$

$$172. \int \frac{dx}{x^n \sqrt{ax^2 + bx + c}} = -\frac{\sqrt{ax^2 + bx + c}}{c(n-1)x^{n-1}} \\ + \frac{b(3-2n)}{2c(n-1)} \int \frac{dx}{x^{n-1} \sqrt{ax^2 + bx + c}} \\ + \frac{a(2-n)}{c(n-1)} \int \frac{dx}{x^{n-2} \sqrt{ax^2 + bx + c}}$$

$$173. \int \frac{dx}{(ax^2 + bx + c)^{3/2}} = \frac{2(2ax + b)}{(b^2 - 4ac)\sqrt{ax^2 + bx + c}} \quad b^2 \neq 4ac$$

$$174. \int \frac{dx}{(ax^2 + bx + c)^{3/2}} = -\frac{1}{2\sqrt{a^3}(x + b/2a)^2} \quad b^2 = 4ac$$

Miscellaneous Algebraic Expressions

$$175. \int \sqrt{2px - x^2} dx = \frac{1}{2} \left[(x-p)\sqrt{2px - x^2} + p^2 \sin^{-1} \frac{x-p}{p} \right]$$

$$176. \int \frac{dx}{\sqrt{2px - x^2}} = \cos^{-1} \frac{p-x}{p}$$

$$177. \int \frac{dx}{\sqrt{ax+b}\sqrt{cx+d}} = \frac{2}{\sqrt{-ac}} \tan^{-1} \sqrt{\frac{-c(ax+b)}{a(cx+d)}} \\ \text{or} \quad \frac{2}{\sqrt{ac}} \tanh^{-1} \sqrt{\frac{c(ax+b)}{a(cx+d)}}$$

$$178. \int \sqrt{ax+b}\sqrt{cx+d} dx \\ = \frac{(2acx + bc + ad)\sqrt{ax+b}\sqrt{cx+d}}{4ac} \\ - \frac{(ad-bc)^2}{8ac} \int \frac{dx}{\sqrt{ax+b}\sqrt{cx+d}}$$

$$179. \int \sqrt{\frac{cx+d}{ax+b}} dx = \frac{\sqrt{ax+b}\sqrt{cx+d}}{a} + \frac{ad-bc}{2a} \int \frac{dx}{\sqrt{ax+b}\sqrt{cx+d}}$$

$$180. \int \sqrt{\frac{x+b}{x+d}} dx = \sqrt{x+d}\sqrt{x+b} + (b-d) \log(\sqrt{x+d} + \sqrt{x+b})$$

$$181. \int \sqrt{\frac{1+x}{1-x}} dx = \sin^{-1} x - \sqrt{1-x^2}$$

$$182. \int \sqrt{\frac{p-x}{q+x}} dx = \sqrt{p-x}\sqrt{q+x} + (p+q) \sin^{-1} \sqrt{\frac{x+q}{p+q}}$$

$$183. \int \sqrt{\frac{p+x}{q-x}} dx = -\sqrt{p+x}\sqrt{q-x} - (p+q) \sin^{-1} \sqrt{\frac{q-x}{p+q}}$$

$$184. \int \frac{dx}{\sqrt{x-p}\sqrt{q-x}} = 2 \sin^{-1} \sqrt{\frac{x-p}{q-p}}$$

Expressions Containing sin ax

$$185. \int \sin u du = -\cos u \quad \text{where } u \text{ is any function of } x$$

$$186. \int \sin ax dx = -\frac{1}{a} \cos ax$$

$$187. \int \sin^2 ax dx = \frac{x}{2} - \frac{\sin 2ax}{4a}$$

$$188. \int \sin^3 ax dx = -\frac{1}{a} \cos ax + \frac{1}{3a} \cos^3 ax$$

$$189. \int \sin^4 ax dx = \frac{3x}{8} - \frac{3 \sin 2ax}{16a} - \frac{\sin^3 ax \cos ax}{4a}$$

$$190. \int \sin^n ax \, dx = -\frac{\sin^{n-1} ax \cos ax}{na} + \frac{n-1}{n} \int \sin^{n-2} ax \, dx$$

n positive integer

$$191. \int \frac{dx}{\sin ax} = \frac{1}{a} \log \tan \frac{ax}{2} = \frac{1}{a} \log (\csc ax - \cot ax)$$

$$192. \int \frac{dx}{\sin^2 ax} = \int \csc^2 ax \, dx = -\frac{1}{a} \cot ax$$

$$193. \int \frac{dx}{\sin^n ax} = -\frac{1}{a(n-1)} \frac{\cos ax}{\sin^{n-1} ax} + \frac{n-2}{n-1} \int \frac{dx}{\sin^{n-2} ax}$$

n integer > 1

$$194. \int \frac{dx}{1 \pm \sin ax} = \mp \frac{1}{a} \tan \left(\frac{\pi}{4} \mp \frac{ax}{2} \right)$$

$$195. \int \frac{dx}{b + c \sin ax} = \frac{-2}{a\sqrt{b^2 - c^2}} \tan^{-1} \left[\sqrt{\frac{b-c}{b+c}} \tan \left(\frac{\pi}{4} - \frac{ax}{2} \right) \right], \quad b^2 > c^2$$

$$196. \int \frac{dx}{b + c \sin ax} = \frac{-1}{a\sqrt{c^2 - b^2}} \log \frac{c + b \sin ax + \sqrt{c^2 - b^2} \cos ax}{b + c \sin ax}, \quad c^2 > b^2$$

$$197. \int \sin ax \sin bx \, dx = \frac{\sin (a-b)x}{2(a-b)} - \frac{\sin (a+b)x}{2(a+b)}, \quad a^2 \neq b^2$$

$$198. \int \sqrt{1 + \sin x} \, dx = \pm 2 \left(\sin \frac{x}{2} - \cos \frac{x}{2} \right); \text{ use } + \text{ sign}$$

when $(8k-1)\frac{\pi}{2} < x \leq (8k+3)\frac{\pi}{2}$, otherwise $-$, k an integer

$$199. \int \sqrt{1 - \sin x} \, dx = \pm 2 \left(\sin \frac{x}{2} + \cos \frac{x}{2} \right); \text{ use } + \text{ sign}$$

when $(8k-3)\frac{\pi}{2} < x \leq (8k+1)\frac{\pi}{2}$, otherwise $-$, k an integer

Expressions Involving $\cos ax$

$$200. \int \cos u \, du = \sin u \quad \text{where } u \text{ is any function of } x$$

$$201. \int \cos ax \, dx = \frac{1}{a} \sin ax$$

$$202. \int \cos^2 ax \, dx = \frac{x}{2} + \frac{\sin 2ax}{4a}$$

$$203. \int \cos^3 ax \, dx = \frac{1}{a} \sin ax - \frac{1}{3a} \sin^3 ax$$

$$204. \int \cos^4 ax \, dx = \frac{3x}{8} + \frac{3 \sin 2ax}{16a} + \frac{\cos^3 ax \sin ax}{4a}$$

$$205. \int \cos^n ax \, dx = \frac{\cos^{n-1} ax \sin ax}{na} + \frac{n-1}{n} \int \cos^{n-2} ax \, dx$$

$$206. \int \frac{dx}{\cos ax} = \frac{1}{a} \log \tan \left(\frac{ax}{2} + \frac{\pi}{4} \right) = \frac{1}{a} \log (\tan ax + \sec ax)$$

$$207. \int \frac{dx}{\cos^2 ax} = \frac{1}{a} \tan ax$$

$$208. \int \frac{dx}{\cos^n ax} = \frac{1}{a(n-1)} \frac{\sin ax}{\cos^{n-1} ax} + \frac{n-2}{n-1} \int \frac{dx}{\cos^{n-2} ax}$$

n integer > 1

$$209. \int \frac{dx}{1 + \cos ax} = \frac{1}{a} \tan \frac{ax}{2}$$

$$210. \int \frac{dx}{1 - \cos ax} = -\frac{1}{a} \cot \frac{ax}{2}$$

$$211. \int \sqrt{1 + \cos x} \, dx = \pm \sqrt{2} \int \cos \frac{x}{2} \, dx = \pm 2\sqrt{2} \sin \frac{x}{2}$$

Use $+$ when $(4k-1)\pi < x \leq (4k+1)\pi$, otherwise $-$, k an integer.

$$212. \int \sqrt{1 - \cos x} \, dx = \pm \sqrt{2} \int \sin \frac{x}{2} \, dx = \mp 2\sqrt{2} \cos \frac{x}{2}$$

Use top signs when $4k\pi < x \leq (4k+2)\pi$, otherwise bottom signs.

$$213. \int \frac{dx}{b + c \cos ax} = \frac{1}{a\sqrt{b^2 - c^2}} \tan^{-1} \frac{\sqrt{b^2 - c^2} \sin ax}{c + b \cos ax} \quad b^2 > c^2$$

$$214. \int \frac{dx}{b + c \cos ax} = \frac{1}{a\sqrt{c^2 - b^2}} \tanh^{-1} \frac{\sqrt{c^2 - b^2} \sin ax}{c + b \cos ax} \quad c^2 > b^2$$

$$215. \int \cos ax \cos bx \, dx = \frac{\sin (a - b)x}{2(a - b)} + \frac{\sin (a + b)x}{2(a + b)} \quad a^2 \neq b^2$$

Expressions Containing sin ax and cos ax

$$216. \int \sin ax \cos bx \, dx = -\frac{1}{2} \left[\frac{\cos (a - b)x}{a - b} + \frac{\cos (a + b)x}{a + b} \right] \quad a^2 \neq b^2$$

$$217. \int \sin^n ax \cos ax \, dx = \frac{1}{a(n + 1)} \sin^{n+1} ax \quad n \neq -1$$

$$218. \int \cos^n ax \sin ax \, dx = -\frac{1}{a(n + 1)} \cos^{n+1} ax \quad n \neq -1$$

$$219. \int \frac{\sin ax}{\cos ax} \, dx = -\frac{1}{a} \log \cos ax$$

$$220. \int \frac{\cos ax}{\sin ax} \, dx = \frac{1}{a} \log \sin ax$$

$$221. \int (b + c \sin ax)^n \cos ax \, dx = \frac{1}{ac(n + 1)} (b + c \sin ax)^{n+1} \quad n \neq -1$$

$$222. \int (b + c \cos ax)^n \sin ax \, dx$$

$$= -\frac{1}{ac(n+1)} (b + c \cos ax)^{n+1} \quad n \neq -1$$

$$223. \int \frac{\cos ax \, dx}{b + c \sin ax} = \frac{1}{ac} \log (b + c \sin ax)$$

$$224. \int \frac{\sin ax}{b + c \cos ax} \, dx = -\frac{1}{ac} \log (b + c \cos ax)$$

$$225. \int \frac{dx}{b \sin ax + c \cos ax}$$

$$= \frac{1}{a \sqrt{b^2 + c^2}} \left[\log \tan \frac{1}{2} \left(ax + \tan^{-1} \frac{c}{b} \right) \right]$$

$$226. \int \frac{dx}{b + c \cos ax + d \sin ax} = \frac{-1}{a \sqrt{b^2 - c^2 - d^2}} \sin^{-1} U$$

$$U \equiv \left[\frac{c^2 + d^2 + b(c \cos ax + d \sin ax)}{\sqrt{c^2 + d^2}(b + c \cos ax + d \sin ax)} \right]$$

$$\text{or} = \frac{1}{a \sqrt{c^2 + d^2 - b^2}} \log V$$

$$V \equiv \left[\frac{c^2 + d^2 + b(c \cos ax + d \sin ax) + \sqrt{c^2 + d^2 - b^2}(c \sin ax - d \cos ax)}{\sqrt{c^2 + d^2}(b + c \cos ax + d \sin ax)} \right]$$

$b^2 \neq c^2 + d^2, -\pi < ax < \pi$

$$227. \int \frac{dx}{b + c \cos ax + d \sin ax}$$

$$= \frac{1}{ab} \left[\frac{b - (c + d) \cos ax + (c - d) \sin ax}{b + (c - d) \cos ax + (c + d) \sin ax} \right], \quad b^2 = c^2 + d^2$$

$$228. \int \frac{\sin^2 ax \, dx}{b + c \cos^2 ax} = \frac{1}{ac} \sqrt{\frac{b+c}{b}} \tan^{-1} \left(\sqrt{\frac{b}{b+c}} \tan ax \right) - \frac{x}{c}$$

$$229. \int \frac{\sin ax \cos ax \, dx}{b \cos^2 ax + c \sin^2 ax} = \frac{1}{2a(c-b)} \log (b \cos^2 ax + c \sin^2 ax)$$

$$230. \int \frac{dx}{b^2 \cos^2 ax - c^2 \sin^2 ax} = \frac{1}{2abc} \log \frac{b \cos ax + c \sin ax}{b \cos ax - c \sin ax}$$

$$231. \int \frac{dx}{b^2 \cos^2 ax + c^2 \sin^2 ax} = \frac{1}{abc} \tan^{-1} \frac{c \tan ax}{b}$$

$$232. \int \sin^2 ax \cos^2 ax \, dx = \frac{x}{8} - \frac{\sin 4ax}{32a}$$

$$233. \int \frac{dx}{\sin ax \cos ax} = \frac{1}{a} \log \tan ax$$

$$234. \int \frac{dx}{\sin^2 ax \cos^2 ax} = \frac{1}{a} (\tan ax - \cot ax)$$

$$235. \int \frac{\sin^2 ax}{\cos ax} \, dx = \frac{1}{a} \left[-\sin ax + \log \tan \left(\frac{ax}{2} + \frac{\pi}{4} \right) \right]$$

$$236. \int \frac{\cos^2 ax}{\sin ax} \, dx = \frac{1}{a} \left(\cos ax + \log \tan \frac{ax}{2} \right)$$

$$237. \int \sin^m ax \cos^n ax \, dx = \frac{\sin^{m-1} ax \cos^{n+1} ax}{a(m+n)} \\ + \frac{m-1}{m+n} \int \sin^{m-2} ax \cos^n ax \, dx \quad m, n > 0$$

$$238. \int \sin^m ax \cos^n ax \, dx = \frac{\sin^{m+1} ax \cos^{n-1} ax}{a(m+n)} \\ + \frac{n-1}{m+n} \int \sin^m ax \cos^{n-2} ax \, dx \quad m, n > 0$$

$$239. \int \frac{\sin^m ax}{\cos^n ax} \, dx = \frac{\sin^{m+1} ax}{a(n-1) \cos^{n-1} ax} \\ - \frac{m-n+2}{n-1} \int \frac{\sin^m ax}{\cos^{n-2} ax} \, dx \quad m, n > 0, n \neq 1$$

$$240. \int \frac{\cos^n ax}{\sin^m ax} \, dx = \frac{-\cos^{n+1} ax}{a(m-1) \sin^{m-1} ax} \\ + \frac{m-n-2}{m-1} \int \frac{\cos^n ax}{\sin^{m-2} ax} \, dx \quad m, n > 0, m \neq 1$$

$$241. \int \frac{dx}{\sin^m ax \cos^n ax} = \frac{1}{a(n-1)} \frac{1}{\sin^{m-1} ax \cos^{n-1} ax} + \frac{m+n-2}{n-1} \int \frac{dx}{\sin^m ax \cos^{n-2} ax}$$

$$242. \int \frac{dx}{\sin^m ax \cos^n ax} = -\frac{1}{a(m-1)} \frac{1}{\sin^{m-1} ax \cos^{n-1} ax} + \frac{m+n-2}{m-1} \int \frac{dx}{\sin^{m-2} ax \cos^n ax}$$

$$243. \int \frac{\sin^{2n} ax}{\cos ax} dx = \int \frac{(1 - \cos^2 ax)^n}{\cos ax} dx$$

(Expand, divide, and use 205.)

$$244. \int \frac{\cos^{2n} ax}{\sin ax} dx = \int \frac{(1 - \sin^2 ax)^n}{\sin ax} dx$$

(Expand, divide, and use 190.)

$$245. \int \frac{\sin^{2n+1} ax}{\cos ax} dx = \int \frac{(1 - \cos^2 ax)^n}{\cos ax} \sin ax dx$$

(Expand, divide, and use 218.)

$$246. \int \frac{\cos^{2n+1} ax}{\sin ax} dx = \int \frac{(1 - \sin^2 ax)^n}{\sin ax} \cos ax dx$$

(Expand, divide, and use 217.)

Expressions Containing tan ax or cot ax (tan ax = 1/cot ax)

$$247. \int \tan u du = -\log \cos u \quad \text{or} \quad \log \sec u$$

where u is any function of x

$$248. \int \tan ax dx = -\frac{1}{a} \log \cos ax$$

$$249. \int \tan^2 ax dx = \frac{1}{a} \tan ax - x$$

$$250. \int \tan^3 ax dx = \frac{1}{2a} \tan^2 ax + \frac{1}{a} \log \cos ax$$

$$251. \int \tan^n ax \, dx = \frac{1}{a(n-1)} \tan^{n-1} ax - \int \tan^{n-2} ax \, dx$$

n integer > 1

$$252. \int \cot u \, du = \log \sin u \quad \text{or} \quad -\log \csc u$$

where u is any function of x

$$253. \int \cot^2 ax \, dx = \int \frac{dx}{\tan^2 ax} = -\frac{1}{a} \cot ax - x$$

$$254. \int \cot^3 ax \, dx = -\frac{1}{2a} \cot^2 ax - \frac{1}{a} \log \sin ax$$

$$255. \int \cot^n ax \, dx = \int \frac{dx}{\tan^n ax}$$

$$= \frac{1}{a(n-1)} \cot^{n-1} ax - \int \cot^{n-2} ax \, dx \quad n \text{ integer } > 1$$

$$256. \int \frac{dx}{b + c \tan ax} = \int \frac{\cot ax \, dx}{b \cot ax + c}$$

$$= \frac{1}{b^2 + c^2} \left[bx + \frac{c}{a} \log (b \cos ax + c \sin ax) \right]$$

$$257. \int \frac{dx}{b + c \cot ax} = \int \frac{\tan ax \, dx}{b \tan ax + c}$$

$$= \frac{1}{b^2 + c^2} \left[bx - \frac{c}{a} \log (c \cos ax + b \sin ax) \right]$$

$$258. \int \frac{dx}{\sqrt{b + c \tan^2 ax}} = \frac{1}{a\sqrt{b-c}} \sin^{-1} \left(\sqrt{\frac{b-c}{b}} \sin ax \right)$$

b positive, $b^2 > c^2$

Expressions Containing $\sec ax = 1/\cos ax$ or $\csc ax = 1/\sin ax$

$$259. \int \sec u \, du = \log (\sec u + \tan u) = \log \tan \left(\frac{u}{2} + \frac{\pi}{4} \right)$$

where u is any function of x

$$260. \int \sec ax \, dx = \frac{1}{a} \log \tan \left(\frac{ax}{2} + \frac{\pi}{4} \right)$$

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$$261. \int \sec^2 ax \, dx = \frac{1}{a} \tan ax$$

$$262. \int \sec^3 ax \, dx = \frac{1}{2a} \left[\tan ax \sec ax + \log \tan \left(\frac{ax}{2} + \frac{\pi}{4} \right) \right]$$

$$263. \int \sec^n ax \, dx = \frac{1}{a(n-1)} \frac{\sin ax}{\cos^{n-1} ax} \\ + \frac{n-2}{n-1} \int \sec^{n-2} ax \, dx \quad n \text{ integer } > 1$$

$$264. \int \csc u \, du = \log (\csc u - \cot u) = \log \tan \frac{u}{2}$$

where u is any function of x

$$265. \int \csc ax \, dx = \frac{1}{a} \log \tan \frac{ax}{2}$$

$$266. \int \csc^2 ax \, dx = -\frac{1}{a} \cot ax$$

$$267. \int \csc^3 ax \, dx = \frac{1}{2a} \left(-\cot ax \csc ax + \log \tan \frac{ax}{2} \right)$$

$$268. \int \csc^n ax \, dx = -\frac{1}{a(n-1)} \frac{\cos ax}{\sin^{n-1} ax} \\ + \frac{n-2}{n-1} \int \csc^{n-2} ax \, dx \quad n \text{ integer } > 1$$

Expressions Containing tan ax and sec ax or cot ax and csc ax

$$269. \int \tan u \sec u \, du = \sec u \quad \text{where } u \text{ is any function of } x$$

$$270. \int \tan ax \sec ax \, dx = \frac{1}{a} \sec ax$$

$$271. \int \tan^n ax \sec^2 ax \, dx = \frac{1}{a(n+1)} \tan^{n+1} ax \quad n \neq -1$$

$$272. \int \tan ax \sec^n ax \, dx = \frac{1}{an} \sec^n ax \quad n \neq 0$$

$$273. \int \cot u \csc u \, du = -\csc u \quad \text{where } u \text{ is any function of } x$$

$$274. \int \cot ax \csc ax \, dx = -\frac{1}{a} \csc ax$$

$$275. \int \cot^n ax \csc^2 ax \, dx = -\frac{1}{a(n+1)} \cot^{n+1} ax \quad n \neq -1$$

$$276. \int \cot ax \csc^n ax \, dx = -\frac{1}{an} \csc^n ax \quad n \neq 0$$

$$277. \int \frac{\csc^2 ax \, dx}{\cot ax} = -\frac{1}{a} \log \cot ax$$

Expressions Containing Algebraic and Trigonometric Functions

$$278. \int x \sin ax \, dx = \frac{1}{a^2} \sin ax - \frac{1}{a} x \cos ax$$

$$279. \int x^2 \sin ax \, dx = \frac{2x}{a^2} \sin ax + \frac{2}{a^3} \cos ax - \frac{x^2}{a} \cos ax$$

$$280. \int x^3 \sin ax \, dx \\ = \frac{3x^2}{a^2} \sin ax - \frac{6}{a^4} \sin ax - \frac{x^3}{a} \cos ax + \frac{6x}{a^3} \cos ax$$

$$281. \int x \sin^2 ax \, dx = \frac{x^2}{4} - \frac{x \sin 2ax}{4a} - \frac{\cos 2ax}{8a^2}$$

$$282. \int x^2 \sin^2 ax \, dx = \frac{x^3}{6} - \left(\frac{x^2}{4a} - \frac{1}{8a^3} \right) \sin 2ax - \frac{x \cos 2ax}{4a^2}$$

$$283. \int x^3 \sin^2 ax \, dx = \frac{x^4}{8} - \left(\frac{x^3}{4a} - \frac{3x}{8a^3} \right) \sin 2ax \\ - \left(\frac{3x^2}{8a^2} - \frac{3}{16a^4} \right) \cos 2ax$$

$$284. \int x \sin^3 ax \, dx = \frac{x \cos 3ax}{12a} - \frac{\sin 3ax}{36a^2} - \frac{3x \cos ax}{4a} + \frac{3 \sin ax}{4a^3}$$

$$285. \int x^n \sin ax \, dx = -\frac{1}{a} x^n \cos ax + \frac{n}{a} \int x^{n-1} \cos ax \, dx$$

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$$286. \int \frac{\sin ax \, dx}{x} = ax - \frac{(ax)^3}{3 \cdot 3!} + \frac{(ax)^5}{5 \cdot 5!} - \dots$$

$$287. \int \frac{\sin ax \, dx}{x^m} = \frac{-1}{m-1} \frac{\sin ax}{x^{m-1}} + \frac{a}{m-1} \int \frac{\cos ax \, dx}{x^{m-1}}$$

$$288. \int x \cos ax \, dx = \frac{1}{a^2} \cos ax + \frac{1}{a} x \sin ax$$

$$289. \int x^2 \cos ax \, dx = \frac{2x}{a^2} \cos ax - \frac{2}{a^3} \sin ax + \frac{x^2}{a} \sin ax$$

$$290. \int x^3 \cos ax \, dx = \frac{(3a^2x^2 - 6) \cos ax}{a^3} + \frac{(a^2x^2 - 6x) \sin ax}{a^2}$$

$$291. \int x \cos^2 ax \, dx = \frac{x^2}{4} + \frac{x \sin 2ax}{4a} + \frac{\cos 2ax}{8a^2}$$

$$292. \int x^2 \cos^2 ax \, dx = \frac{x^3}{6} + \left(\frac{x^2}{4a} - \frac{1}{8a^3} \right) \sin 2ax + \frac{x \cos 2ax}{4a^2}$$

$$293. \int x^3 \cos^2 ax \, dx = \frac{x^4}{8} + \left(\frac{x^3}{4a} - \frac{3x}{8a^3} \right) \sin 2ax \\ + \left(\frac{3x^2}{8a^2} - \frac{3}{16a^4} \right) \cos 2ax$$

$$294. \int x \cos^3 ax \, dx = \frac{x \sin 3ax}{12a} + \frac{\cos 3ax}{36a^2} + \frac{3x \sin ax}{4a} + \frac{3 \cos ax}{4a^2}$$

$$295. \int x^n \cos ax \, dx = \frac{1}{a} x^n \sin ax - \frac{n}{a} \int x^{n-1} \sin ax \, dx, \quad n \text{ positive}$$

$$296. \int \frac{\cos ax \, dx}{x} = \log ax - \frac{(ax)^2}{2 \cdot 2!} + \frac{(ax)^4}{4 \cdot 4!} - \dots$$

$$297. \int \frac{\cos ax \, dx}{x^m} = -\frac{1}{m-1} \frac{\cos ax}{x^{m-1}} - \frac{a}{m-1} \int \frac{\sin ax \, dx}{x^{m-1}}$$

Expressions Containing Exponential and Logarithmic Functions

$$298. \int e^u \, du = e^u \quad \text{where } u \text{ is any function of } x$$

$$299. \int b^u du = \frac{b^u}{\log b} \quad \text{where } u \text{ is any function of } x$$

$$300. \int e^{ax} dx = \frac{1}{a} e^{ax} \quad \int b^{ax} dx = \frac{b^{ax}}{a \log b}$$

$$301. \int x e^{ax} dx = \frac{e^{ax}}{a^2}(ax - 1), \quad \int x b^{ax} dx = \frac{x b^{ax}}{a \log b} - \frac{b^{ax}}{a^2(\log b)^2}$$

$$302. \int x^2 e^{ax} dx = \frac{e^{ax}}{a^3}(a^2 x^2 - 2ax + 2)$$

$$303. \int x^n e^{ax} dx = \frac{1}{a} x^n e^{ax} - \frac{n}{a} \int x^{n-1} e^{ax} dx \quad n \text{ positive}$$

$$304. \int x^n e^{ax} dx = \frac{e^{ax}}{a^{n+1}} \left[(ax)^n - n(ax)^{n-1} + n(n-1)(ax)^{n-2} \right. \\ \left. - \dots + (-1)^n n! \right] \quad n \text{ positive integer}$$

$$305. \int x^n e^{-ax} dx = -\frac{e^{-ax}}{a^{n+1}} \left[(ax)^n + n(ax)^{n-1} + n(n-1)(ax)^{n-2} \right. \\ \left. + \dots + n! \right] \quad n \text{ positive integer.}$$

$$306. \int x^n b^{ax} dx = \frac{x^n b^{ax}}{a \log b} - \frac{n}{a \log b} \int x^{n-1} b^{ax} dx \quad n \text{ positive}$$

$$307. \int \frac{e^{ax}}{x} dx = \log x + ax + \frac{(ax)^2}{2 \cdot 2!} + \frac{(ax)^3}{3 \cdot 3!} + \dots$$

$$308. \int \frac{e^{ax}}{x^n} dx = \frac{1}{n-1} \left(-\frac{e^{ax}}{x^{n-1}} + a \int \frac{e^{ax}}{x^{n-1}} dx \right), \quad n \text{ integ. } > 1$$

$$309. \int \frac{dx}{b + ce^{ax}} = \frac{1}{ab} [ax - \log(b + ce^{ax})]$$

$$310. \int \frac{e^{ax} dx}{b + ce^{ax}} = \frac{1}{ac} \log(b + ce^{ax})$$

$$311. \int \frac{dx}{be^{ax} + ce^{-ax}} = \frac{1}{a\sqrt{bc}} \tan^{-1} \left(e^{ax} \sqrt{\frac{b}{c}} \right), \quad b \text{ and } c \text{ positive}$$

$$312. \int e^{ax} \sin bx dx = \frac{e^{ax}}{a^2 + b^2} (a \sin bx - b \cos bx)$$

$$\begin{aligned}
 313. \int e^{ax} \sin bx \sin cx \, dx \\
 &= \frac{e^{ax}[(b-c) \sin (b-c)x + a \cos (b-c)x]}{2[a^2 + (b-c)^2]} \\
 &\quad - \frac{e^{ax}[(b+c) \sin (b+c)x + a \cos (b+c)x]}{2[a^2 + (b+c)^2]}
 \end{aligned}$$

$$314. \int e^{ax} \cos bx \, dx = \frac{e^{ax}}{a^2 + b^2} (a \cos bx + b \sin bx)$$

$$\begin{aligned}
 315. \int e^{ax} \cos bx \cos cx \, dx \\
 &= \frac{e^{ax}[(b-c) \sin (b-c)x + a \cos (b-c)x]}{2[a^2 + (b-c)^2]} \\
 &\quad + \frac{e^{ax}[(b+c) \sin (b+c)x + a \cos (b+c)x]}{2[a^2 + (b+c)^2]}
 \end{aligned}$$

$$\begin{aligned}
 316. \int e^{ax} \sin bx \cos cx \, dx \\
 &= \frac{e^{ax}[a \sin (b-c)x - (b-c) \cos (b-c)x]}{2[a^2 + (b-c)^2]} \\
 &\quad + \frac{e^{ax}[a \sin (b+c)x - (b+c) \cos (b+c)x]}{2[a^2 + (b+c)^2]}
 \end{aligned}$$

$$\begin{aligned}
 317. \int e^{ax} \sin bx \sin (bx+c) \, dx \\
 &= \frac{e^{ax} \cos c}{2a} - \frac{e^{ax}[a \cos (2bx+c) + 2b \sin (2bx+c)]}{2(a^2 + 4b^2)}
 \end{aligned}$$

$$\begin{aligned}
 318. \int e^{ax} \cos bx \cos (bx+c) \, dx \\
 &= \frac{e^{ax} \cos c}{2a} + \frac{e^{ax}[a \cos (2bx+c) + 2b \sin (2bx+c)]}{2(a^2 + 4b^2)}
 \end{aligned}$$

$$\begin{aligned}
 319. \int e^{ax} \sin bx \cos (bx+c) \, dx \\
 &= \frac{e^{ax} \sin c}{2a} + \frac{e^{ax}[a \sin (2bx+c) - 2b \cos (2bx+c)]}{2(a^2 + 4b^2)}
 \end{aligned}$$

$$320. \int e^{ax} \cos bx \sin (bx + c) dx = \frac{e^{ax} \sin c}{2a} + \frac{e^{ax} [a \sin (2bx + c) - 2b \cos (2bx + c)]}{2(a^2 + 4b^2)}$$

$$321. \int x e^{ax} \sin bx dx = \frac{x e^{ax}}{a^2 + b^2} (a \sin bx - b \cos bx) - \frac{e^{ax}}{(a^2 + b^2)^2} [(a^2 - b^2) \sin bx - 2ab \cos bx]$$

$$322. \int x e^{ax} \cos bx dx = \frac{x e^{ax}}{a^2 + b^2} (a \cos bx + b \sin bx) - \frac{e^{ax}}{(a^2 + b^2)^2} [(a^2 - b^2) \cos bx + 2ab \sin bx]$$

$$323. \int e^{ax} \cos^n bx dx = \frac{e^{ax} \cos^{n-1} bx (a \cos bx + nb \sin bx)}{a^2 + n^2 b^2} + \frac{n(n-1)b^2}{a^2 + n^2 b^2} \int e^{ax} \cos^{n-2} bx dx$$

$$324. \int e^{ax} \sin^n bx dx = \frac{e^{ax} \sin^{n-1} bx (a \sin bx - nb \cos bx)}{a^2 + n^2 b^2} + \frac{n(n-1)b^2}{a^2 + n^2 b^2} \int e^{ax} \sin^{n-2} bx dx$$

$$325. \int \log ax dx = x \log ax - x$$

$$326. \int x \log ax dx = \frac{x^2}{2} \log ax - \frac{x^2}{4}$$

$$327. \int x^2 \log ax dx = \frac{x^3}{3} \log ax - \frac{x^3}{9}$$

$$328. \int (\log ax)^2 dx = x(\log ax)^2 - 2x \log ax + 2x$$

$$329. \int (\log ax)^n dx = x(\log ax)^n - n \int (\log ax)^{n-1} dx \quad n \text{ pos.}$$

$$330. \int x^n \log ax \, dx = x^{n+1} \left[\frac{\log ax}{n+1} - \frac{1}{(n+1)^2} \right] \quad n \neq -1$$

$$331. \int x^n (\log ax)^m \, dx = \frac{x^{n+1}}{n+1} (\log ax)^m - \frac{m}{n+1} \int x^n (\log ax)^{m-1} \, dx$$

$$332. \int \frac{(\log ax)^n}{x} \, dx = \frac{(\log ax)^{n+1}}{n+1} \quad n \neq -1$$

$$333. \int \frac{dx}{x \log ax} = \log (\log ax)$$

$$334. \int \frac{dx}{x(\log ax)^n} = -\frac{1}{(n-1)(\log ax)^{n-1}}$$

$$335. \int \frac{x^n dx}{(\log ax)^m} = \frac{-x^{n+1}}{(m-1)(\log ax)^{m-1}} + \frac{n+1}{m-1} \int \frac{x^n dx}{(\log ax)^{m-1}} \quad m \neq 1$$

$$336. \int \frac{x^n dx}{\log ax} = \frac{1}{a^{n+1}} \int \frac{e^y dy}{y} \quad y = (n+1) \log ax$$

$$337. \int \frac{x^n dx}{\log ax} = \frac{1}{a^{n+1}} \left[\log |\log ax| + (n+1) \log ax + \frac{(n+1)^2 (\log ax)^2}{2 \cdot 2!} + \frac{(n+1)^3 (\log ax)^3}{3 \cdot 3!} + \dots \right]$$

$$338. \int \frac{dx}{\log ax} = \frac{1}{a} \left[\log |\log ax| + \log ax + \frac{(\log ax)^2}{2 \cdot 2!} + \frac{(\log ax)^3}{3 \cdot 3!} + \dots \right]$$

$$339. \int \sin (\log ax) \, dx = \frac{x}{2} [\sin (\log ax) - \cos (\log ax)]$$

$$340. \int \cos (\log ax) \, dx = \frac{x}{2} [\sin (\log ax) + \cos (\log ax)]$$

$$341. \int e^{ax} \log bx \, dx = \frac{1}{a} e^{ax} \log bx - \frac{1}{a} \int \frac{e^{ax}}{x} \, dx$$

Expressions Containing Inverse Trigonometric Functions

$$342. \int \sin^{-1} ax \, dx = x \sin^{-1} ax + \frac{1}{a} \sqrt{1 - a^2 x^2}$$

$$343. \int (\sin^{-1} ax)^2 \, dx = x (\sin^{-1} ax)^2 - 2x + \frac{2}{a} \sqrt{1 - a^2 x^2} \sin^{-1} ax$$

$$344. \int x \sin^{-1} ax \, dx = \frac{x^2}{2} \sin^{-1} ax - \frac{1}{4a^2} \sin^{-1} ax + \frac{x}{4a} \sqrt{1 - a^2 x^2}$$

$$345. \int x^n \sin^{-1} ax \, dx = \frac{x^{n+1}}{n+1} \sin^{-1} ax - \frac{a}{n+1} \int \frac{x^{n+1} \, dx}{\sqrt{1 - a^2 x^2}} \quad n \neq -1$$

$$346. \int \frac{\sin^{-1} ax \, dx}{x} = ax + \frac{1}{2 \cdot 3 \cdot 3} (ax)^3 + \frac{1 \cdot 3}{2 \cdot 4 \cdot 5 \cdot 5} (ax)^5 + \frac{1 \cdot 3 \cdot 5}{2 \cdot 4 \cdot 6 \cdot 7 \cdot 7} (ax)^7 + \dots \quad a^2 x^2 < 1$$

$$347. \int \frac{\sin^{-1} ax \, dx}{x^2} = -\frac{1}{x} \sin^{-1} ax - a \log \left| \frac{1 + \sqrt{1 - a^2 x^2}}{ax} \right|$$

$$348. \int \cos^{-1} ax \, dx = x \cos^{-1} ax - \frac{1}{a} \sqrt{1 - a^2 x^2}$$

$$349. \int (\cos^{-1} ax)^2 \, dx = x (\cos^{-1} ax)^2 - 2x - \frac{2}{a} \sqrt{1 - a^2 x^2} \cos^{-1} ax$$

$$350. \int x \cos^{-1} ax \, dx = \frac{x^2}{2} \cos^{-1} ax - \frac{1}{4a^2} \cos^{-1} ax - \frac{x}{4a} \sqrt{1 - a^2 x^2}$$

$$351. \int x^n \cos^{-1} ax \, dx = \frac{x^{n+1}}{n+1} \cos^{-1} ax + \frac{a}{n+1} \int \frac{x^{n+1} \, dx}{\sqrt{1 - a^2 x^2}} \quad n \neq -1$$

$$352. \int \frac{\cos^{-1} ax \, dx}{x} = \frac{\pi}{2} \log |ax| - ax - \frac{1}{2 \cdot 3 \cdot 3} (ax)^3 \\ - \frac{1 \cdot 3}{2 \cdot 4 \cdot 5 \cdot 5} (ax)^5 - \frac{1 \cdot 3 \cdot 5}{2 \cdot 4 \cdot 6 \cdot 7 \cdot 7} (ax)^7 - \dots \\ a^2 x^2 < 1$$

$$353. \int \frac{\cos^{-1} ax \, dx}{x^2} = -\frac{1}{x} \cos^{-1} ax + a \log \left| \frac{1 + \sqrt{1 - a^2 x^2}}{ax} \right|$$

$$354. \int \tan^{-1} ax \, dx = x \tan^{-1} ax - \frac{1}{2a} \log (1 + a^2 x^2)$$

$$355. \int x^n \tan^{-1} ax \, dx = \frac{x^{n+1}}{n+1} \tan^{-1} ax - \frac{a}{n+1} \int \frac{x^{n+1} \, dx}{1 + a^2 x^2} \\ n \neq -1$$

$$356. \int \frac{\tan^{-1} ax \, dx}{x^2} = -\frac{1}{x} \tan^{-1} ax - \frac{a}{2} \log \frac{1 + a^2 x^2}{a^2 x^2}$$

$$357. \int \cot^{-1} ax \, dx = x \cot^{-1} ax + \frac{1}{2a} \log (1 + a^2 x^2)$$

$$358. \int x^n \cot^{-1} ax \, dx = \frac{x^{n+1}}{n+1} \cot^{-1} ax + \frac{a}{n+1} \int \frac{x^{n+1} \, dx}{1 + a^2 x^2} \\ n \neq -1$$

$$359. \int \frac{\cot^{-1} ax \, dx}{x^2} = -\frac{1}{x} \cot^{-1} ax + \frac{a}{2} \log \frac{1 + a^2 x^2}{a^2 x^2}$$

$$360. \int \sec^{-1} ax \, dx = x \sec^{-1} ax - \frac{1}{a} \log (ax + \sqrt{a^2 x^2 - 1})$$

$$361. \int x^n \sec^{-1} ax \, dx = \frac{x^{n+1}}{n+1} \sec^{-1} ax \pm \frac{1}{n+1} \int \frac{x^n \, dx}{\sqrt{a^2 x^2 - 1}} \\ n \neq -1$$

Use + sign when $\frac{\pi}{2} < \sec^{-1} ax < \pi$;

- sign when $0 < \sec^{-1} ax < \frac{\pi}{2}$.

$$362. \int \csc^{-1} ax \, dx = x \csc^{-1} ax + \frac{1}{a} \log (ax + \sqrt{a^2 x^2 - 1})$$

$$363. \int x^n \csc^{-1} ax \, dx = \frac{x^{n+1}}{n+1} \csc^{-1} ax \pm \frac{1}{n+1} \int \frac{x^n \, dx}{\sqrt{a^2 x^2 - 1}} \quad n \neq -1$$

Use + sign when $0 < \csc^{-1} ax < \frac{\pi}{2}$;

– sign when $-\frac{\pi}{2} < \csc^{-1} ax < 0$.

Remarks on Use of Integral Tables

Many of the integrals apply to both real and complex variables. Often the relations to be integrated may be multiple-valued and have several branches, discontinuities, multiple inverses, etc.; and questions arise concerning the proper signs to use. A graph of the relations concerned helps. As a general rule integration should not be carried out across a point of discontinuity or from a point on one branch of a relation to a point on another branch. [For example, $\int (1/x) \, dx$ should not be integrated from a negative value of x to a positive x , since the interval of integration includes $x = 0$, where $1/x$ is undefined.]

Definite Integrals

$$364. \int_0^\infty \frac{a \, dx}{a^2 + x^2} = \begin{cases} \frac{\pi}{2} & a > 0 \\ 0 & a = 0 \\ -\frac{\pi}{2} & a < 0 \end{cases}$$

$$365. \int_0^\infty x^{n-1} e^{-x} \, dx = \int_0^1 \left(\log_e \frac{1}{x} \right)^{n-1} dx = \Gamma(n)$$

$$\begin{aligned} \Gamma(n+1) &= \begin{cases} n \Gamma(n), & \text{if } n > 0 \\ n!, & \text{if } n \text{ is an integer} \end{cases} & \Gamma(2) &= \Gamma(1) = 1 \\ \Gamma(n) &= \Pi(n-1) & \Gamma(\frac{1}{2}) &= \sqrt{\pi} \\ Z(1) &= -0.5772157 \dots & Z(y) &= D_y [\log_e \Gamma(y)] \\ & & & \text{(see integral 418)} \end{aligned}$$

$$366. \int_0^\infty e^{-zx} z^n x^{n-1} \, dx = \Gamma(n) \quad z > 0$$

$$367. \int_0^1 x^{m-1}(1-x)^{n-1} dx = \int_0^\infty \frac{x^{m-1} dx}{(1+x)^{m+n}} = \frac{\Gamma(m)\Gamma(n)}{\Gamma(m+n)}$$

$$368. \int_0^\infty \frac{x^{n-1}}{1+x} dx = \frac{\pi}{\sin n\pi} \quad 0 < n < 1$$

$$369. \int_0^{\pi/2} \sin^n x dx = \begin{cases} \int_0^{\pi/2} \cos^n x dx & \\ \frac{1}{2} \sqrt{\pi} \frac{\Gamma\left(\frac{n}{2} + \frac{1}{2}\right)}{\Gamma\left(\frac{n}{2} + 1\right)} & \text{if } n > -1 \\ \frac{1 \cdot 3 \cdot 5 \cdots (n-1) \pi}{2 \cdot 4 \cdot 6 \cdots (n) \cdot 2} & \text{if } n \text{ is an even integer} \\ \frac{2 \cdot 4 \cdot 6 \cdots (n-1)}{1 \cdot 3 \cdot 5 \cdot 7 \cdots n} & \text{if } n \text{ is an odd integer} \end{cases}$$

$$370. \int_0^\infty \frac{\sin^2 x}{x^2} dx = \frac{\pi}{2}$$

$$371. \int_0^\infty \frac{\sin ax}{x} dx = \frac{\pi}{2} \quad \text{if } a > 0$$

$$372. \int_0^\infty \frac{\sin x \cos ax}{x} dx = \begin{cases} 0 & \text{if } a < -1 \text{ or } a > 1 \\ \frac{\pi}{4} & \text{if } a = -1 \text{ or } a = 1 \\ \frac{\pi}{2} & \text{if } -1 < a < 1 \end{cases}$$

$$373. \int_0^\pi \sin^2 ax dx = \int_0^\pi \cos^2 ax dx = \frac{\pi}{2}$$

$$374. \int_0^{\pi/a} \sin ax \cos ax dx = \int_0^\pi \sin ax \cos ax dx = 0$$

$$375. \int_0^\pi \sin ax \sin bx dx = \int_0^\pi \cos ax \cos bx dx = 0 \quad a \neq b$$

$$376. \int_0^\pi \sin ax \cos bx \, dx = \begin{cases} \frac{2a}{a^2 - b^2} & \text{if } a - b \text{ is odd} \\ 0 & \text{if } a - b \text{ is even} \end{cases}$$

$$377. \int_0^\infty \frac{\sin ax \sin bx}{x^2} \, dx = \frac{1}{2}\pi a \quad a < b$$

$$378. \int_0^\infty \cos(x^2) \, dx = \int_0^\infty \sin(x^2) \, dx = \frac{1}{2}\sqrt{\frac{\pi}{2}}$$

$$379. \int_0^\infty e^{-a^2x^2} \, dx = \frac{\sqrt{\pi}}{2a} = \frac{1}{2a} \Gamma\left(\frac{1}{2}\right) \quad a > 0$$

$$380. \int_0^\infty x^n e^{-ax} \, dx = \frac{\Gamma(n+1)}{a^{n+1}} \\ = \frac{n!}{a^{n+1}} \quad \text{if } n \text{ is a positive integer, } a > 0$$

$$381. \int_0^\infty x^{2n} e^{-ax^2} \, dx = \frac{1 \cdot 3 \cdot 5 \cdots (2n-1)}{2^{n+1} a^n} \sqrt{\frac{\pi}{a}}$$

$$382. \int_0^\infty \sqrt{x} e^{-ax} \, dx = \frac{1}{2a} \sqrt{\frac{\pi}{a}}$$

$$383. \int_0^\infty \frac{e^{-ax}}{\sqrt{x}} \, dx = \sqrt{\frac{\pi}{a}}$$

$$384. \int_0^\infty e^{(-x^2 - a^2/x^2)} \, dx = \frac{1}{2} e^{-2a} \sqrt{\pi} \quad a > 0$$

$$385. \int_0^\infty e^{-ax} \cos bx \, dx = \frac{a}{a^2 + b^2} \quad a > 0$$

$$386. \int_0^\infty e^{-ax} \sin bx \, dx = \frac{b}{a^2 + b^2} \quad a > 0$$

$$387. \int_0^\infty \frac{e^{-ax} \sin x}{x} \, dx = \cot^{-1} a \quad a > 0$$

$$388. \int_0^\infty e^{-a^2x^2} \cos bx \, dx = \frac{\sqrt{\pi} e^{-b^2/4a^2}}{2a} \quad a > 0$$

$$389. \int_0^1 (\log x)^n \, dx = (-1)^n n! \quad n \text{ positive integer}$$

$$390. \int_0^1 \frac{\log x}{1-x} dx = -\frac{\pi^2}{6}$$

$$391. \int_0^1 \frac{\log x}{1+x} dx = -\frac{\pi^2}{12}$$

$$392. \int_0^1 \frac{\log x}{1-x^2} dx = -\frac{\pi^2}{8}$$

$$393. \int_0^1 \frac{\log x}{\sqrt{1-x^2}} dx = -\frac{\pi}{2} \log 2$$

$$394. \int_0^1 \log \left(\frac{1+x}{1-x} \right) \frac{dx}{x} = \frac{\pi^2}{4}$$

$$395. \int_0^\infty \log \left(\frac{e^x+1}{e^x-1} \right) dx = \frac{\pi^2}{4}$$

$$396. \int_0^1 \frac{dx}{\sqrt{\log(1/x)}} = \sqrt{\pi}$$

$$397. \int_0^1 \log |\log x| dx \\ = \int_0^\infty e^{-x} \log x dx = -\gamma = -0.577\ 2157 \dots$$

$$398. \int_0^{\pi/2} \log \sin x dx = \int_0^{\pi/2} \log \cos x dx = -\frac{\pi}{2} \log_e 2$$

$$399. \int_0^\pi x \log \sin x dx = -\frac{\pi^2}{2} \log_e 2$$

$$400. \int_0^1 \left(\log \frac{1}{x} \right)^{1/2} dx = \frac{\sqrt{\pi}}{2}$$

$$401. \int_0^1 \left(\log \frac{1}{x} \right)^{-1/2} dx = \sqrt{\pi}$$

$$402. \int_0^1 x^m \left(\log \frac{1}{x} \right)^n dx = \frac{\Gamma(n+1)}{(m+1)^{n+1}}, \quad m+1 > 0, n+1 > 0$$

$$403. \int_0^\pi \log(a \pm b \cos x) dx = \pi \log \left(\frac{a + \sqrt{a^2 - b^2}}{2} \right), \quad a \geq b$$

404. $\int_0^\pi \frac{\log(1 + \sin a \cos x)}{\cos x} dx = \pi a$
405. $\int_0^1 \frac{x^b - x^a}{\log x} dx = \log \frac{1+b}{1+a}$
406. $\int_0^\pi \frac{dx}{a + b \cos x} = \frac{\pi}{\sqrt{a^2 - b^2}} \quad a > b > 0$
407. $\int_0^{\pi/2} \frac{dx}{a + b \cos x} = \frac{\cos^{-1} \frac{b}{a}}{\sqrt{a^2 - b^2}} \quad a > b$
408. $\int_0^\infty \frac{\cos ax \, dx}{1 + x^2} = \begin{cases} \frac{\pi}{2} e^{-a} & a > 0 \\ \frac{\pi}{2} e^a & a < 0 \end{cases}$
409. $\int_0^\infty \frac{\cos x \, dx}{\sqrt{x}} = \int_0^\infty \frac{\sin x \, dx}{\sqrt{x}} = \sqrt{\frac{\pi}{2}}$
410. $\int_0^\infty \frac{e^{-ax} - e^{-bx}}{x} dx = \log \frac{b}{a}$
411. $\int_0^\infty \frac{\tan^{-1} ax - \tan^{-1} bx}{x} dx = \frac{\pi}{2} \log \frac{a}{b}$
412. $\int_0^\infty \frac{\cos ax - \cos bx}{x} dx = \log \frac{b}{a}$
413. $\int_0^{\pi/2} \frac{dx}{a^2 \cos^2 x + b^2 \sin^2 x} = \frac{\pi}{2ab}$
414. $\int_0^{\pi/2} \frac{dx}{(a^2 \cos^2 x + b^2 \sin^2 x)^2} = \frac{\pi(a^2 + b^2)}{4a^3b^3}$
415. $\int_0^\pi \frac{(a - b \cos x) \, dx}{a^2 - 2ab \cos x + b^2} = \begin{cases} 0 & a^2 < b^2 \\ \frac{\pi}{a} & a^2 > b^2 \\ \frac{\pi}{2a} & a = b \end{cases}$

$$416. \int_0^1 \frac{1+x^2}{1+x^4} dx = \frac{\pi}{4} \sqrt{2}$$

$$417. \int_0^1 \frac{\log(1+x)}{x} dx = \frac{1}{1^2} - \frac{1}{2^2} + \frac{1}{3^2} - \frac{1}{4^2} + \dots = \frac{\pi^2}{12}$$

$$418. \int_{+\infty}^1 \frac{e^{-xu}}{u} du = \gamma + \log x - x + \frac{x^2}{2 \cdot 2!} - \frac{x^3}{3 \cdot 3!} + \frac{x^4}{4 \cdot 4!} - \dots$$

$$\text{where } \gamma = \lim_{t \rightarrow \infty} \left(1 + \frac{1}{2} + \frac{1}{3} + \dots + \frac{1}{t} - \log t \right) = 0.5772157 \dots \quad 0 < x < \infty$$

$$419. \int_{+\infty}^1 \frac{\cos xu}{u} du = \gamma + \log x - \frac{x^2}{2 \cdot 2!} + \frac{x^4}{4 \cdot 4!} - \frac{x^6}{6 \cdot 6!} + \dots$$

where $\gamma = 0.5772157 \dots \quad 0 < x < \infty$

$$420. \int_0^1 \frac{e^{xu} - e^{-xu}}{u} du = 2 \left(x + \frac{x^3}{3 \cdot 3!} + \frac{x^5}{5 \cdot 5!} + \dots \right)$$

$0 < x < \infty$

$$421. \int_0^1 \frac{1 - e^{-xu}}{u} du = x - \frac{x^2}{2 \cdot 2!} + \frac{x^3}{3 \cdot 3!} - \frac{x^4}{4 \cdot 4!} + \dots$$

$0 < x < \infty$

$$422. \int_0^{\pi/2} \frac{dx}{\sqrt{1 - K^2 \sin^2 x}} = \frac{\pi}{2} \left[1 + \left(\frac{1}{2} \right)^2 K^2 + \left(\frac{1 \cdot 3}{2 \cdot 4} \right)^2 K^4 + \left(\frac{1 \cdot 3 \cdot 5}{2 \cdot 4 \cdot 6} \right)^2 K^6 + \dots \right] \quad K^2 < 1$$

$$423. \int_0^{\pi/2} \sqrt{1 - K^2 \sin^2 x} dx = \frac{\pi}{2} \left[1 - \left(\frac{1}{2} \right)^2 K^2 - \left(\frac{1 \cdot 3}{2 \cdot 4} \right)^2 \frac{K^4}{3} - \left(\frac{1 \cdot 3 \cdot 5}{2 \cdot 4 \cdot 6} \right)^2 \frac{K^6}{5} - \dots \right] \quad K^2 < 1$$

$$424. f(x) = \frac{1}{2}a_0 + a_1 \cos \frac{\pi x}{c} + a_2 \cos \frac{2\pi x}{c} + \dots \\ + b_1 \sin \frac{\pi x}{c} + b_2 \sin \frac{2\pi x}{c} + \dots \quad -c < x < +c$$

$$\text{where } a_m = \frac{1}{c} \int_{-c}^{+c} f(x) \cos \frac{m\pi x}{c} dx$$

$$b_m = \frac{1}{c} \int_{-c}^{+c} f(x) \sin \frac{m\pi x}{c} dx \quad (\text{Fourier series})$$

$$425. \int_0^{\infty} e^{-ax} \cosh bx dx = \frac{a}{a^2 - b^2} \quad 0 \leq |b| < a$$

$$426. \int_0^{\infty} e^{-ax} \sinh bx dx = \frac{b}{a^2 - b^2} \quad 0 \leq |b| < a$$

$$427. \int_0^{\infty} xe^{-ax} \sin bx dx = \frac{2ab}{(a^2 + b^2)^2} \quad a > 0$$

$$428. \int_0^{\infty} xe^{-ax} \cos bx dx = \frac{a^2 - b^2}{(a^2 + b^2)^2} \quad a > 0$$

$$429. \int_0^{\infty} x^2 e^{-ax} \sin bx dx = \frac{2b(3a^2 - b^2)}{(a^2 + b^2)^3} \quad a > 0$$

$$430. \int_0^{\infty} x^2 e^{-ax} \cos bx dx = \frac{2a(a^2 - 3b^2)}{(a^2 + b^2)^3} \quad a > 0$$

$$431. \int_0^{\infty} x^3 e^{-ax} \sin bx dx = \frac{24ab(a^2 - b^2)}{(a^2 + b^2)^4} \quad a > 0$$

$$432. \int_0^{\infty} x^3 e^{-ax} \cos bx dx = \frac{6(a^4 - 6a^2b^2 + b^4)}{(a^2 + b^2)^4} \quad a > 0$$

*The (one-sided) Laplace transform of $f(t)$, t real, is (if it exists)

$$F(s) \equiv \mathcal{L}\{f(t)\} = \int_0^{+\infty} f(t)e^{-st} dt$$

where $s = \sigma + i\omega$ is a complex variable. The inverse Laplace transform $\mathcal{L}^{-1}\{F(s)\}$ of a (suitable) function $F(s)$ is (if it exists) a function $f(t)$ such that $F(s) \equiv \mathcal{L}\{f(t)\}$.

$$433. \int_0^{\infty} x^n e^{-ax} \sin bx \, dx = \frac{n![(a - ib)^{n+1} - (a + ib)^{n+1}]i}{2(a^2 + b^2)^{n+1}}$$

 $a > 0$

$$434. \int_0^{\infty} x^n e^{-ax} \cos bx \, dx = \frac{n![(a - ib)^{n+1} + (a + ib)^{n+1}]}{2(a^2 + b^2)^{n+1}}$$

 $a > 0$

$$435. \int_0^{\infty} e^{-x} \log x \, dx = -\gamma = -0.577 \, 2157 \dots \quad (\text{see 418})$$

$$436. \int_0^{\infty} \left(\frac{1}{1 - e^{-x}} - \frac{1}{x} \right) e^{-x} \, dx = \gamma = 0.577 \, 2157 \dots$$

$$437. \int_0^{\infty} \frac{1}{x} \left(\frac{1}{1 + x} - e^{-x} \right) \, dx = \gamma = 0.577 \, 2157 \dots$$

$$438. \int_0^1 \frac{1 - e^{-x} - e^{-1/x}}{x} \, dx = \gamma = 0.577 \, 2157 \dots$$

8. VECTOR ANALYSIS

149. DEFINITIONS. ANALYTIC REPRESENTATION A vector V is a quantity which is completely specified by a magnitude and a direction. A vector may be represented geometrically by a directed line segment $V = \overrightarrow{OA}$. A scalar S is a quantity which is completely specified by a magnitude.

Let i, j, k represent three vectors of unit magnitude along the three mutually perpendicular lines OX, OY, OZ , respectively. Let V be a vector in space, and a, b, c the magnitudes of the projections of V along the three lines OX, OY, OZ , respectively. Then V may be represented by $V = ai + bj + ck$. The magnitude of V is

$$|V| = \sqrt{a^2 + b^2 + c^2}$$

and the direction cosines of V are such that $\cos \alpha : \cos \beta : \cos \gamma = a : b : c$.

150. VECTOR SUM V OF n VECTORS Let V_1, V_2, \dots, V_n be n vectors given by $V_1 = a_1i + b_1j + c_1k$, etc. Then the sum is

$$V = V_1 + V_2 + \dots + V_n = (a_1 + a_2 + \dots + a_n)i + (b_1 + b_2 + \dots + b_n)j + (c_1 + c_2 + \dots + c_n)k$$

151. PRODUCT OF A SCALAR S AND A VECTOR V

$$SV = (Sa)i + (Sb)j + (Sc)k$$

$$(S_1 + S_2)V = S_1V + S_2V \quad (V_1 + V_2)S = V_1S + V_2S$$

152. SCALAR PRODUCT OF TWO VECTORS, $V_1 \cdot V_2$

$$V_1 \cdot V_2 = |V_1||V_2| \cos \phi \quad \text{where } \phi \text{ is the angle from } V_1 \text{ to } V_2$$

$$V_1 \cdot V_2 = a_1a_2 + b_1b_2 + c_1c_2 = V_2 \cdot V_1 \quad V_1 \cdot V_1 = |V_1|^2$$

$$(V_1 + V_2) \cdot V_3 = V_1 \cdot V_3 + V_2 \cdot V_3$$

$$V_1 \cdot (V_2 + V_3) = V_1 \cdot V_2 + V_1 \cdot V_3$$

$$i \cdot i = j \cdot j = k \cdot k = 1 \quad i \cdot j = j \cdot k = k \cdot i = 0$$

153. VECTOR PRODUCT OF TWO VECTORS, $V_1 \times V_2$

$V_1 \times V_2 = |V_1||V_2| \sin \phi \mathbf{l}$, where ϕ is the angle from V_1 to V_2 and \mathbf{l} is a unit vector perpendicular to the plane of V_1 and V_2 and so directed that a right-handed screw driven in the direction of \mathbf{l} would carry V_1 into V_2 .

$$\begin{aligned} V_1 \times V_2 &= -V_2 \times V_1 \\ &= (b_1c_2 - b_2c_1)\mathbf{i} + (c_1a_2 - c_2a_1)\mathbf{j} + (a_1b_2 - a_2b_1)\mathbf{k} \\ (V_1 + V_2) \times V_3 &= V_1 \times V_3 + V_2 \times V_3 \\ V_1 \times (V_2 + V_3) &= V_1 \times V_2 + V_1 \times V_3 \\ V_1 \times (V_2 \times V_3) &= V_2(V_1 \cdot V_3) - V_3(V_1 \cdot V_2) \\ \mathbf{i} \times \mathbf{i} = \mathbf{j} \times \mathbf{j} = \mathbf{k} \times \mathbf{k} &= \hat{0}, \quad \mathbf{i} \times \mathbf{j} = \mathbf{k}, \quad \mathbf{j} \times \mathbf{k} = \mathbf{i}, \quad \mathbf{k} \times \mathbf{i} = \mathbf{j} \\ V_1 \cdot (V_2 \times V_3) &= (V_1 \times V_2) \cdot V_3 = V_2 \cdot (V_3 \times V_1) = [V_1V_2V_3] \\ &= \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix} \end{aligned}$$

154. DIFFERENTIATION OF VECTORS $V = ai + bj + ck$. If V_1, V_2, \dots are functions of a scalar variable t , then

$$\frac{d}{dt}(V_1 + V_2 + \dots) = \frac{dV_1}{dt} + \frac{dV_2}{dt} + \dots$$

$$\text{where } \frac{dV_1}{dt} = \frac{da_1}{dt}\mathbf{i} + \frac{db_1}{dt}\mathbf{j} + \frac{dc_1}{dt}\mathbf{k}, \text{ etc.}$$

$$\frac{d}{dt}(V_1 \cdot V_2) = \frac{dV_1}{dt} \cdot V_2 + V_1 \cdot \frac{dV_2}{dt}$$

$$\frac{d}{dt}(V_1 \times V_2) = \frac{dV_1}{dt} \times V_2 + V_1 \times \frac{dV_2}{dt}$$

$$V \cdot \frac{dV}{dt} = |V| \frac{d|V|}{dt}. \quad \text{If } |V| \text{ is constant, } V \cdot \frac{dV}{dt} = 0.$$

$$\text{grad } S \equiv \nabla S \equiv \frac{\partial S}{\partial x}\mathbf{i} + \frac{\partial S}{\partial y}\mathbf{j} + \frac{\partial S}{\partial z}\mathbf{k} \quad \text{where } S \text{ is a scalar}$$

$$\text{div } V \equiv \nabla \cdot V \equiv \frac{\partial a}{\partial x} + \frac{\partial b}{\partial y} + \frac{\partial c}{\partial z} \quad \text{(divergence of } V)$$

$$\text{curl } \mathbf{V} \equiv \text{rot } \mathbf{V} \equiv \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ a & b & c \end{vmatrix} \equiv \nabla \times \mathbf{V}$$

$$\text{div grad } S \equiv \nabla^2 S \equiv \frac{\partial^2 S}{\partial x^2} + \frac{\partial^2 S}{\partial y^2} + \frac{\partial^2 S}{\partial z^2}$$

$$\nabla^2 \mathbf{V} \equiv \mathbf{i}\nabla^2 a + \mathbf{j}\nabla^2 b + \mathbf{k}\nabla^2 c$$

$$\text{curl grad } S = 0 \quad \text{div curl } \mathbf{V} = 0$$

$$\text{curl curl } \mathbf{V} = \text{grad div } \mathbf{V} - \nabla^2 \mathbf{V}$$

155. GREEN'S THEOREM Let \mathbf{F} be a vector and V be a volume bounded by a surface S . Then

$$\iiint_V \text{div } \mathbf{F} \, dV = \iiint_V \nabla \cdot \mathbf{F} \, dV = \iint_S \mathbf{F} \cdot d\mathbf{S}$$

where the integrations are to be carried out over the volume V and the surface S .

156. STOKES' THEOREM Let \mathbf{F} be a vector and $d\mathbf{r} = dx \mathbf{i} + dy \mathbf{j} + dz \mathbf{k}$ and S be a surface bounded by a simple closed curve C . Then

$$\int_C \mathbf{F} \cdot d\mathbf{r} = \iint_S \text{curl } \mathbf{F} \cdot d\mathbf{S} = \iint_S \nabla \times \mathbf{F} \cdot d\mathbf{S}$$

9. SETS, RELATIONS, AND FUNCTIONS

157. SETS A set (*class, aggregate, ensemble*) S is a well-defined collection of objects, or symbols, called *elements* or *members* of the set. No restriction is placed on the nature of the objects.

Set A is a *subset* of set B , written $A \subset B$ or $B \supset A$, if and only if each element of A is also an element of B . A is a *proper subset* of B if at least one member of B is not a member of A . $A \not\subset B$ means that at least one element of A is not included in B . Some writers use $A \subseteq B$ to indicate that A is a subset of B and reserve $A \subset B$ to indicate that A is a proper subset of B .

Set A is said to *equal* set B , written $A = B$, if every element of A is an element of B and every element of B is an element of A . Two equal sets are said to be *identical*. If A and B are not equal, we write $A \neq B$.

A *universal set* I (or *universe* or *space*) is the term given to an overall set which includes all the sets of concern in a given study.

An *empty (null) set*, written \emptyset , is a set that has no elements. It is usually assumed that for every set A , $\emptyset \subset A$.

The *complement* A' of a set A with respect to a given universal set I is the set of elements in I that are not in A . [Some writers use \bar{A} or $\complement(A)$ or $I - A$ to denote the complement A' .]

Notation. The elements of a set are usually denoted by small letters. Capital letters are used as the names of sets. The elements of a set are often listed by enclosing them in braces. The notation $x \in S$ means that x is a member of set S ; $x \notin S$ means that x is not a member of S .

To indicate a set of objects x having the property P , the symbol $\{x \mid x \text{ has the property } P\}$ is used. $\{x \mid \}$ is called a *set builder*. The bar " \mid " is read "such that."

Example. $S = \{1, 2, 4, 6\}$ is a set S consisting of the elements 1, 2, 4, 6.

Example. $\{(x, y) \mid (x - y) > 2\}$ denotes the set of number pairs (x, y) such that $(x - y)$ exceeds 2.

158. UNION AND INTERSECTION The *union* (*join*) of sets A and B , written $A \cup B$ or " A cup B ," is the set of all elements that are in A or in B or in both A and B .

The *intersection* (*meet*) of the sets A and B , written $A \cap B$, is the set of all elements which are in both A and B .

The sets A and B are said to be *disjoint* or *mutually exclusive* if they have no elements in common, that is, if their intersection is the empty set. A and B *overlap* if they have some elements in common.

159. VENN DIAGRAM One scheme for representing a subset V of ordered pairs (x, y) consists in using a rectangle to represent the universal set, and sets of points inside the rectangle to represent V . For example, Figs. 58 and 59 illustrate the commutative law; Fig. 60, the complement.



Fig. 58. Crosshatched section illustrates $A \cup B = B \cup A$.

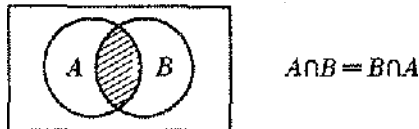


Fig. 59. Crosshatched section illustrates $A \cap B = B \cap A$.

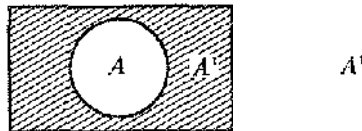


Fig. 60. Crosshatched section illustrates A' .

160. OPERATIONS ON SETS Consider the ordered n -tuple $X = \{x_1, x_2, \dots, x_n\}$ of n elements, where the elements x_i are members of set S_i . An *operation* on X is a *rule* which assigns to each X a uniquely defined element in some set S . If $n = 1$, the operation is *unary*; if $n = 2$, the operation is *binary*; if $n = m$, the operation is *m-ary*.

161. ALGEBRA OF SETS If A, B, C are any three subsets of a universe I , then, for the operations \cup and \cap , the following relationships (laws) hold:

1. Closure laws
 - (a) There exists a unique set $A \cup B$
 - (b) There exists a unique set $A \cap B$
2. Commutative laws
 - (a) $A \cup B = B \cup A$
 - (b) $A \cap B = B \cap A$
3. Associative laws
 - (a) $(A \cup B) \cup C = A \cup (B \cup C)$
 - (b) $(A \cap B) \cap C = A \cap (B \cap C)$
4. Distributive laws
 - (a) $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$
 - (b) $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$
5. Idempotent laws
 - (a) $A \cup A = A$
 - (b) $A \cap A = A$
6. Properties of universe I . Identity laws
 - (a) $A \cup I = I$
 - (b) $A \cap I = A$
7. Properties of null set \emptyset . Identity laws
 - (a) $A \cup \emptyset = A$
 - (b) $A \cap \emptyset = \emptyset$
8. Properties of inclusion \subset
 - (a) $A \subset I$
 - (b) $\emptyset \subset A$
 - (c) If $A \subset B$, then $A \cup B = B$
 - (d) If $B \subset A$, then $A \cap B = B$
 - (e) $A \subset (A \cup B)$
 - (f) $(A \cap B) \subset A$
9. Properties of complement
 - (a) For every set A there exists a unique set A' with respect to I .
 - (b) $A \cup A' = I$
 - (c) $A \cap A' = \emptyset$
 - (d) $(A \cup B)' = A' \cap B'$ De Morgan's law (dualization)
 - (e) $(A \cap B)' = A' \cup B'$ De Morgan's law (dualization)
 - (f) $\emptyset' = I$
 - (g) $I' = \emptyset$
 - (h) $(A')' = A$ (involution)
10. Duality principle

If in any "correct" formula involving the symbols $\cup, \cap, \emptyset, I, \subset$, and \supset we interchange \cup and \cap, \emptyset and I, \subset and \supset throughout the formula, the resulting formula is correct.

11. Absorption laws

$$(a) A \cup (A \cap B) = A \quad (b) A \cap (A \cup B) = A$$

12. Consistency property

The conditions $A \subset B$, $A \cap B = A$, and $A \cup B = B$ are mutually equivalent.

162. ONE-TO-ONE CORRESPONDENCE BETWEEN SETS Two sets A and B are said to be in *one-to-one (reciprocal) correspondence* when a pairing of the elements of A with the elements of B is such that each element of A corresponds to exactly one element of B and each element of B corresponds to exactly one element of A . ("Exactly" means "one and only one.")

163. EQUIVALENT SETS Two sets A and B are said to be *equivalent*, written $A \approx B$, if and only if they can be placed in one-to-one correspondence.

164. CARDINAL NUMBERS Two sets are said to have the same *cardinal number* if and only if they are equivalent.

A set A is said to have the cardinal number n if and only if there exists a one-to-one correspondence between the elements of A and the natural numbers $1, 2, 3, \dots, n$.

165. FINITE AND INFINITE SETS A nonempty set is *finite* if and only if its cardinal number is one of the cardinal numbers $1, 2, 3, \dots$. An *infinite set* is a nonempty set which is *not finite*.

166. \aleph_0 AND \aleph The cardinal number of the set of all natural numbers $1, 2, 3, \dots$ is denoted by \aleph_0 . The cardinal number of the set R of all real numbers is denoted by \aleph . \aleph_0 and \aleph are examples of *transfinite numbers*.

167. DENUMERABLE SETS If a set has the cardinal number \aleph_0 , it is said to be *denumerable (enumerable)*.

168. COUNTABLE SETS A set A is *countable* if and only if there exists a one-to-one correspondence between the elements of A and the elements of a set of natural numbers.

If a set is either finite or denumerable, it is *countable*. The elements of a countable set can be listed in a sequence.

Example. Every finite set A is countable, the number of elements in A being identical with the cardinal number of A .

Example. The set of all rational numbers is countable (and denumerable) with cardinal number \aleph_0 .

Note. The meanings of *denumerable* and *countable* vary with authors.

Theorem. A set A which has the same cardinal number as one of its proper subsets is an infinite set. A set which does not have the same cardinal number as any of its proper subsets is a finite set.

169. NONCOUNTABLE SETS A set which is not countable is said to be *noncountable* (*nondenumerable*).

Example. The set of all real numbers between 0 and 1 is noncountable with cardinal number \aleph .

170. MAPPING OF SET A INTO SET B . Let A and B be two sets not necessarily disjoint. A collection of pairs (a, b) , where $a \in A$ and $b \in B$, such that each a of A occurs in one and only one pair (a, b) and such that any b of B may occur in any number of pairs is called a *mapping* of A into B . Such a mapping is known as a *many-one* (or *many-to-one*) *correspondence* between the elements of A and some or all of the elements of B . A mapping of a set A into a set B is a matching procedure that assigns to each element $a \in A$ a unique element $b \in B$. This matching is written $a \rightarrow b$, b being called the *image* of a .

If every element of B is used as an image, then the mapping is said to be "*A onto B*," as well as "*A into B*."

171. PRODUCT SET Let A_1, A_2, \dots, A_n be n sets. Let $a_i \in A_i$, for $i = 1, 2, \dots, n$. The collection of all ordered n -ples (a_1, a_2, \dots, a_n) is called the *product set* $A_1 \times A_2 \times \dots \times A_n$.

The set of all ordered pairs (x, y) , where $x \in X$ and $y \in Y$, is called the *cartesian product* of X and Y , is denoted by $X \times Y$, and is read "*X cross Y*."

$U \times U$. If $x \in U$ and $y \in U$, the set of all ordered pairs (x, y) that can be found using the elements of U is a *cartesian product set* on U and is denoted by $U \times U$.

Equal Ordered Pairs. The ordered pair (a,b) equals the ordered pair (x,y) ; that is, $(a,b) = (x,y)$, if and only if $a = x$ and $b = y$.

172. VARIABLE A symbol in a sentence, phrase, or expression is a *variable* (or *placeholder*) if and only if it is replaceable by the name of an element (number) of a set called the *replacement set* (or *domain* or *universal set*) specifically defined for the variable. If the set contains only a single element, the symbol used to represent this element is called a *constant*. Variables are used as placeholders for numbers and other mathematical quantities in algebraic sentences, such as equations and inequalities.

173. RELATION A *relation* R in a set U is a subset of ordered pairs (x,y) from $U \times U$. The *domain* of the relation R is the subset of U for which x is a placeholder; the *range* of the relation is the subset of U for which y is a placeholder.

174. GRAPH Let x and y be real numbers. The *graph* of a relation whose ordered pairs are (x,y) is the set of points in the XY plane whose coordinates are the given pairs.

175. LOCUS A *locus* is the set of those points, and no other points, that satisfy a given condition. A locus is a graph of a relation.

176. SOLUTION SET Suppose x belongs to the universal set I . Consider the set $\{x|S\}$, where S is a statement involving x , or a condition placed on x . Any member of I for which the statement S is true is said to *satisfy* S , or to be a *solution set* of S . S is called a *defining condition*, or *set selector*.

Example. If x belongs to the set of real numbers, the solution set of $\{x|2x = 3\}$ is $\{3/2\}$.

Example. If $I = \{1,3,5\}$ and x belongs to I , the solution set of $\{x|x > 2\}$ is $\{3,5\}$.

177. FUNCTION A *function* f in a set U is a subset of ordered pairs (x,y) from $U \times U$ having exactly one y for each x . In other words, a *function* f in U is a relation R in U such that for every x in the domain of R there corresponds exactly one y in the range of R . (This defini-

tion of function corresponds to the notion of a single-valued function given by many writers.)

The notation $f: (x, y)$ is sometimes used to mean "the function f whose ordered pairs are (x, y) ."

The notation $f(x)$, read "f at x," is often used to denote the second element y of the ordered pair (x, y) whose first element is x . $f(x)$ is called "the value of the function f at x ." It is common to write $y = f(x)$, meaning y is the value of f at x .

Example. If $U = \{0, 1\}$, the set of ordered pairs $U \times U$ is the lattice points $(0, 0)$, $(0, 1)$, $(1, 0)$, $(1, 1)$. If the relation R_1 in U is given by $R_1 = \{(x, y) \mid (x^2 + y^2) < 1\}$, then R_1 selects from $U \times U$ the subset $(0, 0)$. If $R_2 = \{(x, y) \mid (x^2 + y^2) \leq 1\}$, then R_2 selects the subset $(0, 0)$, $(0, 1)$, $(1, 0)$. The domain of the relation R_1 is $\{0\}$, the range of R_1 is $\{0\}$; the domain of R_2 is $\{0, 1\}$, the range of R_2 is $\{0, 1\}$. R_1 is a function; R_2 is not a function.

Example. The relation $\{(x, y) \mid y = x^2 + 1\}$ in the set U of real numbers defines a function F . $F(1) = 2$ is the value of $F(x) = x^2 + 1$ at $x = 1$. The domain of the function is U , the range of F is the set of all real numbers equal to or greater than 1.

178. INVERSE RELATION The set R^{-1} obtained by interchanging the coordinates of all the ordered pairs of a given relation R is called the *inverse relation of R* .

Example. The inverse R^{-1} of the relation $R = \{(1, 2), (1, 3), (2, 4)\}$ is $R^{-1} = \{(2, 1), (3, 1), (4, 2)\}$. R is not a function; R^{-1} is a function.

179. INVERSE FUNCTION The inverse of a function is obtained by interchanging the coordinates of each of its ordered pairs; the inverse is a relation, but may or may not be a function.

A function F is a set of ordered pairs (x, y) no two of which have the same first component x with different second components. If a function F is such that no two of its ordered pairs have the same second component with different first components, then the set of ordered pairs obtained from F by interchanging in each ordered pair the first and second elements is called the *inverse function* and is denoted by F^{-1} .

The range of F is the domain of F^{-1} , and the domain of F is the range of F^{-1} . We write $F: (a, b)$ and $F^{-1}: (b, a)$.

Example. If the function F has elements (ordered pairs) of the

form $(a_1, b_1), (a_2, b_2), \dots$, the inverse function F^{-1} , if it exists, has elements (ordered pairs) of the form $(b_1, a_1), (b_2, a_2), \dots$.

Example. Let x and y be real numbers. The inverse to the relation $R = \{(x, y) | y = 2x\}$ is $R^{-1} = \{(y, x) | x = y/2\}$. R^{-1} and R are each functions. The inverse function is then sometimes written $R^{-1} = \{(x, y) | y = x/2\}$.

Example. Let x and y be real numbers. The inverse relation to the relation $R = \{(x, y) | y = 3x^2\}$ is $R^{-1} = \{(y, x) | x = \pm \sqrt{y/3}\}$. R is a function; R^{-1} is not a function. The inverse relation may be written $R^{-1} = \{(x, y) | y = \pm \sqrt{x/3}\}$. If the domain of R is restricted so that $x \geq 0$, then $R^{-1} = \{(x, y) | y = \sqrt{x/3}\}$ is a function.

Example. The inverse relation to $R = \{(x, y) | y - 2 = |x| - x\}$ is $R^{-1} = \{(x, y) | x - 2 = |y| - y\}$. R^{-1} is not a function.

Example. Let x and y be real numbers. The inverse R^{-1} of the relation $R = \{(x, y) | y = \sin x\}$ is $R^{-1} = \{(x, y) | x = \sin y\}$. Here y is a number whose sine is x . R^{-1} is not a function.

If R is restricted to values of x such that $-\pi/2 \leq x \leq \pi/2$, then R^{-1} is a function. In this case we may write (see Sec. 60)

$$R^{-1} = \{(x, y) | y = \text{Sin}^{-1} x\} \quad \frac{-\pi}{2} \leq y \leq \frac{\pi}{2}$$

180. COMPOSITE OF TWO FUNCTIONS Let $f: (x, y)$ and $g: (y, z)$ be two given functions. Construct an ordered pair (x, z) as follows: Select an x ; f assigns to x a value y . If y is in the domain of g , g assigns to y a value z . This gives an ordered pair (x, z) . The set of all such pairs (x, z) which can be constructed by this scheme describes a function known as the *composite function* $g \circ f$. It is written $g \circ f: (x, z)$.

Example. Let the values of the functions $f: (x, y)$ and $g: (y, z)$ be $y = f(x) = x^2$ and $z = g(y) = 3y - 2$, where x, y, z are real numbers. The composite function $g \circ f: (x, z)$ is given by $z = g(y) = 3x^2 - 2$; that is,

$$g \circ f: (x, z) = g \circ f: (x, 3x^2 - 2)$$

The composite function $f \circ g: (x, z) = f \circ g: [x, (3x - 2)^2]$, since the values of $g: (x, y)$ and $f: (y, z)$ are

$$y = g(x) = 3x - 2 \quad \text{and} \quad z = f(y) = y^2 = (3x - 2)^2$$

181. ALGEBRA OF FUNCTIONS Let $f: (x, y)$ and $g: (x, z)$ be two functions whose domains are d_f and d_g , respectively. Let d_f and d_g consist of sets of real or complex numbers for which the elementary operations of $+$, $-$, \times , \div are defined. The sum $f + g$, the difference $f - g$, the product fg , and the quotient f/g are defined as follows:

$$f: (x, y) + g: (x, z) = (f + g): (x, y + z)$$

$$f: (x, y) - g: (x, z) = (f - g): (x, y - z)$$

$$f: (x, y) \times g: (x, z) = (fg): (x, yz)$$

$$\frac{f: (x, y)}{g: (x, z)} = \left(\frac{f}{g}\right): \left(x, \frac{y}{z}\right)$$

Let $y = f(x)$ and $z = g(x)$ be the functional values of f and g , respectively. Then the functional values of the sum, difference, product, and quotient are, respectively,

$$(f + g)(x) = f(x) + g(x)$$

$$(f - g)(x) = f(x) - g(x)$$

$$(fg)(x) = f(x) \times g(x)$$

$$\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)}$$

DOMAIN The domain of $f + g$, $f - g$, and fg is the set of all elements x common to the domains of f and g , that is, the intersection of the sets d_f and d_g . This is the domain of f/g , except that those x s for which $g(x) = 0$ must be excluded since division by zero is not possible.

182. IDENTITY FUNCTION The *identity function* is the function i whose elements are the ordered pairs (x, x) . i is sometimes written $i = \{(x, i(x)) \mid i(x) = x\}$.

183. COMPOSITE OF FUNCTION AND ITS INVERSE Let $F: (x, y)$ and $F^{-1}: (y, x)$ be a function and its inverse, respectively. The composite function $F^{-1} \circ F$ sends (under F) each x into some y , and then (under F^{-1}) sends y back into x . That is, $F^{-1} \circ F = i$. Likewise $F \circ F^{-1} = i$.

184. ADDITIVE SET FUNCTION Let A, B, C, \dots be subsets of a finite set I . Suppose f is a relation that assigns to any subset S of I a unique real number denoted by $f(S)$ such that if A and B are disjoint subsets of I , $f(A \cup B) = f(A) + f(B)$, then f is said to be an *additive set function*.

Theorem. If f is an additive set function and \emptyset is a null set, then $f(\emptyset) = 0$; that is, the value of f at \emptyset is zero.

Theorem. If A and B are any two subsets of I , not necessarily disjoint, and if f is an additive set function, then

$$f(A \cup B) = f(A) + f(B) - f(A \cap B)$$

185. WEIGHT FUNCTION An additive set function w , all of whose values are positive or zero, is called a *weight function*; i.e., for all subsets S of I , $w(S) \geq 0$.

186. PROBABILITY FUNCTION If P is a weight function, I the universal set, and $P(I) = 1$, then P is called a *probability function*.

Theorem. If f is an additive set function, $f(A) + f(A') = f(I)$, where A is a subset of the universal set I and A' the complement of A with respect to I .

Theorem. If P is a probability function, $P(A) + P(A') = 1$.

187. LIMITS Let x take on the sequence of real values $x_1, x_2, \dots, x_n, \dots$. The variable x is said to approach the number a as a limit, written $x \rightarrow a$, if for every positive number ϵ there exists an integer N such that for every $n > N$, $|x_n - a| < \epsilon$.

Let $f(x)$ be the value of the function f at x . $f(x)$ is said to approach L as a limit as $x \rightarrow a$, written $\lim_{x \rightarrow a} f(x) = L$, if for every $\epsilon > 0$ there exists a positive number δ such that for all x satisfying $0 < |x - a| < \delta$, $|f(x) - L| < \epsilon$.

188. CONTINUOUS FUNCTION The function f is *continuous* at $x = a$ if $f(a)$ is defined, $\lim_{x \rightarrow a} f(x) = L$ exists, and $f(a) = L$.

10. LOGIC

189. SYMBOLIC LOGIC A mathematical theory results from the interplay of a set of postulates and a logic. The theory starts with a set of postulates as the basis. The logic constitutes the rules by which such a basis may be developed into a body of theorems. The mathematical theory, or system, consists of the collection of statements composed of the postulates and the theorems.

A *symbolic logic* (*mathematical logic*, *logistics*) is a *skeleton of language* (i.e., a system of symbols and rules for their use). Symbolic logic is not a theory (i.e., a system of assertions about objects). In symbolic logic, relations among propositions, sets, and so on, are represented by symbols and formulas whose interpretations are free from the ambiguities of ordinary language.

The signs of a symbolic logic remain uninterpreted as long as we are concerned only with a *pure logic* (i.e., the building of the language of a symbolic logic). In *applied logic* we are concerned with the application and interpretation of a symbolic logic with respect to a given theory.

190. BASIC SYMBOLS; PROPOSITIONS AND PROPOSITIONAL FORMS

We are concerned with the elementary logical properties of statements. Ordinary declaratory statements (assertions, propositions) are denoted by the letters p, q, r, \dots . Statements may involve constants and variables. A *constant* has a fixed specific meaning. A *variable* serves to refer to unspecified objects, properties, etc., and has associated with it a valid domain of values. A statement which contains variables is said to be *open*; otherwise, it is *closed*. A *proposition* is any statement concerning which it is meaningful to say that its content is "true" or "false". (What is meant by "true" and by "false" will not be discussed here.)

A statement which becomes a proposition when specific values (i.e., constants) are submitted for all variables is called a *propositional form* (*propositional function*, *sentential function*, *sentential formula*). The term *constant* is also called a *propositional constant* or *sentential constant*. The term *propositional* or *sentential variable* is used for a variable whose domain is a set of whole sentences.

Assigned to propositions are *truth-values*, the truth-value of a proposition being *truth* if it is true, and *falsehood* if it is false. A statement about which it is meaningless, or ridiculous, etc., to say that its content is true or false is specifically excluded in our definition of a proposition. Statements requiring other types of symbolic logic, for example, *n*-way logics, and situations where many gradations of possibilities may occur, as contrasted with the exclusive possibility of true or false, yes or no, etc., are specifically excluded here.

When we assert a proposition *p*, without qualification as to its truth or falsity, we mean to imply that *p* is true. The *assertion* of the truth of a proposition *p* is written $\vdash p$.

Statements can be combined in various ways to form new statements. Among the grammatical connectives used to combine phrases, statements, etc., are such common words as "not", "and", "or", "is", "every", "some", "if then", and "if and only if". These words are used in many ways, and their meanings in a given context are often vague and ambiguous. In symbolic logic, the concern is to establish the precise meaning of such terms and to lay down the laws in which these terms are involved. The logical meanings of "and", "or", etc., differ somewhat from the uses of these connectives in ordinary language.

Calculus of Propositions

191. The object of the *calculus of propositions* (*propositional* or *sentential calculus*) is the development of logical inference. As with statements in general, propositions *p*, *q*, *r*, . . . may be combined in many ways to form new propositions, called *composite propositions*. We introduce the following definitions:

1. *Conjunction*. A proposition, represented by $p \wedge q$, which is true when and only when both proposition *p* and proposition *q* are true is called a *conjunction* (*logical product*). $p \wedge q$ is read "*p* and *q*".

2. *Disjunction*. A proposition, represented by $p \vee q$, which is true when and only when at least one of the two propositions *p* and *q* is true is called an (*inclusive*) *disjunction* (*logical sum*). The symbol $p \vee q$ is read "*p* or *q*".

3. *Implication*. A proposition, represented by $p \rightarrow q$, which is false when and only when *p* is true and *q* is false is called a (*material*) *implication* (or *conditional sentence*). $p \rightarrow q$ is read "if *p*, then *q*".

4. *Equivalence.* A proposition, represented by $p \leftrightarrow q$, which is true when and only when p and q are both true or both false is called an *equivalence*. $p \leftrightarrow q$ is read " p if and only if q ".

5. *Negation.* The denial or contradiction of a proposition p is a proposition represented by the symbol p' . p' is read "*not* p ". The proposition p' is true when p is false, and p' is false when p is true. A proposition of the form p' is called a *negation*.

6. *Exclusive Disjunction.* A proposition, represented by $p \veebar q$, which is true when and only when proposition p or proposition q is true, but not both, is called an *exclusive disjunction*.

192. TRUTH TABLES The symbols \wedge , \vee , \rightarrow , and \leftrightarrow denote binary operations performed on two propositions. The symbol $'$ denotes a unary operation performed on propositions.

A compact way of summarizing the definitions of conjunction, etc., is by use of *truth tables*. In such tables the symbol T denotes that the corresponding proposition is true, F that it is false. To form such a table, all possible combinations of T and F for the component propositions involved must be listed and then the truth or falsity of each combination is entered into the table. Table 10.1 gives the truth tables for the operations of conjunction, disjunction, implication, equivalence, negation, and exclusive disjunction. For example, the table states that $p \wedge q$ is true if p and q are each true; that $p \wedge q$ is false in every other case.

TABLE 10.1. TRUTH TABLE FOR OPERATIONS 1 TO 6

p	q	$p \wedge q$	$p \vee q$	$p \rightarrow q$	$p \leftrightarrow q$	p'	$p \veebar q$
T	T	T	T	T	T	F	F
T	F	F	T	F	F	F	T
F	T	F	T	T	F	T	T
F	F	F	F	T	T	T	F

193. RELATIONS TO ORDINARY LANGUAGE OF DEFINITIONS 1 THROUGH 6 "*and*". In logic *and* is used to connect any two propositions whatever without regard to their possible bearing on one another. In ordinary language *and* is commonly used to conjoin propositions

that are relevant to each other, as, for example, in a connected discourse.

"or". In logic *or* is used in the inclusive sense of the Latin word *vel* (which means "*p* or *q* or both *p* and *q*"), and the possible bearing of one proposition on another is disregarded. In ordinary language *or* is used in the exclusive sense of the Latin word *aut* (which means "*p* or *q* but not both *p* and *q*") and in the inclusive sense of *vel*.

"if . . . , then . . . ". In logic "if . . . , then . . ." is considered meaningful even if no connection exists between the *antecedent* (the subordinate clause to which "if" is prefixed) and the *consequent* (the principal clause introduced by "then"). Contemporary logic uses implications in the material meaning defined in 3; the concern is with the truth-values of propositions, not with the meanings of the propositions; whether there is some necessary connection between *p* and *q* is not of importance.

In ordinary language "if . . . , then . . ." is often used only when there is a connection between the forms and contents of the antecedent and consequent, a relation between a premise and a conclusion, or a sort of cause and effect relation. (In mathematics, theorems tend to have the form of an implication; the antecedent may be called a *hypothesis*, the consequent a *conclusion*.) Often in ordinary language if the antecedent is assumed to be true, the consequent must be assumed to be true too; implications are commonly asserted without exact knowledge of whether or not the antecedent and consequent are true.

Implication is commonly used in ordinary language in situations where the presence of a formal connection between antecedent and consequent is a necessary condition for the meaningfulness and truth of the implication. Such an implication is a *formal implication*, or an *implication in formal meaning*. A formal implication which is based on a relation of premise and conclusion, or of cause and effect, really depends upon the structure of the component propositions rather than upon their truth or falsity, and has been termed a *strict implication*.

"if and only if". In logic the assertion $p \leftrightarrow q$ means that *p* and *q* have the same truth-values; that is, $p \leftrightarrow q$ is true when and only when *p* and *q* are both true or both false; $p \leftrightarrow q$ is not intended to imply that *p* and *q* have the same significance or meaning. In ordinary language $p \leftrightarrow q$, that is, "if and only if", is often used with a looser meaning; e.g., it may imply that the antecedent and consequent have the same significance or meaning.

"not". The negation of a sentence is formed with the help of "not". Two sentences, the first being the negation of the other, are called *contradictory sentences*.

194. INVERSE SENTENCE If in the sentence "if p , then q " the antecedent p is replaced by the negation p' and the consequent q is replaced by the negation q' , then the resulting sentence "if p' , then q' " is called the *inverse sentence* to "if p , then q ".

195. COUNTERPOSITIVE SENTENCE If in the inverse sentence "if p' , then q' " the antecedent and consequent are interchanged, the resulting sentence "if q' , then p' " is called the *counterpositive sentence* to "if p , then q ".

196. CONVERSE SENTENCE If in the sentence "if p , then q " the antecedent and consequent are interchanged, the resulting sentence "if q , then p " is called the *converse sentence* to "if p , then q ".

197. TRUTH-FUNCTIONS Many propositional forms can be constructed from simple propositional forms with the help of variables, constants, \wedge , \vee , \rightarrow , etc. Every propositional form is a *truth-function*. The truth or falsehood of any whole sentence obtained from a given propositional form depends exclusively on the truth or falsehood of the sentences substituted for the propositional variables p , q , r , etc.

198. TRUTH-VALUES OF COMPOSITE PROPOSITIONS Propositions may be combined or modified into more complicated *composite* propositions. The truth-value of a composite proposition can be determined for any combination of truth-values of the fundamental propositions p , q , . . . by means of the truth tables for the operations \wedge , \vee , \rightarrow , \leftrightarrow , 'involved.

199. TAUTOLOGIES A composite proposition involving the propositions p , q , . . . which has the truth-value T no matter what the truth values of p , q , . . . are is called a *tautology* and is said to be "logically true." A sentence is called a *tautology*, a *contingency*, or a *contradiction* (*absurdity*, "logically false") as its distribution of truth-values show respectively only T , at least one T , or only F .

TABLE 10.2. SOME LAWS OF LOGIC (TAUTOLOGIES)

For any propositions p, q, r , irrespective of their truth-values, each of the following propositions is a tautology, i.e., has the truth-value T :

- | | | |
|----|------------------------|--|
| 1. | Law of excluded middle | $p \vee p'$ |
| 2. | Law of contradiction | $(p \wedge p)'$ |
| 3. | Law of syllogism | $[(p \rightarrow q) \wedge (q \rightarrow r)] \rightarrow (p \rightarrow r)$ |
| 4. | Law of double negation | $p \leftrightarrow (p')'$ |
| 5. | Law of contraposition | $[p \rightarrow q] \leftrightarrow [q' \rightarrow p']$ |

200. LOGICALLY EQUIVALENT PROPOSITIONS Let m and n be two composite (or fundamental) propositions. Form the truth tables for m and n in terms of p, q, \dots . If the truth tables for m and n agree with each other in point of truth and falsity, the propositions are said to be *logically equivalent*. In such a case the proposition $m \leftrightarrow n$ is a tautology, or law of logic.

Example. The propositions $p \rightarrow q$ and $q' \rightarrow p'$ are logically equivalent; and $[p \rightarrow q] \leftrightarrow [q' \rightarrow p']$ is a tautology.

Example. The proposition $(p')'$ is logically equivalent to p ; and $(p')' \leftrightarrow p$ is a tautology.

TABLE 10.3. PAIRS OF LOGICALLY EQUIVALENT PROPOSITIONS

For any propositions p and q , the propositions m and n listed on a given row are logically equivalent; that is, $m \leftrightarrow n$.

m	\leftrightarrow	n
$p \wedge q$		$(p' \vee q')'$
$p \rightarrow q$		$p' \vee q$
$p \leftrightarrow q$		$[(p' \vee q)' \vee (q' \vee p)']'$
$p \vee q$		$(p' \wedge q')'$
$p \rightarrow q$		$(p \wedge q')'$
$p \leftrightarrow q$		$(p \wedge q)' \wedge (q \wedge p)'$
$p \wedge q$		$(p \rightarrow q)'$
$p \vee q$		$p' \rightarrow q$
$p \leftrightarrow q$		$[(p \rightarrow q) \rightarrow (q \rightarrow p)]'$

201. QUANTIFIERS In addition to "and", "or", etc., there are phrases such as "for all $x \dots$ " and "there exists an x such that \dots "

involving the *quantifiers* "for all" and "there exists" which must be considered in any development of logic.

The following notations for quantifiers are in common use:

- \forall "for every element (of the domain)" or "for any . . ." or "for all . . ." or "for each . . .".
- \exists "there is at least one element (of the domain) such that" or "there exists an element (of the domain)" or "there are . . ." or "for some . . .".

The symbol \forall is called a *universal quantifier*. The symbol \exists is called the *existential quantifier*.

Example. The phrase "for any x , p is true" may be written $\forall x$, p is true.

Example. The phrase "there exists an element x such that p is true" may be written $\exists x$, p is true.

202. UNIVERSAL AND EXISTENTIAL SENTENCES A sentence of the form "for all things x, y, \dots , p is true" is a *universal sentence*.

A sentence of the form "there exists things x, y, \dots such that p is true" is an *existential sentence*.

203. LIMITATIONS OF CALCULUS OF PROPOSITIONS The calculus of propositions is adequate for the rendering of logical connections in which the propositions appear as unanalyzed wholes. However this calculus is not adequate for more general logical purposes, since there are logical inferences which depend on the inner content of the propositions themselves as well as upon the propositions as wholes. To take care of this type of logical structure the calculus of sets (or classes) is needed.

204. OTHER LOGICS Statements for which it is not meaningful to say that their content is true or false lead to various types of logic (e.g., many-valued logics). In such cases the truth tables for a proposition may have entries which are *undecidable*.

Calculus of Relations

205. RELATION A *relation* is a form of connectivity, or association, between two or more things. A relation connecting two elements is

a *dyadic (binary) relation*; connecting three elements, a *triadic relation*; . . . , connecting many elements, a *polyadic relation*.

The phrase "the thing x has the dyadic (binary) relation R to the thing y " may be written $x R y$. x is called a *predecessor* and y a *successor* with respect to the relation R . The set of all predecessors with respect to R is the *domain*, and the set of all successors is the *range* of R .

If x is not related to y by the relation R , we write

$$x \not R y \quad \text{or} \quad \sim(x R y)$$

For mathematical purposes a binary relation R is defined by a set of ordered pairs (see Sec. 160).

206. INCLUSION If, whenever R holds between two things, S also holds between them, the relation R is *included* in the relation S . This is written $R \subset S$.

Example. The relation R of "being smaller" is included in the relation S of "diversity"; that is, $R \subset S$.

207. IDENTICAL RELATIONS If the relations R and S both hold between the same things, that is, if $R \subset S$ and $S \subset R$, then the *relations are identical*, and this is written $R = S$.

208. UNIVERSAL AND NULL RELATIONS A relation which holds between any two individuals is a *universal relation*. A relation which holds between no individuals is a *null relation*.

209. UNION OF RELATIONS The relation $R \cup S$ is said to hold between two things if and only if at least one of the relations R and S holds between the things. $R \cup S$ is the *union* (or *sum*) of two relations R and S .

210. INTERSECTION OF RELATIONS The relation $R \cap S$ is said to hold between two things if and only if each of the relations R and S holds between the things. $R \cap S$ is the *intersection* (or *product*) of two relations R and S .

211. NEGATION OF RELATIONS A relation R' which holds between two things if and only if the relation R does not hold between them is the *negation* R' (or the *complement*) of relation R .

212. IDENTITY AND DIVERSITY If two individuals x and y are identical, the relation (*identity*) is written $x I y$. If x and y are different, the relation (*diversity*) is written $x D y$. The symbols $=$ and \neq are

TABLE 10.4. PROPERTIES OF RELATIONS

Let x, y, z be any elements of set K . With respect to K :

If	Then relation R is
for each $x, x R x$	Reflexive
for each $x, x \not R x$	Irreflexive
for some but not all elements $x, x R x$; and for some but not all $x, x \not R x$	Nonreflexive*
for each x and y , whenever $x R y$, then $y R x$	Symmetric
for each x and y , whenever $x R y$, then $y \not R x$	Asymmetric
for each x and y , $x R y$ and $y R x$, then $x = y$	Antisymmetric
for some but not all elements x and y , $x R y$ implies $y R x$; and for some but not all x and y , $x R y$ implies $y \not R x$	Nonsymmetric*
for each x, y and z , whenever $x R y$ and $y R z$, then $x R z$	Transitive
for each x, y and z , whenever $x R y$ and $y R z$, then $x \not R z$	Intransitive
for some but not all elements x, y and z , $x R y$ and $y R z$ imply $x R z$; and for some but not all x, y and z , $x R y$ and $y R z$ imply $x \not R z$	Nontransitive*
for each x and y , $x \neq y$, then either $x R y$ or $y R x$	Connected or determinate
for each x and y , then either $x R y$ or $y R x$	Strongly connected*

* Some writers define these terms in slightly different ways.

used in the calculus of relations to denote the identity and diversity between relations.

213. CONVERSE The relation \bar{R} , called the *converse* of R , is said to hold between things x and y if and only if R holds between y and x .

214. PROPERTIES OF RELATIONS Table 10.4 lists properties of relations in common use. For example, a relation R is reflexive with respect to set K if each element x of K has the relation R to itself, that is, $x R x$.

215. ORDERING RELATIONS With respect to the dyadic relation R a set K is said to be *ordered* in the sense shown in Table 10.6 if in K the relation R has the properties indicated. For example, the set K is *quasi-ordered* with respect to R if R is reflexive and transitive in K .

216. INDEPENDENCE OF CERTAIN PROPERTIES OF RELATIONS The reflexive, symmetric, and transitive sets of properties are independent of each other. There are 27 triples of these properties that dyadic relations may possess.

TABLE 10.5. EXAMPLES OF RELATIONS

<i>Relation</i>	<i>Properties</i>
"is father of"	Asymmetric, intransitive
"is a brother of"	Nonsymmetric
"admires"	Nonreflexive, nonsymmetric, nontransitive
"is married to"	Symmetric
"implies"	Nonsymmetric
"is son of"	Irreflexive, asymmetric
"is greater than" ($>$)	Irreflexive, asymmetric, transitive
"is smaller than" ($<$)	Irreflexive, asymmetric, transitive
"is different from" (\neq)	Irreflexive, symmetric
"is included in"	Reflexive, transitive
"is older than"	Irreflexive, asymmetric, transitive, not necessarily connected
"is equivalent to"	Reflexive, symmetric, transitive
"is between"	A triadic relation

TABLE 10.6. TYPES OF ORDERING

<i>Type of ordering</i>	<i>Properties of relation R</i>
Quasi-ordering	Reflexive, transitive
Partial ordering	Reflexive, antisymmetric, transitive
Simple ordering	Antisymmetric, transitive, strongly connected
Strict partial ordering	Asymmetric, transitive
Strict simple ordering	Asymmetric, transitive, connected

TABLE 10.7. EXAMPLES OF ORDERING

<i>Type of ordering</i>	<i>Relation</i>
Quasi-ordering	"being at least as tall as" for a set of persons
Partial ordering	"inclusion \subseteq " for set of all sets
Simple ordering	\leq for set of all real numbers
Strict partial ordering	"being taller than" or "of being a husband" for set of human beings
Strict simple ordering	$<$ for set of all real numbers

217. EQUIVALENCE RELATIONS A dyadic relation R is an *equivalence relation* for a set K of elements if and only if R is reflexive, symmetric, and transitive.

Examples. The relations "is similar to", "is identical with", "is the same age as", "equals", "equality", and "congruent to" are equivalence relations.

An equivalence relation \approx in a set K separates the elements of K into subsets. All elements equivalent to some particular element x of K belong to one subset; all elements equivalent to some other element y of K , not already in the first subset, belong to a second subset, and so on. Such a separation of K into subsets is called the (*class or subset*) *decomposition* of K corresponding to the equivalence relation \approx .

11. ALGEBRAIC STRUCTURES

Groups

218. TYPICAL MATHEMATICAL SYSTEM A typical *mathematical system* M consists of *elements* a, b, c, \dots , an *equals relation*, and certain *operations*. The equals relation separates the elements into classes. Two elements a and b are *equal* if and only if they belong to the same class.

An equals relation $a = b$ is characterized by the following properties: (a) *Determinative*. Either $a = b$ or $a \neq b$. (b) *Reflexive*. $a = a$. (c) *Symmetric*. If $a = b$, then $b = a$. (d) *Transitive*. If $a = b$ and $b = c$, then $a = c$. (See Sec. 214.)

219. BINARY OPERATION Consider a binary operation \circ which relates to every ordered pair of elements a and b of M a uniquely defined element c of some set N , written $a \circ b = c$. If $N \subseteq M$, then M is *closed under the operation* \circ . (In defining operation some writers require that set N be included in set M .) The operation \circ is said to be *properly or well defined* relative to the "equals" relation if, when either a or b or both are replaced by elements equal to them respectively, c is replaceable by an element equal to it. A variety of symbols are used for an operation \circ , such as $\odot, \oplus, \otimes, +, -, \cdot, \dots$.

220. GROUPS Consider a mathematical system G containing the elements a, b, c, \dots and a binary operation \circ , which is used to pair two elements of G , $a \circ b$, in the order indicated. The operation is assumed to be well defined relative to the equals relation. The system G is called a *group* if the following four postulates hold. For each a, b, c of G :

1. *Closure*. $a \circ b$ is a unique element c in G .
2. *Associative law*. $(a \circ b) \circ c = a \circ (b \circ c)$.
3. *Identity element*. There exists in G an element i , called the *identity* (or *unit*), such that for every a in G , $a \circ i = a$.
4. *Inverse element*. For each a in G there exists in G an element a^{-1} , called the *inverse* of a , such that $a \circ a^{-1} = i$.

If in addition a group satisfies the postulate

5. *Commutativity.* $a \circ b = b \circ a$,

it is called a *commutative* (or *abelian*) group.

The relation $a \circ b$ is called the "product of a and b ."

221. A group consisting of but a finite number of elements is called a *finite group*. The *order* of a finite group is the number of elements which it contains.

A *subgroup* of G is a set of elements of G which by themselves constitute a group. A subgroup of G different from G and from the identity is called a *proper subgroup* of G . A subgroup of G which is not proper is *improper*.

If G_1 and G_2 are subgroups of G , the elements common to G_1 and G_2 form a group called the *intersection* of G_1 and G_2 , written $G_1 \wedge G_2$.

The symbol for $a \circ a \circ a \circ \dots \circ a$ to n factors is a^n .

The least positive integer r such that $a^r = i$ is called the *period* of a .

A set of elements of a group G are said to be *independent* if no element of the set can be expressed as a product of the remaining elements of the set, allowing any number of repetitions.

A set of independent elements of G which have the property that every element of G can be represented in terms of them as a finite product is called a set of *independent generators* of G .

A *cyclic group* is a group generated by a single element.

Examples. The following sets form a group under the operation indicated:

(a) The set of all integers, under addition; the unit element is 0, the inverse of a is $-a$.

(b) The set of all positive rational numbers, under multiplication; the unit element is 1.

(c) The elements of a ring, under addition (see Sec. 226).

(d) The nonzero elements of a field, under multiplication (see Sec. 226).

(e) The set of three letters a, b, c , under permutations.

(f) The set of all euclidean rotations of the plane about the origin.

$$R: x' = x \cos \theta + y \sin \theta \qquad y' = -x \sin \theta + y \cos \theta$$

the operation being composition of transformations. (The result of performing first rotation R_1 and then rotation R_2 is denoted by $R_2 \circ R_1$.)

222. THEOREMS ON GROUPS For each a, b, c, \dots in group G :

- (a) $a \circ i = i \circ a = a$.
- (b) $a \circ a^{-1} = a^{-1} \circ a = i$.
- (c) $(a \circ b)^{-1} = b^{-1} \circ a^{-1}$.
- (d) $(a^{-1})^{-1} = a$.
- (e) The identity i is unique.
- (f) The inverse of a is unique.
- (g) If $a \circ b = a \circ c$, then $b = c$.
- (h) If $b \circ a = c \circ a$, then $b = c$.
- (i) The equation $a \circ x = b$ has a unique solution $x = a^{-1} \circ b$ in G .
- (j) The equation $y \circ a = b$ has a unique solution $y = b \circ a^{-1}$ in G .
- (k) If $a = b$, then for any c in G , $a \circ c = b \circ c$.
- (l) If $a = b$ and $c = d$, then $a \circ c = b \circ d$.
- (m) If a is an element of a finite group, there exists a positive integer r such that $a^r = i$ and $a^{-1} = a^{r-1}$.
- (n) The order of a subgroup of a group G is a divisor of the order of G .
- (o) Every finite group possesses a set of independent generators.

Algebra of Positive Integers

223. ADDITION Let a and b be any two positive integers. To each ordered pair a and b there is assigned a unique positive integer e , called the *sum* of a and b , denoted by $e = a + b$. The operation $+$ of *addition* is a binary operation on the set of positive integers and possesses properties 1, 2, and 3 of Table 11.1. Here $=$ means identity.

MULTIPLICATION To each ordered pair a and b of positive integers there is assigned a unique positive integer f , called the *product* of a and b , denoted by $f = a \times b$. The operation \times of *multiplication* is a binary operation on the set of positive integers and possesses properties 4, 5, and 6 of Table 11.1.

DISTRIBUTIVE LAW The operations of addition and multiplication possess property 7 of Table 11.1.

ALGEBRA OF POSITIVE INTEGERS The statements made in Table 11.1 always hold in the algebra of positive integers.

224. OTHER ALGEBRAS The statements of Table 11.1 hold in other situations where the elements a, b, c, \dots belong to a set S other than the positive integers and the operations $+$ and \times are suitably defined.

Examples. If S is the set of all real numbers with $+$ and \times denoting the usual addition and multiplication operations, then the statements of Table 11.1 always hold in the algebra of real numbers. When S is the set of all real polynomials in the real variable x , the statements of Table 11.1 always hold in the algebra of real polynomials.

TABLE 11.1. BASIC PROPERTIES OF BINARY OPERATIONS OF ADDITION AND MULTIPLICATION OF POSITIVE INTEGERS

Let a, b , and c denote arbitrary positive integers and $=$ be the symbol for identity.

1. *Closure law for addition.* $a + b$ is a positive integer.
2. *Commutative law for addition.* $a + b = b + a$.
3. *Associative law for addition.* $(a + b) + c = a + (b + c)$.
4. *Closure law for multiplication.* $a \times b$ is a positive integer.
5. *Commutative law for multiplication.* $a \times b = b \times a$.
6. *Associative law for multiplication.* $(a \times b) \times c = a \times (b \times c)$.
7. *Distributive law for multiplication and addition.*
 $a \times (b + c) = (a \times b) + (a \times c)$.

Postulates for Common Types of Algebraic Structures

225. There are a number of algebraic structures in common use which have associated with them one or more of the postulates listed in Table 11.2.

TABLE 11.2. LIST OF POSTULATES, ONE OR MORE OF WHICH ARE ASSOCIATED WITH ALGEBRAIC STRUCTURES IN COMMON USE

Let S be a set of elements a, b, c, \dots . Let \oplus and \otimes denote two binary operations on the set S . Let the "equality" symbol $=$ be used in the sense of identity; that is, $a = b$ means that a and b are the same element and $a \neq b$ means that a and b are not the same element.

(The operations \oplus and \otimes are called "addition" and "multiplication," respectively, but this does not mean that \oplus and \otimes always stand for ordinary addition and ordinary multiplication.)

For each a, b, c, \dots of S :

- A₁. *Closure.* $a \oplus b$ is a unique element in S .
- A₂. *Associativity.* $(a \oplus b) \oplus c = a \oplus (b \oplus c)$.
- A₃. *Identity element.* There exists in S an element z , called the *identity* or *unit*, such that for any element in S ; $a \oplus z = a$. (z is called "zero," or 0 .)
- A₄. *Inverse element under \oplus .* For each element a in S there exists an element \bar{a} in S , called the *inverse* of a with respect to operation \oplus , such that $a \oplus \bar{a} = z$. [\bar{a} is often written $(-a)$, so that $a \oplus (-a) = 0$.]
- A₅. *Commutativity.* $a \oplus b = b \oplus a$.
- M₁. *Closure.* $a \otimes b$ is a unique element in S .
- M₂. *Associativity.* $(a \otimes b) \otimes c = a \otimes (b \otimes c)$.
- M₃. *Identity element.* There exists in S an element u , called the *identity* or *unit* for the operation \otimes , such that for every element a in S , $a \otimes u = a$. (u is called "unity.")
- M₄. *Inverse element under \otimes .* For each element a of S , except z , there exists an element a^{-1} in S , called the *inverse* of a with respect to operation \otimes , such that $a \otimes a^{-1} = u$.
- M₅. *Commutativity.* $a \otimes b = b \otimes a$.
- M₆. *No proper divisors.* The set S contains no proper divisors of zero.
- D₁. *Left-hand distributive law of \otimes over \oplus .*
 $a \otimes (b \oplus c) = (a \otimes b) \oplus (a \otimes c)$.
- D₂. *Right-hand distributive law of \otimes over \oplus .*
 $(b \oplus c) \otimes a = (b \otimes a) \oplus (c \otimes a)$.
- C₁. *Left-hand cancellation law for the operation \otimes .* If $c \neq z$ and $c \otimes a = c \otimes b$, then $a = b$.
- C₂. *Right-hand cancellation law for the operation \otimes .* If $c \neq z$ and $a \otimes c = b \otimes c$, then $a = b$.

Laws of Order

- O₁. There exists a subset P of the set S such that if $a \neq z$, then one and only one of a and \bar{a} is in P .
- O₂. If a and b are in P , then $a \oplus b$ is in P .
- O₃. If a and b are in P , then $a \otimes b$ is in P .

Definitions of Symbols \otimes and \ominus . The elements of set P are called the *positive* elements of S ; all other elements of S , except the *zero* z , are called the *negative* elements of S . If a and b are in S and if $a \oplus \bar{b}$ is positive, then we write $a \otimes b$ and $b \ominus a$.

Divisors of Zero. If $a \otimes b = z$ and $b \neq z$, a is called a *divisor of zero*. A number $a \neq z$ which has this property is called a *proper divisor of zero*. Nonzero elements a and b having product $a \otimes b = z$ are called "divisors of zero."

226. POSTULATE SETS FOR SEVERAL TYPES OF ALGEBRAIC STRUCTURES Different selections from the postulates listed in Table 11.2 have been used to define a variety of algebraic structures. Several systems are defined below. The sets of postulates listed are in some cases redundant (i.e., fewer postulates could be used).

Ordinary Elementary Algebra. The 18 postulates of Table 11.2 constitute a postulate set for ordinary elementary algebra.

Basic Properties of the Positive Integers. The basic properties of the positive integers are given by postulates $A_1, A_2, A_5; M_1, M_2, M_3, M_5; D_1, D_2$.

Group. M_1, \dots, M_4 constitute a postulate set for a group. If M_5 holds, the group is an *abelian* or *commutative group*.

Field. $A_1, \dots, A_5; M_1, \dots, M_5; D_1, D_2$ constitute a postulate set for a *field*.

Ordered Field. $A_1, \dots, A_5; M_1, \dots, M_5; D_1, D_2; O_1, O_2, O_3$ constitute a postulate set for an *ordered field*.

Noncommutative Field. $A_1, \dots, A_5; M_1, M_2, M_3, M_4; D_1, D_2$ constitute a postulate set for a *noncommutative field*.

Ring. $A_1, \dots, A_5; M_1, M_2; D_1, D_2$ constitute a postulate set for a *ring*.

Commutative Ring. A ring satisfying postulate M_5 is a *commutative ring*.

Ring with Unit (Identity) Element. A ring satisfying postulate M_3 is a *ring with unit element*.

Integral Domain. $A_1, \dots, A_5; M_1, M_2, M_3, M_5; C_1, C_2; D_1, D_2$ constitute a postulate set for an *integral domain*, or *domain of integrity*.

227. ALTERNATE DEFINITIONS A more informative way of defining the structures listed above is as follows:

A *field* is a set S of at least two elements for which two binary operations \oplus and \otimes are defined such that (1) the elements of S constitute an abelian group under addition \oplus ; (2) the nonzero elements of S constitute an abelian group under multiplication \otimes ; and (3) multiplication is distributive over addition. An *ordered field* is a field for which postulates O_1, O_2, O_3 hold.

A *ring* is a set R of at least two elements for which two binary operations \oplus and \otimes are defined such that (1) the elements of R constitute an abelian group under addition \oplus , the identity element being denoted by z , and the inverse of a by $-a$; (2) multiplication is distributive with respect to addition; (3) multiplication is associative.

An *integral domain* D is a commutative ring with unit element which is not necessarily a field because postulate M_4 is replaced by postulate M_6 . $A_1, \dots, A_5; M_1, M_2, M_3, M_5, M_6; D_1, D_2$ constitute a postulate set for an integral domain. Instead of M_6 , the postulates C_1 and C_2 can be used.

A *noncommutative field* is a ring with unit element "unity" satisfying M_4 .

Remark. Slight variations of the above definitions are given by different writers.

Examples: With respect to addition and multiplication, each of the indicated sets forms the following:

Field: (a) all rational numbers; (b) all real numbers; (c) all numbers $a + b\sqrt{2}$, where a and b are rational.

Ring: (a) the positive and negative integers and zero; (b) the integers modulo m ; (c) the even integers.

Integral domain: (a) the polynomials whose coefficients lie in an integral domain; (b) the set of all numbers $a + b\sqrt{2}$, where a and b are integers.

228. THEOREMS ON FIELDS Let a, b, c, d, \dots be elements of a field F , with identity element ("zero") z and "unity" element u ; let $-a$ denote the addition inverse \bar{a} of a and a^{-1} denote the multiplicative inverse of a :

(a) $a \otimes b = z$ if and only if $a = z$ or $b = z$, or both $a = z$ and $b = z$.

(b) $a \oplus x = b$ has a unique solution in F .

(c) $a \otimes x = b$ has a unique solution in F if $a \neq z$.

- (d) If $a = b$ and $c = d$, then $a \oplus c = b \oplus d$.
- (e) If $a = b$ and $c = d$, then $a \otimes c = b \otimes d$.
- (f) $[-(-a)] = a$.
- (g) $[a^{-1}]^{-1} = a$, provided $a \neq x$.
- (h) $a \otimes z = z$ for all a in F .
- (i) $a \otimes (-b) = -(a \otimes b)$.
- (j) $(-a) \otimes (-b) = a \otimes b$, or $\bar{a} \otimes \bar{b} = a \otimes b$.

229. THEOREMS ON INTEGRAL DOMAINS Let a, b, c, \dots be elements of an integral domain D :

- (a) The zero element z is unique.
- (b) The unit element u is unique.
- (c) If $a \oplus b = a \oplus c$, then $b = c$.
- (d) The additive inverse \bar{a} of a is unique.
- (e) $a \oplus x = b$ has exactly one solution x .
- (f) For all a in D , $a \otimes z = z \otimes a = z$.
- (g) For all a and b in D , $\bar{a} \otimes \bar{b} = a \otimes b$.
- (h) $a \otimes b = z$ if and only if $a = z$ or $b = z$ or both $a = z$ and $b = z$.

230. THEOREMS ON RINGS Many of the theorems on integral domains, which do not depend on the cancellation law or the commutative law, apply to rings.

231. ISOMORPHIC MODELS OF POSTULATE SYSTEMS Two models (interpretations) M_1 and M_2 of a postulate (axiom) system are said to be *isomorphic with respect to the postulate system* Σ if there exists a one-to-one correspondence between the elements of M_1 and M_2 which preserves the postulate statements of Σ (i.e., preserves the relations and the operations of Σ).

232. HOMOMORPHIC CORRESPONDENCE Let A and B denote two mathematical systems. Let \circ_A be a well-defined operation which relates to every ordered pair of elements a_i, a_j of A a unique element a_{ij} of A , and denote this by $a_i \circ_A a_j = a_{ij}$. Similarly, let \circ_B be a well-defined operation relating every ordered pair of elements b_i, b_j of B with a unique element b_{ij} of B , written $b_i \circ_B b_j = b_{ij}$. Suppose there is a correspondence, denoted by $a_i \rightarrow b_i$, by means of which every

element a_i of A determines a unique element b_i of B , such that every element b_i of B is the correspondent of at least one a_i of A . If the correspondence $a_i \rightarrow b_i, a_j \rightarrow b_j$ implies the correspondence $a_{ij} \rightarrow b_{ij}$, that is, the correspondence $a_i \circ_A a_j \rightarrow b_i \circ_B b_j$, then the correspondence is called a *homomorphism* (or a *mapping*) of A onto B . We write $A \sim B$ to mean that a homomorphism of A onto B exists.

In case each element b_i of B is the correspondent of one and only one element a_i of A , the correspondence is called a *biunique correspondence*, or a *one-to-one reciprocal correspondence*. This is written $a_i \leftrightarrow b_i$. If the correspondence $a_i \leftrightarrow b_i, a_j \leftrightarrow b_j$ implies $a_i \circ_A a_j = a_{ij} \leftrightarrow b_i \circ_B b_j = b_{ij}$, the homomorphism is an *isomorphism*. In this case we write $A \cong B$.

An *endomorphism* is a homomorphism of a system onto itself, or onto a part of itself.

An *automorphism* is an isomorphism of a system onto (or with) itself.

Boolean Algebra

233. A POSTULATE SET FOR BOOLEAN ALGEBRA A Boolean algebra is a set B of elements a, b, c, \dots with two binary operations \cup and \cap , satisfying the following postulates. (Some writers use the symbols \oplus and \otimes , others $+$ and \times , to denote the operation \cup and \cap .)

For all elements a, b, \dots of B :

1. $a \cup b$ and $a \cap b$ are each unique elements in B . (*closure*)
2. $a \cup b = b \cup a$ and $a \cap b = b \cap a$. (*commutative*)
3. There exists an element (*zero*) z in B and an element u (*unity*) in B such that $a \cup z = a, a \cap u = a, z \neq u$.
4. $a \cup (b \cap c) = (a \cup b) \cap (a \cup c)$ (*distributive*),
 $a \cap (b \cup c) = (a \cap b) \cup (a \cap c)$.
5. For each a there exists an element a' of B such that $a \cup a' = u$ and $a \cap a' = z$. (a' is *complement* of a .)

The elements a, b, c, \dots and operations \cup and \cap may be considered undefined. $a \cup b$ is called the (logical) *sum* and $a \cap b$ the (logical) *product* of a and b . For any two elements a and $b, a \cup b$ and $a \cap b$ are each unique elements of B .

234. The subsets of a given set B under the operations of union and intersection satisfy the postulates for a Boolean algebra. In this application, a, b, c, \dots are the subsets, z is represented by the null set \emptyset ,

and u by the universe I , and $=$ means "is" or "equals." The algebra of sets is one representation of a Boolean algebra.

235. THEOREMS Each formula listed in Sec. 161, with A, B, C, \dots , \emptyset , and I replaced by a, b, c, \dots, z , and u , respectively, is a theorem derivable from the postulates for a Boolean algebra. Thus:

Duality Principle. If in any theorem of Boolean algebra the symbols \cup and \cap , the elements z and u , and \subset and \supset are interchanged throughout, the resulting statement is valid.

236. FURTHER PROPERTIES

$$\begin{aligned} (a \cup b) \cup c &= a \cup (b \cup c) && \text{(associative law)} \\ (a \cap b) \cap c &= a \cap (b \cap c) && \text{(associative law)} \\ a \cup a &= a, a \cap a = a && \text{(idempotent-property)} \\ u \cup u &= u, a \cap z = z \\ a \cap u &= a, a \cup z = a \\ a \cap (a \cup b) &= a && \text{(absorption law)} \\ a \cup (a \cap b) &= a && \text{(absorption law)} \\ a' &\text{ is unique} \\ (a \cup b)' &= a' \cap b' && \text{(dualization)} \\ (a \cap b)' &= a' \cup b' && \text{(dualization)} \\ (a')' &= a, u' = z, z' = u \\ a \cup (a' \cap b) &= a \cup b \\ (a \cap b) \cup (a \cap c) \cup (b \cap c') &= (a \cap c) \cup (b \cap c') \end{aligned}$$

237. DEFINITION OF INCLUSION \subset Let a and b be elements of B . $a \subset b$ if and only if $a \cup b = b$.

Theorems. Let a, b, c, \dots be elements of B :

- (a) $a \subset a$ for all a . (reflexive)
- (b) If $a \subset b$ and $b \subset a$, then $a = b$. (antisymmetric)
- (c) If $a \subset b$ and $b \subset c$, then $a \subset c$. (transitive)
- (d) The elements of a Boolean algebra are partially ordered with respect to relation \subset .
- (e) For each a of B , $z \subset a \subset u$. (universal bound)
- (f) If $a \subset x$ and $b \subset x$ where x is in B , then $(a \cup b) \subset x$.
- (g) $a \cup b = b$ if and only if $a \cap b = a$. (consistency)
- (h) $a \subset b$ if and only if $a \cap b = a$. (consistency)

238. BOOLEAN FUNCTIONS Let x_1, x_2, \dots, x_n be n Boolean variables, each of which can equal any element of a Boolean algebra B . A *Boolean function* (or *Boolean polynomial*), written $Y = F(x_1, x_2, \dots, x_n)$, is an expression built up from x_1, x_2, \dots, x_n through the operations of \cup , \cap and complementation '.

A *minimal polynomial* in n variables x_1, \dots, x_n is a logical product of n letters in which the i th letter is either x_i or the complement x_i' .

Theorem. Every Boolean function can be expressed uniquely as a logical sum of minimal polynomials or is identically equal to 0.

239. MEASURE ALGEBRA A Boolean algebra B with elements A_1, A_2, A_3, \dots is *completely additive* if and only if every sum $A_1 \cup A_2 \cup \dots$ is uniquely defined in B . A completely additive Boolean algebra B is a *measure algebra* if and only if there exists a real valued *measure function* M defined for all elements A of B , such that

$$\begin{aligned} M(A) &\geq 0 & M(0) &= 0 \\ M(A_1 \cup A_2 \cup \dots) &= M(A_1) \cup M(A_2) \cup \dots \end{aligned}$$

for every countable set of disjoint elements A_1, A_2, \dots . Here $M(A)$ denotes the *value* of the measure M associated with the element A .

240. ALGEBRA OF EVENTS ASSOCIATED WITH A GIVEN EXPERIMENT

Let S denote a set (space) of sample descriptions of all possible *outcomes* (states, events) of a given experiment. An *event* E is any subset of S . To say that the event E is *realized*, or *has occurred*, is to say that the outcome of a specific experiment under consideration has a description that is a member of subset E .

Suppose the events E_1, E_2, \dots are such as to permit the following definitions:

1. The *union* (*logical sum*, or *join*) of a set of events E_1, E_2, \dots , written $E_1 \cup E_2 \cup E_3 \cup \dots$, is the outcome of realizing *at least one* of the events E_1, E_2, \dots .
2. The *intersection* (*logical product*, or *meet*) of two events E_1 and E_2 , written $E_1 \cap E_2$, is the joint outcome of realizing *both* E_1 and E_2 .
3. The (*logical*) *complement* of an event E is the outcome E' of not realizing E . $E' + E = S$. $E \cap E' = \emptyset$.
4. The *certain event* I is the set S (i.e., at least one of the outcomes of S is realized in a specific experiment).

5. The *impossible event* \emptyset is an event that contains no descriptions in S , and therefore cannot occur in the experiment. $S' = \emptyset$.

241. DEFINITIONS Events E_1 and E_2 are said to be *mutually exclusive*, or *disjoint*, if and only if $E_1 \cap E_2$ is \emptyset , that is, if and only if the joint event of simultaneously realizing *both* E_1 and E_2 is impossible.

If E_1 and E_2 are events of the set S , we say $E_1 \subset E_2$ if and only if $E_1 \cup E_2 = E_2$. ($E_1 \subset E_2$ is read " E_1 is included in E_2 ."") The symbol $=$ here means "is". In other words, the event E_1 is included in the event E_2 if and only if the event of realizing at least one of the two events E_1 and E_2 is the realization of the event E_2 .

Theorem. $E_1 \subset E_2$ if and only if $E_1 \cap E_2 = E_1$. In other words, the event E_1 is included in the event E_2 if and only if the event of realizing both events E_1 and E_2 is the realization of the event E_1 .

Theorem. For any event E , $\emptyset \subset E \subset I$.

242. DEFINITION If a set B , consisting of set S and the impossible event \emptyset , has the structure of a completely additive Boolean algebra and has all the properties defined in Secs. 233 and 239, then B is called an *algebra of events* associated with the given experiment.

The algebra of events associated with the given experiment under the hypothesis that event E_1 occurs is the set B_1 of joint events $E \cap E_1$. In this case the certain event in B_1 is $E_1 \cap E_1 = E_1$.

243. EVENT ALGEBRAS AND SYMBOLIC LOGIC In two-valued (Aristotelian) logic an algebra \mathfrak{B} of events (logical propositions, assertions) is related to a simpler Boolean algebra \mathfrak{T} of truth-values. A truth-value $T[E]$ is taken to be equal to 1 if an event E is realized (i.e., if E is true) and equal to 0 if the event E is not realized (i.e., if E is false). Let I denote a certain event and \emptyset denote the impossible event. The relation between the algebras \mathfrak{B} and \mathfrak{T} is given by the homomorphism

$$(1) \quad \begin{aligned} T[I] &= 1 & T[\emptyset] &= 0 \\ T[E_1 \cup E_2] &= T[E_1] \oplus T[E_2] & T[E_1 \cap E_2] &= T[E_1] \otimes T[E_2] \end{aligned}$$

The truth tables for the events $E_1 \cup E_2$ and $E_1 \cap E_2$ and the truth-values T , with associated arithmetic, are shown in Tables 11.3 and 11.4.

TABLE 11.3. TRUTH TABLE

E_1	E_2	$E_1 \cup E_2$	$E_1 \cap E_2$
T	T	T	T
T	F	T	F
F	T	T	F
F	F	F	F

TABLE 11.4. ARITHMETIC OF TRUTH-VALUES

$T[E_1] \oplus T[E_2] = T[E_1 \cup E_2]$				$T[E_1] \otimes T[E_2] = T[E_1 \cap E_2]$					
1	\oplus	1	=	1	1	\otimes	1	=	1
1	\oplus	0	=	1	1	\otimes	0	=	0
0	\oplus	1	=	1	0	\otimes	1	=	0
0	\oplus	0	=	0	0	\otimes	0	=	0

The truth-value $T[E]$ of any proposition E logically related to a set of events E_1, E_2, \dots , and expressible as a (Boolean) function $E = F(E_1, E_2, \dots)$ of events E_1, E_2, \dots , may be written $T[E] = T[G\{T[E_1], T[E_2], \dots\}]$, where G is the homomorphic image of F obtained by replacing \cup and \cap by \oplus and \otimes , respectively. In the arithmetic of truth-values, $0' = 1, 1' = 0$. In view of the assumptions made with regard to the algebras used here, any proposition E (i.e., an event) is either true or false.

244. APPLICATION TO SWITCHING CIRCUITS The two-valued Boolean algebras mentioned above have been used extensively in the theory of electric switching. Consider a simple electric circuit X joining terminal point α to terminal point β (Fig. 61). The circuit X is either *open* or *closed*. In the event E that circuit X is open, the truth value $T[E]$ of the event E is taken to be 0; and in the event E that the circuit X is closed, the truth-value $T[E]$ is taken to be 1. A similar



Fig. 61

statement holds if the word "circuit" is replaced by the word "switch." The box C in Fig. 61 may be considered a network of circuits with a terminal at each end.

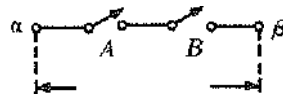


Fig. 62. Series circuit. Truth-value for circuit joining α to β is the truth-value of $A \cap B$, that is, $T[A \cap B] = T[A] \otimes T[B]$. When $T[A \cap B] = 0$, the circuit is open; when $T[A \cap B] = 1$, the circuit is closed.

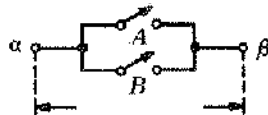
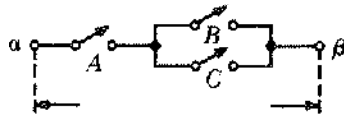


Fig. 63. Parallel circuit. Truth-value for circuit joining α to β is the truth-value of $A \cup B$, that is, $T[A \cup B] = T[A] \oplus T[B]$. When $T[A \cup B] = 0$, the circuit is open; when $T[A \cup B] = 1$, the circuit is closed.



Boolean Polynomial: $A \cap (B \cup C)$

Fig. 64. Series-parallel circuit. When $T[A \cap (B \cup C)] = 0$, the circuit joining α and β is open; when $T[A \cap (B \cup C)] = 1$, the circuit is closed.

245. SERIES CIRCUIT Consider the series circuit in Fig. 62. In the event switch A is open, the corresponding truth-value is $T[A] = 0$; if A is closed, $T[A] = 1$. If B is open, $T[B] = 0$; if B is closed, $T[B] = 1$. In the event both A and B are closed, the series circuit joining α to β of Fig. 62 is closed; if either A or B or both A and B are open, then the circuit joining α and β is open. In other words the truth-value for the series circuit joining α and β is $T[A \cap B] =$

$T[A] \otimes T[B]$: When $T[A \cap B] = 1$, the series circuit is closed; when $T[A \cap B] = 0$, the series circuit is open. (Compare with Table 11.4.)

246. PARALLEL CIRCUIT The parallel circuit of Fig. 63 may be treated similarly.

247. OTHER TYPES OF CIRCUITS In electrical problems the actual circuit diagrams are used to help set up the corresponding Boolean functions (polynomials) whose truth-values reflect when a circuit is open or closed in terms of the state of the open or closed conditions of the component circuit or switches. Figure 64 is such an example.

12. NUMBER SYSTEMS

248. There are several elementary theorems involving number systems which are fundamental in the field of digital computers.

Algorithms. An *algorithm* is a step-by-step iteration process for solving a given problem. Some algorithms involve trial or tentative steps (e.g., division), others are straightforward (e.g., multiplication). Before a digital computer can be programmed to solve problems of a given type, a complete algorithm must be devised.

249. EUCLIDEAN ALGORITHM Let m and n be two positive integers, $m > n$. By Euclid's theorem, integers q_i and n_i exist, $q_i \geq 0$, such that

$$\begin{aligned}
 (1) \quad & m = q_0 n + n_1 && 0 < n_1 < n \\
 & n = q_1 n_1 + n_2 && 0 < n_2 < n_1 \\
 & \dots && \dots \\
 & n_{k-2} = q_{k-1} n_{k-1} + n_k && 0 < n_k < n_{k-1} \\
 & n_{k-1} = q_k n_k
 \end{aligned}$$

If m is a multiple of n , write $n = n_0$, $k = 0$ in (1).

Theorem. The number n_k to which the above process leads is the *greatest common divisor* of m and n .

250. SCALES OF NOTATION If m and r are positive integers and $r > 1$, then m can be represented in terms of r uniquely in the form

$$(2) \quad m = p_h r^h + p_{h-1} r^{h-1} + \dots + p_1 r + p_0$$

where $p_h \neq 0$ $0 \leq p_i < r$ $i = 0, 1, \dots, h$

Representation (2) may be written

$$(3) \quad m = (p_h p_{h-1} \dots p_1 p_0)_r$$

Here r , called the *base* or *radix*, is the number of different digits used. In any positional number system the digits $0, 1, 2, \dots, (r - 1)$ have an order. *Advancing the digit 0* means replacing it by the digit 1;

advancing the digit 1 means replacing it by the digit 2 (if $r > 2$), . . . ; advancing the digit $(r - 1)$ means replacing it by the digit 0. If any digit advances to 0, the digit at its left advances.

251. If N is any real positive number and r is a positive integer, $r > 1$, then N can be represented uniquely as

$$(4) \quad N = p_n r^n + p_{n-1} r^{n-1} + \dots + p_1 r + p_0 + p_{-1} r^{-1} + \dots + p_{-m} r^{-m} + \dots$$

and (4) may be written

$$(5) \quad N = (p_n p_{n-1} \dots p_1 p_0 p_{-1} p_{-2} \dots p_{-m} \dots)_r$$

where $p_n \neq 0$ $0 \leq p_i < r$
 $i = n, n - 1, \dots, 1, 0, -1, -2, \dots, -m, \dots$

If N is a rational fraction, that is, the ratio of two integers, (4) either terminates or recurs; if N is irrational, (4) is nonrecurring and non-terminating.

The principal systems in use are those having radix 2, 3, 8, 10, 12, 16, and 20.

Table 12.1 gives examples of several common number systems. The decimal system with radix 10 is the one most commonly used. From the elementary notions of addition and multiplication and rules for counting, tables for addition and multiplication can be constructed. Thus, in the ternary system $2 + 1 = 10$, $2 + 2 = 11$, $2 \times 1 = 2$, $2 \times 2 = 11$. In the octal system, $6 + 7 = 15$, $7 \times 2 = 16$. Samples of addition and multiplication tables are given in Table 12.2.

252. CONVERSION RULES To convert a representation of an integer m :

1. In a system with radix r to the equivalent integer in decimal system, use Eq. (2).
2. In a decimal system to equivalent representation in a system with radix r , divide m repeatedly by r and take the remainders in reverse order (i.e., use a division algorithm).

To convert a representation of a real positive number:

3. From a system with radix r to equivalent representation in decimal system, use Eq. (4).

TABLE 12.1. EXAMPLES OF NUMBER SYSTEMS*

	<i>Binary</i>	<i>Ternary</i>	<i>Octal</i>	<i>Decimal</i>	<i>Duodecimal</i>
Radix	2	3	8	10	12
Digits used	0, 1	0, 1, 2	0, 1, 2, 3, 4, 5, 6, 7	0, 1, 2, 3, 4, 5, 6, 7, 8, 9	0, 1, . . . , 8, 9, <i>t</i> , <i>e</i>
	00000	0000	0000	0000	0000
	00001	0001	0001	0001	0001
	00010	0002	0002	0002	0002
	00011	0010	0003	0003	0003
	00100	0011	0004	0004	0004
	00101	0012	0005	0005	0005
	00110	0020	0006	0006	0006
	00111	0021	0007	0007	0007
	01000	0022	0010	0008	0008
	01001	0100	0011	0009	0009
	01010	0101	0012	0010	000 <i>t</i>
	01011	0102	0013	0011	000 <i>e</i>
	01100	0110	0014	0012	0010
	01101	0111	0015	0013	0011
	01110	0112	0016	0014	0012

* The quantities in the same horizontal row have the same cardinal value.

4. From a decimal system to equivalent representation in system of radix r , convert its integral and fractional parts separately. Use Rule 2 to convert integral part. To convert (decimal) fractional part, multiply repeatedly by r and take out the resulting integral parts in *forward order* (i.e., use multiplication algorithm).

Examples. (a) Convert $(1012)_7$ to decimal system:
 $(1012)_7 = 1(7)^3 + 0(7)^2 + 1(7)^1 + 2(7)^0 = (352)_{10}$.

(b) Convert $(18)_{10}$ to binary system:
 $18 = 2(9) + 0$, $9 = 2(4) + 1$, $4 = 2(2) + 0$, $2 = 2(1) + 0$,
 $1 = 2(0) + 1$. Hence $(18)_{10} = (10010)_2$.

(c) Convert $(21.201)_5$ to decimal system:
 $(21.201)_5 = 2(5)^1 + 1(5)^0 + 2(5)^{-1} + 0(5)^{-2} + 1(5)^{-3} = (11.408)_{10}$.

(d) Convert $(0.375)_{10}$ to binary system: $2(0.375) = 0 + 0.75$,

TABLE 12.2. SAMPLES OF ADDITION AND MULTIPLICATION TABLES

+	0	1
0	0	1
1	1	10

+	0	1	2
0	0	1	2
1	1	2	10
2	2	10	11

×	0	1	10
0	0	0	0
1	0	1	10
10	0	10	100

×	0	1	2	10
0	0	0	0	0
1	0	1	2	10
2	0	2	11	20
10	0	10	20	100

$2(0.75) = 1 + 0.5$, $2(0.5) = 1 + 0.0$, $2(0.0) = 0 + 0.0$. Hence $(0.375)_{10} = (.0110)_2$.

(e) Convert $(18.375)_{10}$ to binary system: By Examples (b) and (d), $(18)_{10} = (10010)_2$ and $(0.375)_{10} = (0.0110)_2$. Hence $(18.375)_{10} = (10010)_2 + (0.0110)_2 = (10010.0110)_2$.

Remark. In converting from a binary to a decimal representation it is often simpler to convert from binary to octal to decimal.

Exactly eight numbers can be represented with three binary digits. Triplets of binary digits may be used as marks to represent octal digits. Conversion from binary to octal notation simply implies changing the notation used for the marks. The change is made triplet by triplet, beginning at the right. Thus

$$(10,001,001)_2 = (211)_8 \quad (101,100,111)_2 = (547)_8$$

253. SUBTRACTION BY COMPLEMENTS Some computers subtract by adding the complement of the subtrahend to the minuend and then making suitable adjustments.

Let k be any n -order decimal integer. The *tens complement* of k is $10^n - k$. The *nines complement* of k is $(10^n - k) - 1$.

Let b be any n -order binary integer. The *twos complement* of b is $2^n - b$. The *ones complement* of b is $(2^n - b) - 1$.

TABLE 12.3. EXAMPLES ILLUSTRATING COMPLEMENTS

Decimal number k	Tens complement*	Nines complement†
021	979	978
0021	9979	9978
05555	94445	94444

Binary number b	Twos complement‡	Ones complement§
1	1	0
100	100	011
101	011	010
0101	1011	1010
10110	01010	01001

* *Algorithm:* Subtract each digit in k from 9; then add 1 to the resulting number.

† *Algorithm:* Subtract each digit in k from 9.

‡ *Algorithm:* Change each digit in b ; then add 1 to the resulting number.

§ *Algorithm:* Change each digit in b .

254. RULE FOR SUBTRACTION BY TENS (TWOS) COMPLEMENT To the minuend add the tens (twos) complement of the subtrahend. If there is an extra column with digit 1, replace 1 by +; otherwise tens-(twos-)complement the result and prefix -.

255. RULE FOR SUBTRACTION BY NINES (ONES) COMPLEMENT To the minuend add the nines (ones) complement of the subtrahend. If there is an extra column with digit 1, then "end-around-carry"; otherwise, nines- (ones-)complement the result and prefix -. To "end-around-carry" means to replace the extra column with digit 1 by + and carry the 1 to the units column, where it is added to the number.

Examples. (a) Subtract $(076)_{10}$ from $(324)_{10}$. The nines complement of 076 is 923. Add. $923 + 324 = 1247$. Replace extra column 1 by +, getting 247. Carry 1 to units column and add, giving the answer 248.

(b) Subtract $(324)_{10}$ from $(076)_{10}$. The nines complement of 324 is 675. Add. $675 + 076 = 751$. There is no extra column 1. The nines complement of 751 is 248. Prefix -, giving the answer -248.

(c) Subtract $(01110001)_2$ from $(11001101)_2$. The ones complement of 01110001 is 10001110 . Add. $10001110 + 11001101 = 101011011$. There is an extra column 1. Hence the answer is $+01011011 + 1 = (01011100)_2$.

(d) Subtract $(11001101)_2$ from $(01110001)_2$. The ones complement of 11001101 is 00110010 . Add. $00110010 + 01110001 = 10100011$. There is no extra column 1. Hence the answer is $-(01011100)_2$.

256. FIXED- AND FLOATING-POINT ARITHMETIC The decimal (n -ary) point divides the integral and fractional parts of a decimal (n -ary) number. In fixed-point notation, all zeros between the point and the first significant digit must be written down. In floating-point notation, only the significant digits and a power of the base of the number system are indicated. Thus:

Notation for	Fixed-point		Floating-point
Decimal	3,078,400.0	0.30784×10^7	$30,784 + 7$
	0.0034123	0.34123×10^{-2}	$34,123 - 2$
Binary	10101000.0	0.10101×2^8	$10,101 + 8$
	0.00010101	0.10101×2^{-3}	$10,101 - 3$

257. CONGRUENCES If a and b are any two integers (positive, zero, or negative) whose difference is divisible by m , a and b are said to be *congruent modulo m* , written

$$a \equiv b \pmod{m}$$

Each of the numbers a and b is said to be a *residue* of the other. m is called the *modulus*.

Theorems. If $a \equiv b \pmod{m}$, $d \equiv b \pmod{m}$, $\alpha \equiv \beta \pmod{m}$, c and n are any two integers, $n > 0$, and $f(x)$ is any polynomial in x with integer coefficients, then

$$\begin{array}{ll} a \equiv d \pmod{m} & a \pm \alpha \equiv (b \pm \beta) \pmod{m} \\ ca \equiv cb \pmod{m} & a\alpha \equiv b\beta \pmod{m} \\ a^n \equiv b^n \pmod{m} & f(a) \equiv f(b) \pmod{m} \end{array}$$

13. MATRICES

258. A *matrix* is an array of $m \times n$ numbers (real or complex)

$$\begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \cdots & \cdots & \cdots & \cdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{pmatrix}$$

arranged in m rows and n columns which obey the rules of addition and multiplication given in Sec. 259. The matrix may be denoted by $[a_{ij}]$ or (a_{ij}) or $\|a_{ij}\|$ or A , where a_{ij} denotes the element in the i th row and j th column. The *order* of the matrix is the number pair (m, n) , written $m \times n$. The matrix is *square* of order n if $m = n$.

A *column matrix*, or *column vector*, is an $m \times 1$ matrix, written $X = \{x_i\}$ or $x_i \uparrow$.

A *row matrix*, or *row vector*, is a $1 \times n$ matrix, written $U = (u_i)$ or $[u_j]$ or \bar{u}_i or $U = (u_1, u_2, \dots, u_n)$.

259. OPERATIONS Let $A = [a_{ij}]$ and $B = [b_{ij}]$ be two matrices of the same order $m \times n$.

1. *Equals.* A and B are *equal* if and only if the elements in like positions in the arrays are equal (that is, if $a_{ij} = b_{ij}$ for $i = 1, \dots, m$ and $j = 1, \dots, n$).

2. *Addition.* The *sum* of A and B is the matrix $C = A + B$, where $C \equiv [c_{ij}]$, with $c_{ij} = a_{ij} + b_{ij}$.

3. *Multiplication.* If A is of order $m \times n$ and B is of order $n \times p$, the *product* AB is the matrix $D = AB$, where

$$D \equiv [d_{ij}] \quad \text{with } d_{ij} = \sum_{k=1}^n a_{ik} b_{kj}$$

4. *Product of Scalar and Matrix.* The product of a (real or complex) number λ , called a *scalar*, and a matrix A is the matrix in which each element is λ times the corresponding element of A ; that is,

$$\lambda A \equiv \lambda[a_{ij}] = [(\lambda a_{ij})] = A\lambda$$

260. A *null* or *zero matrix*, 0 , is a matrix in which every element is zero.

An *identity (unit) matrix* I is a square matrix $[a_{ij}]$ in which $a_{ij} = \delta_{ij}$, where $\delta_{ij} = 1$ if $i = j$, $\delta_{ij} = 0$ if $i \neq j$. (δ_{ij} is the *Kronecker delta*.)

A *diagonal matrix* is a matrix all of whose elements off the *main diagonal* (the set of positions a_{ii}) are zero.

A *scalar matrix* is a diagonal matrix whose main diagonal elements are all equal. If λ is a scalar, λI is a scalar matrix.

261. DETERMINANT Associated with a square matrix A is a number $d(A)$, called the *determinant* of A , defined by

$$d(A) = \sum (-1)^h a_{1i_1} a_{2i_2} \cdots a_{ni_n}$$

where the summation is taken over all permutations i_1, i_2, \dots, i_n of the integers $1, 2, \dots, n$, and h is the number of successive interchanges of two i s which would be necessary to permute the order i_1, i_2, \dots, i_n into the order $1, 2, \dots, n$. See Sec. 17 for theorems on determinants.

262. RANK OF MATRIX Let $A_{\substack{r_1 \cdots r_k \\ s_1 \cdots s_k}}$ be the square *minor matrix* (array) formed from the square matrix A by selecting from A the elements in rows $r_1 \cdots r_k$ and columns $s_1 \cdots s_k$, in those orders. If the same columns as rows are selected, the minor matrix is called a *principal minor matrix*.

The order ρ of a minor square array of A of maximum order whose determinant is not zero is the *rank* of A . A matrix whose order exceeds its rank is *singular*. If $d(A) \neq 0$, the rank of A is equal to its order.

Theorem. The rank of AB cannot exceed the rank of either A or B . If B and C are nonsingular, then A, AB, CA have the same rank.

263. LAPLACE'S EXPANSION Let any distinct rows r_1, r_2, \dots, r_k be selected from square matrix A . Then

$$d(A) = \sum (-1)^h d \left(A_{\substack{r_1 \cdots r_k \\ i_1 \cdots i_k}} \right) d \left(A_{\substack{r_{k+1} \cdots r_n \\ i_{k+1} \cdots i_n}} \right)$$

where h is equal to the number of any set of interchanges necessary to bring the order i_1, i_2, \dots, i_n into the order r_1, r_2, \dots, r_n , and where

the summation is taken over the $n!/k!(n-k)!$ ways of choosing the combinations i_1, i_2, \dots, i_k from the integers $1, 2, \dots, n$.

The *cofactor* of a_{rs} is defined as

$$A_{rs} = (-1)^{r+s} d \left(A \begin{array}{c} 1, 2, \dots, r-1, r+1, \dots, n \\ 1, 2, \dots, s-1, s+1, \dots, n \end{array} \right)$$

$$\begin{aligned} \text{Theorem. } d(A) &= \sum_{i=1}^n a_{ri} A_{ri} & d(A) &= \sum_{i=1}^n a_{is} A_{is} \\ \sum_{i=1}^n a_{ri} A_{pi} &= 0 \text{ if } r \neq p & \sum_{i=1}^n a_{is} A_{ip} &= 0 \text{ if } s \neq p \end{aligned}$$

264. SPECIAL MATRICES The *transpose* A^T of a matrix A is the matrix resulting from A by interchanging rows and columns of A ; that is, if $A = [a_{rs}]$, $A^T = [a_{sr}]$.

The *adjoint* or *adjugate* of a square matrix $A = [a_{rs}]$ is the matrix

$$\text{adj } A = [A_{sr}] = [A_{rs}]^T$$

where A_{rs} is the cofactor of a_{rs} in A .

The *inverse* or *reciprocal* matrix A^{-1} of A exists only if $d(A) \neq 0$. A^{-1} is defined by the equation $A^{-1}A = I$ and is unique. Furthermore, $AA^{-1} = I$ and $d(A) A^{-1} = \text{adj } A$.

If $a_{ij} = a_{ji}$, $A^T = A$ and A is *symmetric*.

If $a_{ij} = -a_{ji}$, $a_{ii} = 0$, $A^T = -A$, and A is *skew*.

If \bar{a}_{ij} is the complex conjugate of a_{ij} , $\bar{A} \equiv [\bar{a}_{ij}]$ is the *complex conjugate* of $A = [a_{ij}]$.

If $\bar{A}^T = A$, A is said to be *hermitian*.

If $\bar{A}^T = -A$, A is *skew-hermitian*.

If $A^T A = I$, where I is the unit matrix, A is *orthogonal*.

If $\bar{A}^T A = I$, A is *unitary*.

265. PROPERTIES OF MATRICES Matrices obey many of the laws of Table 11.2, Chap. 11. In general, multiplication is not commutative.

$$A + B = B + A \quad A + (B + C) = (A + B) + C \quad (AB)C = A(BC)$$

$$A(B + C) = AB + AC \quad (B + C)A = BA + CA$$

$$A - B = A + (-1)B = A + (-I)B$$

$$AA^{-1} = I \quad A^{-1}A = I$$

$$d(AB) = d(A) d(B) \quad d(A^{-1}) = 1/d(A)$$

$$\begin{aligned}
 d(AA^{-1}) &= d(A) d(A^{-1}) = d(I) = 1 & (A^T)^T &= A & (A^{-1})^{-1} &= A \\
 (A + B)^T &= A^T + B^T & (AB)^T &= B^T A^T & (A^T)^{-1} &= (A^{-1})^T \\
 (AB)^{-1} &= B^{-1} A^{-1} & \text{adj}(AB) &= \text{adj } B \text{ adj } A
 \end{aligned}$$

If A is square of order n

$$d(\text{adj } A) = (d(A))^{n-1}$$

If α and β are scalars,

$$\begin{aligned}
 \alpha(A + B) &= \alpha A + \alpha B, & (\alpha + \beta)A &= \alpha A + \beta A, \\
 \alpha(\beta A) &= \alpha\beta A, & \alpha(AB) &= (\alpha A)B = A(\alpha B)
 \end{aligned}$$

266. ELEMENTARY MATRICES An elementary operation (transformation) upon the rows (columns) of a matrix A is a transformation of one or more of the following types:

1. The interchange of rows (columns)
2. The addition to the elements of a row (column) of h times the corresponding elements of another row (column)
3. The multiplication of the elements of a row (column) by a number k

An elementary matrix is a matrix obtained by performing any elementary operation on the identity matrix I .

Let $I_{(ij)}$ be the matrix obtained from I by interchanging i th and j th rows without disturbing remaining rows. Multiplying $I_{(ij)}$ on the left (right) of A interchanges row (column) i and row (column) j of A .

Let $H_{(ij)} = I + (h)_{(ij)}$, ($i \neq j$), be the matrix obtained from I by inserting element h in i th row, j th column. Multiplying $H_{(ij)}$ on the left (right) of A adds to each element of i th row (j th column) of A , h times the corresponding element of j th row (i th column) of A .

Let $J_{(ii)} = I + (k)_{(ii)}$ be the matrix obtained from I by replacing element in i th row, column i , by k . Multiplying $J_{(ii)}$ on the left (right) of A multiplies each element in the i th row (column) of A by k .

$I_{(ij)}$, $H_{(ij)}$, $J_{(ii)}$ are elementary matrices.

Theorem. If D is obtained from A by a finite number of elementary transformations upon the rows and columns of A , then $D = RAS$, where R and S are the products of elementary matrices.

267. LINEAR DEPENDENCE The elements z_1, z_2, \dots, z_n (for example, matrices, the rows of matrices, polynomials, numbers, solutions to set of equations) are said to be *linearly independent* with respect to a field F if no linear combination of them, $\sum_{i=1}^n a_i z_i$, with coefficients a_i in F , is zero unless $a_1 = a_2 = \dots = a_n = 0$.

If numbers a_i not all zero exist such that $\sum_{i=1}^n a_i z_i = 0$, then the z_1, z_2, \dots, z_n are said to be *linearly dependent* relative to F .

268. SET OF LINEAR EQUATIONS Consider the linear equations

$$(1) \quad \sum_{s=1}^n a_{rs} x_s = y_r \quad (r = 1, \dots, m) \quad \text{written } A\{x\} = \{y\}$$

of matrix $A = [a_{rs}]$, with a_{rs} in field F . Let B be the matrix derived from A by annexing the column $\{y\}$ on the right side of A ; ρ is the rank of A , and ρ' the rank of B .

Theorem. (a) If $\rho' > \rho$, Eq. (1) is *inconsistent*.

(b) If $\rho' = \rho$, certain ρ of the equations determine uniquely ρ of the unknowns as linear functions of the remaining $(n - \rho)$ unknowns; for all values of the latter the expressions for these ρ unknowns satisfy also the remaining $(m - \rho)$ equations.

Theorem. If Eq. (1) is *homogeneous*, i.e., if

$$(2) \quad \sum_{s=1}^n a_{rs} x_s = 0 \quad (r = 1, \dots, m) \quad \text{written } A\{x\} = \{0\}$$

(a) Equation (2) has $(n - \rho)$ linearly independent solutions in F , and every other solution is *linearly dependent* on them [that is, ρ equations may be selected from (2), whose matrix has a nonvanishing ρ -rowed determinant and these ρ equations determine ρ of the unknowns as homogeneous linear functions, with coefficients in F , of the remaining $(n - \rho)$ unknowns; for all values of the latter, the expressions for the ρ unknowns satisfy (2).]

(b) Equation (2) has solutions not all zero if and only if $\rho < n$; the unique solution $x_j = 0$, ($j = 1, \dots, n$), if $\rho = n$.

(c) n homogeneous linear equations (2) in n unknowns have solutions not all zero if and only if $d(A) = 0$; the unique solution $x_j = 0$, ($j = 1, \dots, n$), if $d(A) \neq 0$.

Remark. The finding of solutions to $A\{x\} = \{y\}$ can be facilitated by performing a sequence of elementary transformations on the rows of A , reducing A to a triangular matrix (one in which blocks of zeros

appear above or below the main diagonal) or a diagonal matrix D .

If A is nonsingular, A can be reduced to the identity matrix I . The result of performing the same sequence of operations on the rows of I will be the inverse matrix A^{-1} . In other words, if $E_k E_{k-1} \dots E_1 = E$ is the left product of the elementary matrices which reduces A to the identity matrix I , that is, if $EA = I$, then $E = A^{-1}$. Furthermore, $\{x\} = A^{-1}\{y\} = E\{y\}$ is the solution to the equation $A\{x\} = \{y\}$ in case $d(A) \neq 0$.

269. EQUIVALENCE The square matrix A is *equivalent* to the square matrix B if there exist nonsingular matrices P and Q such that $A = PBQ$.

Theorem. Every matrix A of rank ρ is equivalent to a diagonal matrix $\|h_1, h_2, \dots, h_\rho, 0, \dots, 0\|$, $h_i \neq 0$.

A is *congruent* to B if there exists a nonsingular matrix P such that $A = P^T B P$.

Theorem. Every symmetric matrix A of rank ρ with elements in a field F is congruent in F to a diagonal matrix

$$\|d_1, d_2, \dots, d_\rho, 0, \dots, 0\| \quad d_i \neq 0$$

A is said to be *similar* to B if there exists a nonsingular matrix P such that $A = P^{-1} B P$.

270. BILINEAR FORMS If the variables in the bilinear form

$$(1) \quad G \equiv \sum_{i,j=1}^n a_{ij} x_i y_j \quad \text{written } G = (x)A\{y\}$$

are subjected to the nonsingular transformations

$$(2) \quad \{x\} = P\{\xi\} \quad \{y\} = Q\{\eta\}$$

where $P = [p_{ij}]$ and $Q = [q_{ij}]$, then G becomes

$$(3) \quad G = (\xi)P^T A Q\{\eta\}$$

Matrix $B = P^T A Q$ is equivalent to matrix A .

271. QUADRATIC FORMS If the variables in the symmetric quadratic form

$$Q \equiv \sum_{i,j=1}^n a_{ij} x_i x_j \quad \text{written } Q = (x)A\{x\} \quad (a_{ij} = a_{ji})$$

are subjected to the nonsingular transformations

$$\{x\} = C\{y\} \quad C = [C_{ij}]$$

then Q becomes

$$Q = (y)C^TAC\{y\}$$

Matrix $B = C^TAC$ is congruent to matrix A .

272. TRANSFORMATION OF REFERENCE FRAMES Suppose vectors $\{x\}$ and $\{y\}$, referred respectively to reference frames X and Y , are related by

$$(1) \quad \{x\} = A\{y\} \quad A = [a_{rs}]$$

Change from frame X to frame \bar{X} and from frame Y to frame \bar{Y} by the nonsingular transformations

$$\{x\} = P\{\bar{x}\}, \quad P = [p_{rs}]; \quad \text{and} \quad \{y\} = Q\{\bar{y}\}, \quad Q = [q_{rs}]$$

Then

$$(2) \quad \{\bar{x}\} = P^{-1}AQ\{\bar{y}\}$$

Matrix $B = P^{-1}AQ$ of Eq. (2) is equivalent to matrix A of Eq. (1).

14. STATISTICS*

273. FREQUENCY DISTRIBUTIONS Let x_1, x_2, \dots, x_k be a set of numbers (*variates*, observations, measurements) which occur with frequencies (*weighting factors*) f_1, f_2, \dots, f_k , respectively, where each $f_i \geq 0$, and $x_1 < x_2 < \dots < x_k$. Such a set is a *frequency distribution*. The *weighted arithmetic mean*, or *weighted average*, is

$$\bar{x} = (1/n) \sum_{i=1}^k f_i x_i \quad \text{where } n = \sum_{i=1}^k f_i \text{ is the total weight}$$

Let L be the *least* and U the *greatest* of x_1, \dots, x_k . ($U - L$) is the *range* of the distribution. [Some writers call the interval (L, U) the range.] Other parameters in common use are listed in Table 14.1.

A plot of f_i in terms of x_i as abscissa is a *frequency diagram*. The quantity $F_j = \sum_{i=1}^j f_i \equiv \text{cum } f|_j$ is the *cumulative frequency* associated with item x_j . F_j is the number of items having measurements X less than or equal to x_j . A plot of F_j in terms of x_j is a *cumulative frequency graph*.

274. GROUPED DATA When the number of items X_1, X_2, \dots, X_n in the data is large, the data may be grouped by dividing the range into some number k of equal intervals of length c , called *class intervals*, these intervals being located so as to have a convenient midpoint x_i , called a *class mark* or *midvalue*. Every item in the data falling in a given class interval i is assigned a value equal to the x_i of the interval. The boundaries of class interval i are $(x_i - c/2)$ and $(x_i + c/2)$. The class intervals are ordered as 1, 2, \dots , k and the corresponding *class frequencies* f_1, \dots, f_k listed. The *cumulative frequency* associated with the upper boundary of j th class interval is $F_j = \text{cum } f|_j$. The plots of f_j and F_j in terms of x_j give a *frequency histogram* and a *cumulative frequency polygon*, respectively.

* A convenient summary of theories, formulas, definitions, working rules, and tabular material useful in practical problems in probability and statistics is given in Richard S. Burington and Donald C. May, *Handbook of Probability and Statistics with Tables*, McGraw-Hill Book Company, New York, 1953.

TABLE 14.1. STATISTICAL PARAMETERS FOR DISCRETE FREQUENCY DISTRIBUTIONS

<i>Measures of Location</i>	
$n = \sum_{i=1}^k f_i$	Total weight
$\bar{x} = \frac{1}{n} \sum_{i=1}^k f_i x_i$	Weighted arithmetic mean
$x_i - \bar{x}$	Deviation (or error) of x_i with respect to \bar{x}
$ x_i - \bar{x} $	Absolute deviation (or absolute error) of x_i with respect to \bar{x}
$\left(\sum_{i=1}^k \frac{f_i x_i^2}{n} \right)^{1/2}$	Root mean square (rms) value of x_1, \dots, x_k
A value of x_i having a maximum frequency f_i	Mode (Mo)
In list of ordered variates x , n in number, the <i>median</i> is the mid-value (if n odd) or the average of the two middle values (if n even)	Median
<i>Moments</i>	
$\mu_r(x_0) = \frac{1}{n} \sum_{i=1}^k f_i (x_i - x_0)^r$	r th moment about $x = x_0$
$\mu_2(x_0)$	Mean square deviation from x_0
$\sqrt{\mu_2(x_0)}$	Root mean square deviation from x_0
$\nu_r = \frac{1}{n} \sum_{i=1}^k f_i x_i^r$	r th moment about $x = 0$
$\mu_r = \frac{1}{n} \sum_{i=1}^k f_i (x_i - \bar{x})^r$	r th (central) moment about weighted mean \bar{x}
$\sigma^2 = \mu_2 = \nu_2 - \bar{x}^2$	Variance, 2d moment about the mean
$\sigma = \sqrt{\mu_2}$	Standard deviation
$\alpha_r = \frac{1}{n} \sum_{i=1}^k f_i t_i^r$	r th moment in standard t units about $t = 0$
where $t_i = \frac{x_i - \bar{x}}{\sigma}$	

TABLE 14.1. Continued

$\frac{1}{n} \sum_{i=1}^k f_i x_i - x_0 ^r$	r th absolute moment about x_0
$MD _0 = \frac{1}{n} \sum_{i=1}^k f_i x_i - x_0 $	Mean deviation from x_0
$MD = \frac{1}{n} \sum_{i=1}^k f_i x_i - \bar{x} $	Mean (absolute) deviation from the mean, mean absolute error (m.a.e.)

Measures of Skewness and Kurtosis

$\gamma_1 = \frac{\mu_3}{\sigma^3} = \alpha_3$	Coefficient of skewness = γ_1
$\frac{\mu_3}{2\sigma^3}$	Momental skewness
$\gamma_2 = \frac{\mu_4}{\sigma^4} - 3 = \alpha_4 - 3$	Coefficient of excess = γ_2 , "flatness"
$\beta_2 = \frac{\mu_4}{\sigma^4} = \gamma_2 + 3 = \alpha_4$	Kurtosis = β_2 , "flatness"

Measures of Dispersion

Consider the ordered variates, $x_1 < x_2 < \dots < x_k$, where f_i is frequency of x_i , \bar{x} is mean, and σ is standard deviation. Let $n = \sum_{i=1}^k f_i$, $\text{cum } f|_j = \sum_{i=1}^j f_i$. Plot $\text{cum } f|_j$ as a function of x . The largest x_j for which $\text{cum } f|_j \leq p/100$ is called a p th percentile.

Lower (first) quartile	Q_1 : the 25th percentile
Median (second) quartile	Q_2 : the 50th percentile
Upper (third) quartile	Q_3 : the 75th percentile
Semi-interquartile range	$Q: Q = \frac{Q_3 - Q_1}{2}$

Variance σ^2 and standard deviation σ are measures of dispersion. $V = \sigma/\bar{x}$, or $100\sigma/\bar{x}$ per cent, is the *coefficient of variation*.

275. DISCRETE PROBABILITY DISTRIBUTIONS Suppose X takes on only the discrete values x_1, x_2, \dots, x_k , with $x_1 < x_2 < \dots < x_k$. Let the probability that X takes the value x_i be

$$P[X = x_i] = f(x_i) \quad i = 1, \dots, k$$

Then

$$\sum_{i=1}^k f(x_i) = 1$$

The probability that $X \leq x'$ is

$$P[X \leq x'] = \sum_{i=1}^{i=j} f(x_i) = F(x')$$

where x_j is the largest discrete value of X less than or equal to x' . $F(x_k) = 1$. $F(x')$ is the *cumulative discrete probability*.

The probability that $x' < X \leq x''$ is

$$P[x' < X \leq x''] = F(x'') - F(x')$$

The *discrete distribution* $f(x)$ is describable by parameters similar to those given in Table 14.1. The mean μ and variance σ^2 of X are, respectively,

$$\mu = \sum_{i=1}^k x_i f(x_i)$$

$$\sigma^2 = \sum_{i=1}^k (x_i - \mu)^2 f(x_i) = \sum_{i=1}^k x_i^2 f(x_i) - \mu^2$$

Probability distributions and sample frequency distributions are similar in form, the $f(x_i)$ corresponding to f_i/n .

Mathematical Expectation. Let $g(X)$ be a function of the discrete chance variable X , X having probability distribution $f(x)$. The expected value (mean value) of $g(X)$ is

$$E[g(X)] = \sum_{i=1}^k g(x_i) f(x_i)$$

The mean μ is the mathematical expectation $E[X]$ of X .

The r th moment μ_r about the mean μ is

$$E[(X - \mu)^r] = \mu_r = \sum_{i=1}^k (x_i - \mu)^r f(x_i)$$

The variance of X is $\mu_2 = \sigma^2$; σ is the standard deviation of X .

276. CONTINUOUS PROBABILITY DISTRIBUTION Let X be a chance variable which is certain to fall in the interval (α, β) . A *continuous cumulative distribution function* $F(x)$ is a function which gives the probability that the chance variable X is less than or equal to any particular value x which may be selected, that is,

$$P[X \leq x] = F(x)$$

$F(x)$ is called the *probability function* of X .

The probability that $x' < X \leq x''$ is

$$P[x' < X \leq x''] = F(x'') - F(x')$$

If the derivative $F'(x)$ of $F(x)$ with respect to x exists, then $f(x) = F'(x)$ is the *continuous probability density function* (*frequency function, distribution*), and

$$P[x' < X \leq x''] = \int_{x'}^{x''} f(x) dx = F(x'') - F(x')$$

$$F(x') = \int_{\alpha}^{x'} f(x) dx = P[X \leq x']$$

$$F(\beta) = \int_{\alpha}^{\beta} f(x) dx = 1 \quad F(\alpha) = 0 \quad 0 \leq F(x) \leq 1$$

The principal features of a continuous distribution $f(x)$ may be described by parameters similar to those given for discrete distributions.

The mean μ and variance σ^2 of X are, respectively,

$$\mu = \int_{\alpha}^{\beta} xf(x) dx$$

$$\sigma^2 = \int_{\alpha}^{\beta} (x - \mu)^2 f(x) dx = \int_{\alpha}^{\beta} x^2 f(x) dx - \mu^2$$

Mathematical Expectation. Let $g(X)$ be a function of the continuous variable X , X having probability distribution $f(x)$ and being certain to fall in (α, β) . The *expected value* (*mean value*) of $g(X)$ is

$$E[g(X)] = \int_{\alpha}^{\beta} g(x)f(x) dx$$

The mean μ is the mathematical expectation $E[X]$ of X .

The r th moment μ_r of the distribution about the mean μ is

$$E[(X - \mu)]^r = \mu_r = \int_{\alpha}^{\beta} (x - \mu)^r f(x) dx$$

$\mu_2 = \sigma^2$ is the variance; σ , the standard deviation.

277. BINOMIAL DISTRIBUTION If X is such that the probability that X takes the value x is

$$(1) \quad P\{X = x\} = f(x) = C_x^n p^x q^{n-x} \quad (x = 0, 1, \dots, n)$$

where

$$q = 1 - p \quad \text{and} \quad C_x^n = n!/x!(n-x)! \equiv \binom{n}{x}$$

then X is said to have a *binomial probability distribution*.

The probability that $X \leq x'$, where x' is any value such that $x' \leq n$, is

$$P\{X \leq x'\} = F_p(x') = C_0^n p^0 q^n + C_1^n p^1 q^{n-1} + \dots + C_{x^*}^n p^{x^*} q^{n-x^*}$$

where x^* is the largest integer which does not exceed x' .

The $(x+1)$ th term of the binomial expansion

$$(q+p)^n = q^n + C_1^n p q^{n-1} + \dots + C_x^n p^x q^{n-x} + \dots + p^n$$

is the right-hand side of (1).

Properties. The mean, variance, . . . , of X having distribution (1) are

$$\text{Mean} = \mu = np \quad \text{Variance} = \sigma^2 = npq$$

$$\text{Standard deviation} = \sigma = \sqrt{npq}$$

$$\text{Coefficient of skewness} = \gamma_1 = \frac{q-p}{\sqrt{npq}}$$

$$\text{Coefficient of excess} = \gamma_2 = \frac{1-6pq}{npq}$$

Mode = positive integral value (or values) of x for which

$$np - q \leq x \leq np + p$$

r th moment μ_r about mean μ :

$$\mu_1 = 0 \quad \mu_2 = \sigma^2 = npq \quad \mu_3 = npq(q-p)$$

$$\mu_4 = npq[1 + 3(n-2)pq]$$

The normal probability distribution (Sec. 280) gives a good approximation to the binomial distribution for n sufficiently large. To fit normal curve $y = \psi(t)/\sigma$, where $\psi(t) = (1/\sqrt{2\pi}) \exp[-t^2/2]$, to

binomial distribution $f(x) = C_x^n p^x q^{n-x}$, set $\mu = np$, $\sigma = \sqrt{npq}$, $t = (x - \mu)/\sigma$.

278. REPEATED TRIALS Let p be the probability of occurrence of an event E ("success") on each of n trials and q the probability of non-occurrence of E ("failure"). $p + q = 1$ and $0 \leq p \leq 1$. The probability of x successes followed by $(n - x)$ failures is $p^x q^{n-x}$. If the order of occurrence of successes is of no consequence, the number of orders in which x successes and $(n - x)$ failures can occur is $C_x^n = n!/x!(n - x)!$.

The probability of *exactly* x successes (in n independent trials) irrespective of order is

$$P(X = x) = f(x) = C_x^n p^x q^{n-x}$$

The probability of *at least* x occurrences in n trials is

$$P[X \geq x] = \sum_{i=x}^n C_i^n p^i q^{n-i} \quad \text{for } x = 0, 1, \dots, n$$

The probability of *at most* x occurrences in n trials is

$$P[X \leq x] = \sum_{i=0}^x C_i^n p^i q^{n-i} \quad \text{for } x = 0, 1, \dots, n$$

The mean number of occurrences of event E is $\mu = np$; the mean number of nonoccurrences of event E is nq (see Table 22).

279. POISSON DISTRIBUTION If X is such that the probability that X takes the value x is

$$(1) P[X = x] = f(x) = e^{-m} m^x / x!, \quad m > 0, \quad \text{for } x = 0, 1, 2, \dots$$

the variable X is said to have a *Poisson distribution*. Note: $f(0) = e^{-m}$.

The probability that $X \geq x'$ is

$$P[X \geq x'] = \sum_{x=x'}^{x=\infty} e^{-m} m^x / x!$$

Note: $P[X \geq 0] = 1$.

The probability that $X \leq x^*$ is

$$P[X \leq x^*] = \sum_{x=0}^{x=x^*} e^{-m} m^x / x!$$

Properties. The mean, variance, . . . , of X having distribution (1) are

$$\text{Mean} = \mu = m \qquad \text{Variance} = \sigma^2 = m$$

$$\text{Standard deviation} = \sigma = \sqrt{m}$$

$$\text{Coefficient of skewness} = \gamma_1 = \frac{1}{\sqrt{m}}$$

$$\text{Coefficient of excess} = \gamma_2 = \frac{1}{m}$$

r th moment μ_r about mean μ :

$$\mu_2 = \sigma^2 = m \qquad \mu_3 = m \qquad \mu_4 = 3m^2 + m \qquad \dots$$

r th moment ν_r about $x = 0$:

$$\begin{aligned} \nu_1 = m, \quad \nu_2 = m(m+1), \quad \nu_3 = m(m^2 + 3m + 1) \\ \nu_4 = m(m^3 + 6m^2 + 7m + 1) \quad \dots \end{aligned}$$

The binomial distribution $C_x^n p^x q^{n-x}$ approaches the Poisson distribution as a limit when $n \rightarrow \infty$ and $p \rightarrow 0$ such that $np = m$ is a constant. The Poisson distribution may be used to approximate the binomial distribution when n is large enough and p is small enough (e.g., when $n > 50$, $p < 0.1$, $0 < np < 10$).

280. NORMAL DISTRIBUTION If X is such that the probability that X is less than or equal to x is

$$P[X \leq x] = F_N(x) = \frac{1}{\sqrt{2\pi}\sigma} \int_{-\infty}^x \exp\left[-\frac{(X-\mu)^2}{2\sigma^2}\right] dX$$

where μ and σ are constants, X is distributed normally, and $F_N(x)$ is the *cumulative normal (Gaussian) distribution*.

The normal probability density function $f_N(x)$ is

$$(1) \qquad f_N(x) = \frac{1}{\sqrt{2\pi}\sigma} \exp\left[-\frac{(x-\mu)^2}{2\sigma^2}\right]$$

The probability that X falls between x' and x^* is

$$P[x' < X \leq x^*] = F_N(x^*) - F_N(x')$$

(See Table 22 for comprehensive table of values for the normal distribution.)

(1) may be written

$$f_N(x) = \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{t^2}{2}\right) \quad \text{where } t = \frac{x - \mu}{\sigma}$$

Properties. The mean, variance, . . . , of X having distribution (1) are

Mean = μ Variance = σ^2

Standard deviation = σ

$F_N(-\infty) = 0$ $F_N(+\infty) = 1$

Coefficient of skewness = $\gamma_1 = 0$

Coefficient of excess = $\gamma_2 = 0$

Mode = μ Median = μ

r th moment μ_r about mean μ :

$\mu_1 = 0$ $\mu_2 = \sigma^2$ $\mu_3 = 0$ $\mu_4 = 3\sigma^4$. . .

r th moment ν_r about $x = 0$:

$\nu_1 = \mu$ $\nu_2 = \mu^2 + \sigma^2$ $\nu_3 = \mu(\mu^2 + 3\sigma^2)$

$\nu_4 = \mu^4 + 6\mu^2\sigma^2 + 3\sigma^4$

Mean (absolute) deviation from mean

= MD = m.a.e. = $\sigma\sqrt{2/\pi} = 0.7979\sigma$

Modulus of precision = $h = 1/\sigma\sqrt{2} = 0.7071\sigma$

Probable error = p.e. = $0.6745\sigma = 0.8453$ (m.a.e.)

Upper and lower quartiles: $Q_3 = \mu + \text{p.e.}$ $Q_1 = \mu - \text{p.e.}$

The fractions of the areas under the curve $f_N(x)$ above x axis, plotted as a function of x between lines parallel to y axis, are:

Zone, %	Range
50	$\mu \pm 0.6745\sigma$
68.27	$\mu \pm \sigma$
90	$\mu \pm 1.645\sigma$
95.45	$\mu \pm 2\sigma$
99.73	$\mu \pm 3\sigma$

281. PROBABILITY OF OCCURRENCE ON CERTAIN DEVIATIONS The probability that X will differ from μ by not more than σ is

$$P[|X - \mu| \leq \sigma] = P[-1 \leq t \leq 1] \\ = \frac{1}{\sqrt{2\pi}} \int_{t=-1}^{t=1} \exp\left(-\frac{t^2}{2}\right) dt = 0.6827$$

[Here $t = (X - \mu)/\sigma$.]

The probability that x will deviate more than $\lambda\sigma$ from the mean μ is

$$P = P[|x - \mu| > \lambda\sigma] = \frac{2}{\sqrt{2\pi}} \int_{\lambda}^{\infty} \exp\left(-\frac{t^2}{2}\right) dt$$

Example. Values of λ for 0.1, 1, and 5 per cent *tolerance limits* on x are, respectively,

λ	3.29	2.58	1.96
P	0.001	0.01	0.05

282. FITTING A POLYNOMIAL TO DATA REPRESENTED BY A SET OF POINTS Consider the set of points $(x_1, y_1), (x_2, y_2), \dots, (x_k, y_k)$. It is desired to find among the curves

$$y(x) = a_0 + a_1x + \dots + a_nx^n \quad k > n$$

a curve which gives a "best possible fit" to the given points. By the method of least squares, a "best" system of values for the coefficients (namely, A_0, A_1, \dots, A_n) is one that renders $M = \sum_{i=1}^k d_i^2$ a minimum.

Here $d_i = y_i - y(x_i)$ is the *deviation* of y_i from the value $y(x_i)$. A curve C with coefficients $a_j = A_j$ ($j = 0, 1, \dots, n$) is a "best-fitting curve." C is called a *mean square regression curve*. When $n = 1$, C is a *mean square regression line*.

The coefficients of C may be found by solving the $n + 1$ "normal" equations $\partial M / \partial a_j = 0$ ($j = 0, 1, \dots, n$) for a_j . The coefficients A_0, \dots, A_n of C are the roots of the simultaneous equations:

$$\begin{cases} A_0 S_0 + A_1 S_1 + \dots + A_n S_n = T_0 \\ A_0 S_1 + A_1 S_2 + \dots + A_n S_{n+1} = T_1 \\ \dots\dots\dots \\ A_0 S_n + A_1 S_{n+1} + \dots + A_n S_{2n} = T_n \end{cases}$$

where $S_j = \sum_{i=1}^{i=k} x_i^j$ $S_0 = k$ $T_j = \sum_{i=1}^{i=k} x_i^j y_i$

283. SIMPLE (LINEAR) CORRELATION One measure of the degree of association, or correlation, between the x s and the corresponding y s in the data $(x_1, y_1), (x_2, y_2), \dots, (x_k, y_k)$ is the ratio

$$r = \frac{\text{cov}(x, y)}{\sigma_x \sigma_y}$$

where \bar{x} and σ_x are the mean and standard deviation, respectively, of the x s; \bar{y} and σ_y are the corresponding parameters for the y s; and

$$\text{cov}(x, y) = \frac{\sum_{i=1}^k (x_i - \bar{x})(y_i - \bar{y})}{k} = \frac{\sum_{i=1}^k x_i y_i - k\bar{x}\bar{y}}{k}$$

is the covariance between the x s and y s. r is called the *correlation coefficient* (or *product moment about the mean*) of the variables x and y . *Note:* $\text{var } x = \text{cov}(x, x) = \sigma_x^2$.

284. FITTING A LINE TO POINTS A line $y(x) = a_0 + a_1 x$ may be fitted to the points $(x_1, y_1), \dots, (x_k, y_k)$ in many ways. If the fit is made to minimize the sum of the squares of the vertical distances from the line (assuming the errors to be in the y coordinates only), the line so obtained is known as the (*mean*) *regression line of y on x* and has the equation

$$y - \bar{y} = \frac{r\sigma_y}{\sigma_x}(x - \bar{x})$$

The slope of this line is $m_1 = r\sigma_y/\sigma_x$, which is called the *regression coefficient of y on x* .

The standard deviation of the error of estimate in y is S_y , where

$S_y^2 = \sigma_y^2(1 - r^2)$. S_y^2 is the *residual variance* of y ; S_y , the *standard error of estimate in y* .

If the fit is made so as to minimize the sum of the squares of the horizontal distances from the line (assuming the errors to be in the x coordinates only), the line so obtained is the (*mean*) *regression line of x on y* and has the equation

$$y - \bar{y} = \frac{\sigma_y}{r\sigma_x}(x - \bar{x})$$

The *regression coefficient of x on y* is $r\sigma_x/\sigma_y$.

S_x is the standard deviation of the errors of estimate in x and is called the *standard error of estimate in x* . The *residual variance of x* is S_x^2 , where $S_x^2 = \sigma_x^2(1 - r^2)$.

When $r = \pm 1$, the lines of regression coincide and the correlation is perfect; if $r = 0$, the lines are at maximum divergence and x and y are *uncorrelated*. When $r \neq 0$, the variables are *correlated*. If $r < 0$, on the average y decreases as x increases; if $r > 0$, y increases with x .

235. SAMPLE DISTRIBUTIONS A *population* consists of a collection $\{x\}$ of the values of a variable X . A *sample* is a set of values x_1, x_2, x_3, \dots taken from population $\{x\}$. Let $f(x)$ be the frequency distribution function of X in $\{x\}$; with \bar{x} = mean of X and σ_x^2 = variance of X . In the set of all possible random samples of size n , the values x_1, x_2, \dots, x_n may be considered statistically independent. If each variable x_i has the frequency distribution function $f(x)$, the sample set (x_1, x_2, \dots, x_n) has the frequency distribution function $f(x_1)f(x_2) \dots f(x_n)$.

For the sample x_1, x_2, \dots, x_n , let the mean \bar{x} and variance s^2 be, respectively,

$$\bar{x} = \sum_{i=1}^n \frac{x_i}{n} \quad s^2 = \sum_{i=1}^n \frac{(x_i - \bar{x})^2}{n}$$

Theorem 1. An "unbiased" estimate of the population mean \bar{x} is the sample mean \bar{x} ; of the variance σ_x^2 of a population is $s^2n/(n - 1)$, where $s^2 = \sum_{i=1}^n (x_i - \bar{x})^2/n$ is the sample variance.

Theorem 2. If an infinite population of values x is normally distributed with mean \bar{x} and standard deviation σ_x , then the sampling

distribution of means \bar{x} is normally distributed with mean \bar{x} and standard deviation σ_x/\sqrt{n} .

The probability p that \bar{x} for any one sample will fall in the interval $\bar{x} - \delta \leq \bar{x} \leq \bar{x} + \delta$ is

$$p = \int_{-\beta}^{+\beta} \psi(u) du \quad \text{where} \quad \beta = \frac{\delta\sqrt{n}}{\sigma_x}$$

$$u = (\bar{x} - \bar{x}) \frac{\sqrt{n}}{\sigma_x} \quad \text{and} \quad \psi(u) = \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{u^2}{2}\right)$$

286. GOODNESS OF FIT (CHI-SQUARED TEST) There are various statistical ways to test how well a sample distribution agrees with an assumed population distribution $f(x)$. One scheme for doing this makes use of a chi-squared test. The comparison makes use of the observed and the population frequencies for a suitable set of class intervals of the variable of the distribution.

Consider a population of items falling into r mutually exclusive class intervals (*cells*). Given a sample x_1, x_2, \dots, x_n of n items having observed frequency f_i in i th cell ($n = \sum_{i=1}^r f_i$), suppose the assumed theoretical frequency for the i th class is np_i . The χ^2 test is concerned with the difference between f_i and np_i for all cells. This difference is measured by

$$(1) \quad \chi^2 = \sum_{i=1}^r \frac{(f_i - np_i)^2}{np_i} = \left(\sum_{i=1}^r \frac{f_i^2}{np_i} \right) - n$$

The *hypothesis H* of the test is that the data x_1, \dots, x_n are a sample of a random variable with distribution $f(x)$.

The *purpose of the test* is to determine whether the observed data may be considered consistent with *H*.

The *steps in the test* are (a) Calculate (1). (b) Determine the number of degrees of freedom m . (c) Select a level of significance ϵ . (d) From tables of χ^2 distribution find a χ_0^2 such that $P[\chi^2 \geq \chi_0^2] \approx \epsilon$ (Table 24).

The *interpretation of the test* is as follows: (a) If for the sample a value $\chi^2 > \chi_0^2$ is found, the sample x_1, \dots, x_n shows a significant deviation from hypothesis *H*, and *H* is rejected at the ϵ level of sig-

nificance. (b) If for the sample a value $\chi^2 \leq \chi_0^2$ is found, the sample x_1, \dots, x_n is considered consistent with hypothesis H .

In using Table 24, an estimate of the number m of degrees of freedom must be made, where $m = r - 1 - b$, r is the number of cells, and b is the number of parameters in the population distribution determined from the sample. When the population distribution is completely known, $m = r - 1$.

The chi-squared test may be applied to data grouped in cells provided n is large enough (say, $n \geq 50$), the theoretical and sample cell frequencies are not too small (f_i and np_i each greater than, say, 5 to 10), and the constraints on the cell frequencies are linear.

Table 24 is applicable only when $m \leq 30$. When $m > 30$, $\sqrt{2\chi^2}$ is, approximately, normally distributed, with mean $\sqrt{2m - 1}$ and unit standard deviation.

For further material and tables on probability and statistics see Richard S. Burington and Donald C. May, *Handbook of Probability and Statistics with Tables*, McGraw-Hill Book Company, New York, 1953.

PART TWO

Tables

EXPLANATION OF TABLES

Logarithms

TABLE I. FIVE-PLACE COMMON LOGARITHMS OF NUMBERS FROM 1 TO 10,000

Definition of Logarithm. The logarithm x of the number N to the base b is the exponent of the power to which b must be raised to give N . That is,

$$\log_b N = x \quad \text{or} \quad b^x = N$$

The number N is positive, and b may be any positive number except 1.

Properties of Logarithms

(a) The logarithm of a product is equal to the sum of the logarithms of the factors; thus

$$\log_b MN = \log_b M + \log_b N$$

(b) The logarithm of a quotient is equal to the logarithm of the numerator minus the logarithm of the denominator; thus

$$\log_b \frac{M}{N} = \log_b M - \log_b N$$

(c) The logarithm of a power of a number is equal to the logarithm of the number multiplied by the exponent of the power; thus

$$\log_b M^p = p \log_b M$$

(d) The logarithm of a root of a number is equal to the logarithm of the number divided by the index of the root; thus

$$\log_b \sqrt[q]{M} = \frac{1}{q} \log_b M$$

Other properties of logarithms:

$$\log_b b = 1 \qquad \log_b \sqrt[q]{M^p} = \frac{p}{q} \log_b M$$

$$\log_b 1 = 0 \qquad \log_b N = \log_a N \log_b a = \frac{\log_a N}{\log_a b}$$

$$\log_b b^N = N \qquad b^{\log_b N} = N$$

Systems of Logarithms. There are two common systems of logarithms in use: (1) the *natural* (Napierian or hyperbolic) system which uses the base $e = 2.71828 \dots$; (2) the *common* (Briggsian) system which uses the base 10.

We shall use the abbreviation* $\log N \equiv \log_{10} N$ in this section.

Unless otherwise stated, tables of logarithms are always tables of common logarithms.

Characteristic of a Common Logarithm of a Number. Every real positive number has a real common logarithm such that if $a < b$, $\log a < \log b$. Neither zero nor any negative number has a real logarithm.

A common logarithm, in general, consists of an integer, which is called the *characteristic*, and a decimal (usually endless), which is called the *mantissa*. The characteristic of any number may be determined from the following rules.

Rule I. The characteristic of any number greater than 1 is one less than the number of digits before the decimal point.

Rule II.† The characteristic of a number less than 1 is found by subtracting from 9 the number of ciphers between the decimal point and the first significant digit and writing -10 after the result.

Thus the characteristic of $\log 936$ is 2; the characteristic of 9.36 is 0; of $\log 0.936$ is $9 - 10$; of $\log 0.00936$ is $7 - 10$.

Mantissa of a Common Logarithm of a Number. An important consequence of the use of base 10 is that the mantissa of a number is independent of the position of the decimal point. Thus 93,600, 93,600, 0.000936, all have the same mantissa. Hence in Tables of Common Logarithms only mantissas are given. This is done in Table 1. A five-place table gives the values of the mantissa correct to five places of decimals.

To Find the Logarithm of a Given Number N . By means of Rules I and II determine the characteristic. Then use Table 1 to find mantissa.

To find mantissa when the given number (neglecting decimal point) consists of four or fewer digits (exclusive of ciphers at the beginning or end), look in the column marked N for the first three significant digits

* Note, however, that $\log N \equiv \log_e N$ is used in the Table of Integrals.

† Some writers use a dash over the characteristic to indicate a negative value, for example,

$$\log .004657 = 7.66811 - 10 = \bar{3}.66811$$

and pick the column headed by the fourth digit—the mantissa is the number appearing at the intersection of this row and column. Thus to find the logarithm of 64030, first note (by Rule I) that the characteristic is 4. Next in Table 1, find 640 in column marked N and opposite it in column 3 is the desired mantissa, .80638. Hence $\log 64030 = 4.80638$. Likewise, $\log 0.0064030 = 7.80638 - 10$; $\log 0.64030 = 9.80638 - 10$.

*Interpolation.** The mantissa of a number of more than four significant figures can be found approximately by assuming that the mantissa varies directly as the number in the small interval not tabulated. Thus if N has five digits (significant) and f is the fifth digit of N , the mantissa of N is

$$m = m_1 + \frac{f}{10}(m_2 - m_1)$$

where m_1 is the mantissa corresponding to the first four digits of N and m_2 is the next larger mantissa in the table. $(m_2 - m_1)$ is called a *tabular difference*. The proportional part of the difference $m_2 - m_1$ is called the *correction*. These proportional parts are printed without zeros at the right-hand side of each page as an aid to mental multiplications.

For example, find $\log 64034$. Here $f = 4$. From the table we see $m_1 = .80638$, $m_2 = .80645$, whence

$$m = .80638 + (4/10)(.00007), \log 64034 = 4 + m = 4.80641$$

To Find the Number N When Its Logarithm Is Known. (The number N whose logarithm is k is called the *antilogarithm* of k .)

Case 1. If the mantissa m is found exactly in Table 1, join the figure at the top of the column containing m to the right of the figures in the column marked N and in the same row as m , and place the decimal point according to the characteristic of the logarithm.

Case 2. If the mantissa m is not found exactly in the table, interpolate as follows: Find the next smaller mantissa m_1 to m ; the first four significant digits of N correspond to the mantissa m_1 , and the fifth digit f equals the nearest whole number to

$$f = 10 \left(\frac{m - m_1}{m_2 - m_1} \right)$$

* A more accurate method of interpolation is given on page 214.

where m_2 is the next larger mantissa to m appearing in the table. Then locate the decimal point according to the characteristic.

The decimal point may be located by means of the following rules:

Rule III. If the characteristic of the logarithm is positive (then the mantissa is not followed by -10), begin at the left, count digits one more than the characteristic, and place the decimal point to the right of the last digit counted.

Rule IV. If the characteristic is negative (then the mantissa will be preceded by an integer n and followed by -10), prefix $(9 - n)$ ciphers, and place the decimal point to the left of these ciphers.

Illustrations of the Use of Logarithms.

Example 1. Given $\log x = 2.91089$, find x . The mantissa .91089 appears in the table. Join the figure 5 which appears at the top of the column to the right of the number 814 in the column N , giving the number 8145. By Rule III, the decimal point is placed to the right of 4, thus giving $x = 814.5$.

Example 2. Given $\log x = 2.34917$, find x . The mantissa $m = .34917$ does not appear in the table. The next smaller and next larger mantissas are m_1 and m_2 ,

$$m_1 = .34908 \qquad m = .34917 \qquad m_2 = .34928$$

The first four digits of N , corresponding to m_1 , are 2234, and the fifth digit is the nearest whole number (5) to

$$10 \left(\frac{m - m_1}{m_2 - m_1} \right) = 10 \left(\frac{.00009}{.00020} \right) = 4.5$$

By Rule III, we locate decimal point, thus giving $x = 223.45$.

Example 3. Find $x = (396.21)(.004657)(21.21)$:

$$\begin{aligned} \log 396.21 &= 2.59792 \\ \log .004657 &= 7.66811 - 10 \\ \log 21.210 &= \underline{1.32654} \quad (\text{add}) \\ \log x &= \underline{11.59257} - 10, x = 39.135 \end{aligned}$$

Example 4. Find $x = \frac{396.21^*}{24.3}$;
 $\log 396.21 = 2.59792$
 $\log 24.3 = 1.38561$ (subtract)
 $\log x = 1.21231, x = 16.305$

Example 5. Find $x = (3.5273)^4$;
 $\log 3.5273 = 0.54745$
 $\log x = 2.18980, x = 154.81$ (multiply)

Example 6. Given $\log x = -2.23653$, find x . To convert this logarithm to one with a positive mantissa, add algebraically -2.23653 to $10.00000 - 10$. Thus

$$\begin{array}{r} 10.00000 - 10 \\ - 2.23653 \\ \hline \log x = 7.76347 - 10, \text{ hence } x = 0.0058006 \end{array}$$

Example 7. Find $x = \sqrt[3]{.04657}$;
 $\log x = \frac{1}{3} \log (.04657)$
 $= \frac{1}{3}(8.66811 - 10) = \frac{1}{3}(-1.33189)$
 $\log x = -0.44396 = 9.55604 - 10, x = 0.35978$
 or $\log x = \frac{1}{3}(8.66811 - 10) = \frac{1}{3}(28.66811 - 30)$
 $\log x = 9.55604 - 10, x = 0.35978$

Example 8. Find $x = \frac{1}{21.210}$;
 $\log x = \log 1 - \log 21.210$
 $\log 1 = 10.00000 - 10$
 $\log 21.210 = 1.32654$ (subtract)
 $\log x = 8.67346 - 10, x = 0.047148$

* Some writers use *cologarithms*. The cologarithm of a number N is the negative of the logarithm of N ; that is, $\text{colog } N = 10.00000 - \log N - 10$. Adding the cologarithm is equivalent to subtracting the logarithm. Thus in our example,

$$\begin{aligned} \text{colog } 24.3 &= (10.00000 - 1.38561) - 10 = 8.61439 - 10 \\ \log 396.21 - \log 24.3 &= \log 396.21 + \text{colog } 24.3 = 2.59792 + 8.61439 - 10 \\ &= 11.21231 - 10 = \text{antilog } 16.305 \end{aligned}$$

TABLE 2. SEVEN-PLACE COMMON LOGARITHMS OF NUMBERS This table gives seven-place logarithms for numbers of five significant figures from 10,000 to 12,000.

The following tables are self-explanatory: Table 3, Common Logarithms of Primes; Table 4, Common Logarithms of Gamma Functions.

Trigonometric Functions

TABLE 6. COMMON LOGARITHMS OF TRIGONOMETRIC FUNCTIONS In this table, the logarithmic values of the sine, cosine, tangent, and cotangent of angles at intervals of one minute from 0° to 90° are given. Since $\log \sec A = -\log \cos A$ and $\log \csc A = -\log \sin A$, $\log \sec A$ and $\log \csc A$ are omitted. For angles between 0° and 45° , the number of the degrees and the name of the trigonometric function are read at the top of the page and the number of minutes in the left-hand column. The corresponding information for angles between 45° and 90° is found at the bottom of the page and in the right-hand column.

The arrangement and the principles of interpolation are similar to those given in the explanation for Table 1. The -10 portion of the characteristic is not printed in the table but must be written down whenever such a logarithm is used.

Although the logarithmic values of the trigonometric functions may be interpolated to the nearest second, interpolation for the logarithms of the sine and tangent of small angles from 0° to 3° and for the logarithms of the cosine and cotangent of angles from 87° to 90° is not accurate. Table 5 should be used in these cases.

Table 5 gives

$$S = \log \sin A - \log A' \text{ and } T = \log \tan A - \log A'$$

where A is the given angle and A' is the number of minutes in A , for values of A from 0° to 3° . Then

$$\log \sin A = \log A' + S \text{ and } \log \tan A = \log A' + T$$

Likewise, $\log \cos A = \log (90^\circ - A)' + S$ and $\log \cot A = \log (90^\circ - A)' + T$, for values of A from 87° to 90° , the S and T corresponding to $(90^\circ - A)'$.

For the functions of angles greater than 90° , use the relations given in Sec. 57.

Tables of proportional parts are provided in Table 6 which may be used in the interpolation of logarithmic values of the functions between 4° and 86° .

Example 1. Find $\log \sin 21^\circ 13' 26''$.

Turn to the page having 21° at the top. In the row having $13'$ on the left, find in the column marked "L Sin" at the top $\log \sin 21^\circ 13' = 9.55858 - 10$. (The -10 portion of the characteristic is omitted in the table.) The tabular difference is 33 and is given in the column marked *d*. In the table of proportional parts for 33, the correction for $20''$ is 11.0 and for $6''$ is 3.3. These two corrections must be added to the value of $\log \sin 21^\circ 13'$. Hence $\log \sin 21^\circ 13' 26'' = 9.55872 - 10$.

Example 2. Find $\log \cot 56^\circ 23' 37''$.

On the page having 56° at the bottom, and in the row having 23 on the right, find, in the column that is marked "L Ctn" at the bottom, $\log \cot 56^\circ 23' = 9.82270 - 10$. The tabular difference is 27, given in the column marked *c.d.* In the table of proportional parts for 27, we obtain 17 as the correction for $37''$. Since the logarithmic value of the cotangent decreases as the angle increases from 0° to 90° , this correction for $37''$ must be subtracted from $\log \cot 56^\circ 23'$. Hence $\log \cot 56^\circ 23' 37'' = 9.82253 - 10$.

Example 3. Find the acute angle A if $\log \tan A = 9.67341 - 10$.

The tabulated value of the logarithm of the tangent just smaller than $9.67341 - 10$ is $9.67327 - 10$. This corresponds to $25^\circ 14'$. The difference between $9.67341 - 10$ and $9.67327 - 10$ is 14, and the tabular difference to be used is 33. In the proportional parts table for 33, we find the largest value less than 14 is 11.0, which corresponds to $20''$. The difference $14 - 11.0 = 3.0$, which corresponds to $5''$. Hence, $25''$ is the approximate correction and $A = 25^\circ 14' 25''$.

Example 4. Find the acute angle A if $\log \cos A = 9.89317 - 10$.

The value of the logarithm of the cosine decreases as the angle increases from 0° to 90° . Hence we must find in the column marked "L Cos" a value just larger than $9.89317 - 10$. This value is $9.89324 - 10$ and corresponds to $38^\circ 33'$. The difference between

9.89324 - 10 and 9.89317 - 10 is 7, and the tabular difference is 10. From the table of proportional parts we see that the largest value just smaller than 7 is 6.7, which corresponds to 40". The difference $7 - 6.7 = 0.3$, which corresponds to 2". Hence the correction is 42", approximately. Therefore $A = 38^\circ 33' 42''$.

Example 5. Find $\log \sin 0^\circ 35' 30''$.

Convert 35'30" to minutes. In Table 5, column *A*, we find, for 35.5 minutes, $S = 6.46372 - 10$. By Table 1, $\log 35.5 = 1.55023$.

Hence,

$$\begin{aligned}\log \sin 0^\circ 35' 30'' &= \log 35.5 + S \\ &= 1.55023 + 6.46372 - 10 = 8.01395 - 10\end{aligned}$$

TABLE 7. NATURAL TRIGONOMETRIC FUNCTIONS This table gives the values of the sine, cosine, tangent, and cotangent at intervals of one minute from 0° to 90° . The method of interpolation is similar to that given in the explanation of Table 6. Table 8 gives values of the natural secants and cosecants.

The following tables are self-explanatory: Table 9, Minutes and Seconds to Decimal Parts of a Degree; Table 10, Natural Trigonometric Functions for Decimal Fractions of a Degree; Table 11, Common Logarithms of Trigonometric Functions in Radian Measure; Table 12, Degrees, Minutes, and Seconds to Radians; Table 13, Trigonometric Functions in Radian Measure; Table 14, Radians to Degrees, Minutes, and Seconds.

Exponential and Hyperbolic Functions

TABLE 15. NATURAL LOGARITHMS OF NUMBERS This table gives the logarithms of N to the Napierian base e ($= 2.71828 \dots$) for equidistant values of N from 0.00 to 10.09 and from 10 to 1109. For values of N greater than 1109, use the formula $\log_e 10 N = \log_e N + \log_e 10 = \log_e N + 2.30258509$ or the formula $\log_e N = (\log_e 10) (\log_{10} N) = 2.30258509 (\log_{10} N)$.

TABLE 16. VALUES AND COMMON LOGARITHMS OF EXPONENTIAL AND HYPERBOLIC FUNCTIONS This table gives the values of e^x , e^{-x} , $\sinh x$, $\cosh x$, $\tanh x$, and the common logarithms of e^x , $\sinh x$, and $\cosh x$ for

values of x equally spaced from 0.00 to 3.00 and for certain values of x from 3.00 to 10.00. The common logarithm of e^{-x} may be found by the relation $\log_{10} e^{-x} = -\log_{10} e^x = \text{colog } e^x$. To find $\log_{10} \tanh x$, use $\log_{10} \tanh x = \log_{10} \sinh x - \log_{10} \cosh x$. This table may be extended indefinitely by means of Table 17, since $\log_{10} e^x = Mx$.

TABLE 17. MULTIPLES OF M AND $1/M$ The purpose of this table is to facilitate the multiplication of a number N by M and $1/M$. This occurs whenever it is desired to change from common logarithms to natural logarithms, and conversely. Thus

$$\log_{10} x = (\log_e x)(\log_{10} e) = M \log_e x; \log_e x = \log_{10} x/M$$

These multiples are also required in

$$\log_{10} e^x = Mx, \log_e (10^n x) = \log_e x + n(1/M)$$

and in the approximate formulas

$$\log_{10} (1 \pm x) = \pm xM \text{ and } 10^{\pm x} = 1 \pm (1/M)x$$

Powers, Roots, Reciprocals, Areas, and Circumferences

The following tables are self-explanatory: Table 18, Squares, Cubes, Square Roots, and Cube Roots; Table 19, Reciprocals, Circumferences, and Areas of Circles.

Statistics and Probability

Table 20, Logarithms of Factorial n ; Factorials and Their Reciprocals; Table 21, Binomial Coefficients; Probability; Table 22, Probability Functions (Normal Distribution); Table 23, Factors for Computing Probable Errors. Table 24, χ^2 Distribution.

Interest and Actuarial Experience

The following tables are self-explanatory: Table 25, Amount of 1 at Compound Interest; Table 26, Present Value of 1 at Compound Interest; Table 27, Amount of an Annuity of 1; Table 28, Present Value of an Annuity of 1; Table 29, The Annuity That 1 Will Purchase;

Table 30, Logarithms for Interest Computation; Table 31, American Experience Mortality Table; Constants, Equivalents, and Conversion Factors; Table 32, Decimal Equivalents of Common Fractions.

Square Roots of Certain Common Fractions

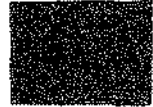
TABLE 33. IMPORTANT CONSTANTS This table gives the values and logarithms of a number of important constants.

The following tables are self-explanatory: Table 34, Complete Elliptic Integrals, K and E , for Different Values of the Modulus, k ; Table 35, A Table of Conversion Factors (Weights and Measures).

Four-place Tables

The following tables are self-explanatory: Table 36, Common Logarithms of Trigonometric Functions; Table 37, Natural Trigonometric Functions; Table 38, Common Logarithms of Numbers; Table 39, Common Antilogarithms.

LOGARITHMS



- TABLE 1. *Five-place Common Logarithms of Numbers, 191*
- TABLE 2. *Seven-place Common Logarithms of Numbers, 209*
- TABLE 3. *Common Logarithms of Primes, 213*
- TABLE 4. *Common Logarithms of Gamma
Function $\Gamma(n)$, 214
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Table 1 FIVE-PLACE COMMON LOGARITHMS OF NUMBERS† 191

100-150

N.											Proportional parts		
	0	1	2	3	4	5	6	7	8	9	44	43	42
100	00 000	043	087	130	173	217	260	303	346	389			
101	432	475	518	561	604	647	689	732	775	817	1	4.4	4.3
102	860	903	946	988	*030	*072	*115	*157	*199	*242	2	8.8	8.6
103	01 284	326	368	410	452	494	536	578	620	662	3	13.2	12.9
104	703	745	787	828	870	912	953	995	*036	*076	4	17.6	17.2
105	02 119	160	202	243	284	325	366	407	449	490	5	22.0	21.5
106	531	572	612	653	694	735	776	816	857	898	6	26.4	25.8
107	938	979	*019	*060	*100	*141	*181	*222	*262	*302	7	30.8	30.1
108	03 342	383	423	463	503	543	583	623	663	703	8	35.2	34.4
109	743	782	822	862	902	941	981	*021	*060	*100	9	39.6	38.7
110	04 139	179	218	258	297	336	376	415	454	493			
111	532	571	610	650	689	727	766	805	844	883	1	41	40
112	922	961	999	*038	*077	*115	*154	*192	*231	*269	2	4.1	4.0
113	06 308	346	385	423	461	500	538	576	614	652	3	8.1	7.9
114	690	729	767	805	843	881	918	956	994	*032	4	12.1	11.7
115	06 070	108	145	183	221	258	296	333	371	408	5	16.4	16.0
116	446	483	521	558	595	633	670	707	744	781	6	20.8	20.0
117	819	856	893	930	967	*004	*041	*078	*115	*151	7	25.2	24.0
118	07 188	225	262	298	335	372	408	445	482	518	8	29.6	28.0
119	556	591	628	664	700	737	773	809	846	882	9	34.0	32.0
120	08 918	954	990	*027	*063	*099	*135	*171	*207	*243			
121	406	441	477	512	547	582	617	652	687	722	1	38	37
122	896	931	966	*001	*036	*071	*106	*141	*176	*211	2	3.8	3.7
123	09 342	377	412	447	482	517	552	587	621	656	3	7.6	7.4
124	691	726	760	795	830	864	899	934	968	*003	4	11.4	11.1
125	09 691	726	760	795	830	864	899	934	968	*003	5	15.2	14.8
126	10 037	072	106	140	175	209	243	278	312	346	6	19.6	18.8
127	360	415	449	483	517	551	585	619	653	687	7	24.0	22.2
128	721	765	799	833	867	899	924	958	992	*025	8	28.4	25.9
129	11 059	093	126	160	193	227	261	294	327	361	9	32.8	29.5
130	09 394	428	461	494	528	561	594	628	661	694			
131	877	910	943	976	1009	1042	1075	1108	1141	*024	1	35	34
132	12 057	090	123	156	189	222	254	287	320	352	2	3.5	3.4
133	365	418	450	483	516	548	581	613	646	679	3	7.0	6.8
134	719	743	775	808	840	872	905	937	969	*001	4	10.5	10.2
135	13 933	066	098	130	162	194	226	258	290	322	5	14.0	13.6
136	354	386	418	450	481	513	545	577	608	640	6	17.5	17.0
137	672	704	735	767	799	830	862	893	925	956	7	21.0	20.4
138	988	*019	*051	*082	*114	*145	*176	*208	*239	*270	8	24.5	23.8
139	14 301	333	364	395	426	457	489	520	551	582	9	28.0	27.2
140	09 613	644	675	706	737	768	799	829	860	891			
141	922	953	983	*014	*045	*076	*106	*137	*168	*198	1	32	31
142	15 229	259	290	320	351	381	412	442	473	503	2	3.2	3.1
143	534	564	594	625	655	685	715	746	776	806	3	6.4	6.2
144	836	866	897	927	957	987	*017	*047	*077	*107	4	9.6	9.3
145	16 137	167	197	227	256	286	316	346	376	406	5	12.8	12.6
146	435	465	495	524	554	584	613	643	673	702	6	16.0	15.5
147	722	761	791	820	850	879	909	938	967	997	7	19.2	18.6
148	17 026	066	085	114	143	173	202	231	260	289	8	22.4	21.7
149	319	348	377	406	435	464	493	522	551	580	9	25.6	24.8
150	609	638	667	696	725	754	782	811	840	869			
N.	0	1	2	3	4	5	6	7	8	9	Proportional parts		

.00 000-.17 869

† Entries in Table 1 are values of $\log_{10} N$ for the indicated values of N .

Table 1 FIVE-PLACE COMMON LOGARITHMS OF NUMBERS (continued)

150-200

N.	0	1	2	3	4	5	6	7	8	9	Proportional parts															
150	17 609	628	667	686	725	754	782	811	840	869	<table border="1"> <tr><td>29</td><td>28</td></tr> <tr><td>1 3.9</td><td>2.8</td></tr> <tr><td>2 5.8</td><td>5.6</td></tr> <tr><td>3 8.7</td><td>8.4</td></tr> <tr><td>4 11.6</td><td>11.2</td></tr> </table>	29	28	1 3.9	2.8	2 5.8	5.6	3 8.7	8.4	4 11.6	11.2					
29	28																									
1 3.9	2.8																									
2 5.8	5.6																									
3 8.7	8.4																									
4 11.6	11.2																									
151	898	926	955	984	*013	*041	*070	*099	*127	*155																
152	18 184	213	241	270	298	327	355	384	412	441																
153	469	498	526	554	583	611	639	667	696	724																
154	752	780	808	837	865	893	921	949	977	*005																
155	19 033	051	089	117	145	173	201	229	257	285	<table border="1"> <tr><td>5</td><td>14.5</td><td>14.0</td></tr> <tr><td>6</td><td>17.4</td><td>16.8</td></tr> <tr><td>7</td><td>20.3</td><td>19.6</td></tr> <tr><td>8</td><td>23.2</td><td>22.4</td></tr> <tr><td>9</td><td>26.1</td><td>25.2</td></tr> </table>	5	14.5	14.0	6	17.4	16.8	7	20.3	19.6	8	23.2	22.4	9	26.1	25.2
5	14.5	14.0																								
6	17.4	16.8																								
7	20.3	19.6																								
8	23.2	22.4																								
9	26.1	25.2																								
156	312	340	368	396	424	451	479	507	535	562																
157	590	618	646	673	700	728	756	783	811	838																
158	866	893	921	948	976	*003	*030	*058	*085	*112																
159	20 140	167	194	222	249	276	303	330	356	385																
160	412	439	466	493	520	546	575	602	629	656																
161	683	710	737	763	790	817	844	871	898	925																
162	952	978	*005	*032	*059	*085	*112	*139	*165	*192																
163	21 210	246	272	299	325	352	378	405	431	458																
164	484	511	537	564	590	617	643	669	696	722																
165	746	775	801	827	854	880	905	932	958	985	<table border="1"> <tr><td>27</td><td>26</td></tr> <tr><td>1 2.7</td><td>2.6</td></tr> <tr><td>2 5.4</td><td>5.2</td></tr> <tr><td>3 8.1</td><td>7.8</td></tr> <tr><td>4 10.8</td><td>10.4</td></tr> </table>	27	26	1 2.7	2.6	2 5.4	5.2	3 8.1	7.8	4 10.8	10.4					
27	26																									
1 2.7	2.6																									
2 5.4	5.2																									
3 8.1	7.8																									
4 10.8	10.4																									
166	22 011	037	063	089	115	141	167	194	220	246																
167	272	298	324	350	376	401	427	453	479	505																
168	531	557	583	608	634	660	685	712	737	763																
169	789	814	840	866	891	917	943	968	994	*019																
170	23 045	070	096	121	147	172	198	223	249	274																
171	300	325	350	376	401	426	452	477	502	528																
172	563	578	603	629	654	679	704	729	754	779																
173	805	830	856	880	905	930	955	980	*006	*030																
174	24 055	080	105	130	155	180	204	229	254	279																
175	304	329	353	378	403	428	452	477	502	527	<table border="1"> <tr><td>25</td><td></td></tr> <tr><td>1 2.5</td><td></td></tr> <tr><td>2 5.0</td><td></td></tr> <tr><td>3 7.5</td><td></td></tr> <tr><td>4 10.0</td><td></td></tr> </table>	25		1 2.5		2 5.0		3 7.5		4 10.0						
25																										
1 2.5																										
2 5.0																										
3 7.5																										
4 10.0																										
176	561	576	601	625	650	674	699	724	748	773																
177	797	822	846	871	895	920	944	969	993	*018																
178	25 042	066	091	115	139	164	188	212	237	261																
179	285	310	334	358	382	406	431	455	479	503																
180	527	551	575	600	624	648	672	696	720	744																
181	768	792	816	840	864	888	912	935	959	983																
182	26 007	031	055	079	102	126	150	174	198	221																
183	245	269	293	316	340	364	387	411	435	458																
184	482	505	529	553	576	600	623	647	670	694																
185	717	741	764	788	811	834	858	881	905	928	<table border="1"> <tr><td>24</td><td>23</td></tr> <tr><td>1 2.4</td><td>2.3</td></tr> <tr><td>2 4.8</td><td>4.6</td></tr> <tr><td>3 7.2</td><td>6.9</td></tr> <tr><td>4 9.6</td><td>9.2</td></tr> </table>	24	23	1 2.4	2.3	2 4.8	4.6	3 7.2	6.9	4 9.6	9.2					
24	23																									
1 2.4	2.3																									
2 4.8	4.6																									
3 7.2	6.9																									
4 9.6	9.2																									
186	951	975	998	*021	*045	*068	*091	*114	*138	*161																
187	27 184	207	231	254	277	300	323	346	370	393																
188	416	439	462	485	508	531	554	577	600	623																
189	646	669	692	715	738	761	784	807	830	852																
190	875	898	921	944	967	989	*012	*035	*058	*081																
191	28 103	126	149	171	194	217	240	262	285	307																
192	330	353	375	398	421	443	466	488	511	533																
193	566	578	601	623	646	668	691	713	735	758																
194	780	803	825	847	870	892	914	937	959	981																
195	29 003	026	048	070	092	115	137	159	181	203																
196	226	248	270	292	314	336	358	380	403	425																
197	447	469	491	513	535	557	579	601	623	645																
198	867	888	910	932	954	976	998	820	842	863																
199	885	907	929	951	973	994	*016	*038	*060	*081																
200	30 103	125	146	168	190	211	233	255	276	298																
N.	0	1	2	3	4	5	6	7	8	9	Proportional parts															

Table I FIVE-PLACE COMMON LOGARITHMS OF NUMBERS (continued)

200-250

N.	0	1	2	3	4	5	6	7	8	9	Proportional parts																				
200	30 103	125	146	168	190	211	233	255	276	298	<table border="1"> <tr><td>22</td><td>21</td></tr> <tr><td>1 2.2</td><td>2.1</td></tr> <tr><td>2 4.4</td><td>4.2</td></tr> <tr><td>3 6.6</td><td>6.3</td></tr> <tr><td>4 8.8</td><td>8.4</td></tr> <tr><td>5 11.0</td><td>10.5</td></tr> <tr><td>6 13.2</td><td>12.6</td></tr> <tr><td>7 15.4</td><td>14.7</td></tr> <tr><td>8 17.6</td><td>16.8</td></tr> <tr><td>9 19.8</td><td>18.9</td></tr> </table>	22	21	1 2.2	2.1	2 4.4	4.2	3 6.6	6.3	4 8.8	8.4	5 11.0	10.5	6 13.2	12.6	7 15.4	14.7	8 17.6	16.8	9 19.8	18.9
22	21																														
1 2.2	2.1																														
2 4.4	4.2																														
3 6.6	6.3																														
4 8.8	8.4																														
5 11.0	10.5																														
6 13.2	12.6																														
7 15.4	14.7																														
8 17.6	16.8																														
9 19.8	18.9																														
201	320	341	363	384	405	428	449	471	492	514																					
202	535	557	578	600	621	643	664	685	707	728																					
203	750	771	792	814	835	856	878	899	920	942																					
204	963	984	*006	*027	*048	*069	*091	*112	*133	*154	<table border="1"> <tr><td>20</td><td></td></tr> <tr><td>1 2.0</td><td></td></tr> <tr><td>2 4.0</td><td></td></tr> <tr><td>3 6.0</td><td></td></tr> <tr><td>4 8.0</td><td></td></tr> <tr><td>5 10.0</td><td></td></tr> <tr><td>6 12.0</td><td></td></tr> <tr><td>7 14.0</td><td></td></tr> <tr><td>8 16.0</td><td></td></tr> <tr><td>9 18.0</td><td></td></tr> </table>	20		1 2.0		2 4.0		3 6.0		4 8.0		5 10.0		6 12.0		7 14.0		8 16.0		9 18.0	
20																															
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8 16.0																															
9 18.0																															
205	31 175	197	218	239	260	281	302	323	345	366																					
206	387	408	429	450	471	492	513	534	555	576																					
207	597	618	639	660	681	702	723	744	765	785																					
208	806	827	848	869	890	911	931	952	973	994																					
209	22 015	036	056	077	098	118	139	160	181	201	<table border="1"> <tr><td>19</td><td></td></tr> <tr><td>1 1.9</td><td></td></tr> <tr><td>2 3.8</td><td></td></tr> <tr><td>3 5.7</td><td></td></tr> <tr><td>4 7.5</td><td></td></tr> <tr><td>5 9.4</td><td></td></tr> <tr><td>6 11.4</td><td></td></tr> <tr><td>7 13.3</td><td></td></tr> <tr><td>8 15.2</td><td></td></tr> <tr><td>9 17.1</td><td></td></tr> </table>	19		1 1.9		2 3.8		3 5.7		4 7.5		5 9.4		6 11.4		7 13.3		8 15.2		9 17.1	
19																															
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6 11.4																															
7 13.3																															
8 15.2																															
9 17.1																															
210	222	243	263	284	305	325	346	366	387	408																					
211	428	449	469	490	510	531	552	572	593	613																					
212	634	654	675	695	715	736	756	777	797	818																					
213	838	858	878	899	919	940	960	980	*001	*021																					
214	33 041	062	082	102	122	143	163	183	203	224	<table border="1"> <tr><td>18</td><td></td></tr> <tr><td>1 1.8</td><td></td></tr> <tr><td>2 3.6</td><td></td></tr> <tr><td>3 5.4</td><td></td></tr> <tr><td>4 7.2</td><td></td></tr> <tr><td>5 9.0</td><td></td></tr> <tr><td>6 10.8</td><td></td></tr> <tr><td>7 12.6</td><td></td></tr> <tr><td>8 14.4</td><td></td></tr> <tr><td>9 16.2</td><td></td></tr> </table>	18		1 1.8		2 3.6		3 5.4		4 7.2		5 9.0		6 10.8		7 12.6		8 14.4		9 16.2	
18																															
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3 5.4																															
4 7.2																															
5 9.0																															
6 10.8																															
7 12.6																															
8 14.4																															
9 16.2																															
215	244	264	284	304	325	345	365	385	405	425																					
216	445	465	485	505	525	545	565	585	605	625																					
217	646	666	686	706	726	746	766	786	806	826																					
218	846	866	885	905	925	945	965	985	*005	*025																					
219	34 044	064	084	104	124	143	163	183	203	223	<table border="1"> <tr><td>17</td><td></td></tr> <tr><td>1 1.7</td><td></td></tr> <tr><td>2 3.4</td><td></td></tr> <tr><td>3 5.1</td><td></td></tr> <tr><td>4 6.8</td><td></td></tr> <tr><td>5 8.5</td><td></td></tr> <tr><td>6 10.2</td><td></td></tr> <tr><td>7 11.9</td><td></td></tr> <tr><td>8 13.6</td><td></td></tr> <tr><td>9 15.3</td><td></td></tr> </table>	17		1 1.7		2 3.4		3 5.1		4 6.8		5 8.5		6 10.2		7 11.9		8 13.6		9 15.3	
17																															
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7 11.9																															
8 13.6																															
9 15.3																															
220	242	262	282	301	321	341	361	380	400	420																					
221	439	459	479	498	518	537	557	577	596	616																					
222	635	655	674	694	713	733	753	772	792	811																					
223	830	850	869	889	908	928	947	967	986	*005																					
224	35 025	044	064	083	102	122	141	160	180	199	<table border="1"> <tr><td>16</td><td></td></tr> <tr><td>1 1.6</td><td></td></tr> <tr><td>2 3.2</td><td></td></tr> <tr><td>3 4.8</td><td></td></tr> <tr><td>4 6.4</td><td></td></tr> <tr><td>5 8.0</td><td></td></tr> <tr><td>6 9.6</td><td></td></tr> <tr><td>7 11.2</td><td></td></tr> <tr><td>8 12.8</td><td></td></tr> <tr><td>9 14.4</td><td></td></tr> </table>	16		1 1.6		2 3.2		3 4.8		4 6.4		5 8.0		6 9.6		7 11.2		8 12.8		9 14.4	
16																															
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9 14.4																															
225	218	238	257	276	295	315	334	353	372	392																					
226	411	430	449	468	488	507	526	545	564	583																					
227	603	622	641	660	679	698	717	735	755	774																					
228	793	813	832	851	870	889	908	927	946	965																					
229	984	*003	*021	*040	*059	*078	*097	*115	*135	*154	<table border="1"> <tr><td>15</td><td></td></tr> <tr><td>1 1.5</td><td></td></tr> <tr><td>2 3.0</td><td></td></tr> <tr><td>3 4.5</td><td></td></tr> <tr><td>4 6.0</td><td></td></tr> <tr><td>5 7.5</td><td></td></tr> <tr><td>6 9.0</td><td></td></tr> <tr><td>7 10.5</td><td></td></tr> <tr><td>8 12.0</td><td></td></tr> <tr><td>9 13.5</td><td></td></tr> </table>	15		1 1.5		2 3.0		3 4.5		4 6.0		5 7.5		6 9.0		7 10.5		8 12.0		9 13.5	
15																															
1 1.5																															
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6 9.0																															
7 10.5																															
8 12.0																															
9 13.5																															
230	36 173	192	211	229	246	267	286	305	324	342																					
231	361	380	399	418	436	455	474	493	511	530																					
232	549	568	586	605	624	642	661	680	698	717																					
233	736	754	773	791	810	829	847	865	884	903																					
234	922	940	959	977	996	*014	*033	*051	*070	*089	<table border="1"> <tr><td>14</td><td></td></tr> <tr><td>1 1.4</td><td></td></tr> <tr><td>2 2.8</td><td></td></tr> <tr><td>3 4.2</td><td></td></tr> <tr><td>4 5.6</td><td></td></tr> <tr><td>5 7.0</td><td></td></tr> <tr><td>6 8.4</td><td></td></tr> <tr><td>7 9.8</td><td></td></tr> <tr><td>8 11.2</td><td></td></tr> <tr><td>9 12.6</td><td></td></tr> </table>	14		1 1.4		2 2.8		3 4.2		4 5.6		5 7.0		6 8.4		7 9.8		8 11.2		9 12.6	
14																															
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235	37 107	125	144	162	181	199	218	236	254	273																					
236	291	310	328	346	365	383	401	420	438	457																					
237	475	493	511	530	548	566	585	603	621	639																					
238	658	676	694	712	731	749	767	785	803	822																					
239	840	858	876	894	912	931	949	967	985	*003	<table border="1"> <tr><td>13</td><td></td></tr> <tr><td>1 1.3</td><td></td></tr> <tr><td>2 2.6</td><td></td></tr> <tr><td>3 3.9</td><td></td></tr> <tr><td>4 5.2</td><td></td></tr> <tr><td>5 6.5</td><td></td></tr> <tr><td>6 7.8</td><td></td></tr> <tr><td>7 9.1</td><td></td></tr> <tr><td>8 10.4</td><td></td></tr> <tr><td>9 11.7</td><td></td></tr> </table>	13		1 1.3		2 2.6		3 3.9		4 5.2		5 6.5		6 7.8		7 9.1		8 10.4		9 11.7	
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7 9.1																															
8 10.4																															
9 11.7																															
240	38 021	039	057	075	093	112	130	148	166	184																					
241	202	220	238	256	274	292	310	328	346	364																					
242	362	399	417	435	453	471	489	507	525	543																					
243	561	578	596	614	632	650	668	686	703	721																					
244	739	757	775	792	810	828	846	863	881	899	<table border="1"> <tr><td>12</td><td></td></tr> <tr><td>1 1.2</td><td></td></tr> <tr><td>2 2.4</td><td></td></tr> <tr><td>3 3.6</td><td></td></tr> <tr><td>4 4.8</td><td></td></tr> <tr><td>5 6.0</td><td></td></tr> <tr><td>6 7.2</td><td></td></tr> <tr><td>7 8.4</td><td></td></tr> <tr><td>8 9.6</td><td></td></tr> <tr><td>9 10.8</td><td></td></tr> </table>	12		1 1.2		2 2.4		3 3.6		4 4.8		5 6.0		6 7.2		7 8.4		8 9.6		9 10.8	
12																															
1 1.2																															
2 2.4																															
3 3.6																															
4 4.8																															
5 6.0																															
6 7.2																															
7 8.4																															
8 9.6																															
9 10.8																															
245	917	934	952	970	987	*005	*023	*041	*059	*076																					
246	39 064	111	129	146	164	182	199	217	235	252																					
247	270	287	305	322	340	358	375	393	410	428																					
248	445	463	480	498	515	533	550	568	585	602																					
249	620	637	655	672	690	707	724	742	759	777																					
250	794	811	829	846	863	881	898	915	933	950																					
N.	0	1	2	3	4	5	6	7	8	9	Proportional parts																				

Table 1 FIVE-PLACE COMMON LOGARITHMS OF NUMBERS (continued)

250-300

N.	0	1	2	3	4	5	6	7	8	9	Proportional parts																				
250	39 794	811	829	846	863	881	898	915	933	950	<table border="1"> <tr><td>1</td><td>18</td></tr> <tr><td>2</td><td>1.6</td></tr> <tr><td>3</td><td>3.6</td></tr> <tr><td>4</td><td>5.4</td></tr> <tr><td>5</td><td>7.2</td></tr> <tr><td>6</td><td>9.0</td></tr> <tr><td>7</td><td>10.8</td></tr> <tr><td>8</td><td>12.6</td></tr> <tr><td>9</td><td>14.4</td></tr> <tr><td>9</td><td>16.2</td></tr> </table>	1	18	2	1.6	3	3.6	4	5.4	5	7.2	6	9.0	7	10.8	8	12.6	9	14.4	9	16.2
1	18																														
2	1.6																														
3	3.6																														
4	5.4																														
5	7.2																														
6	9.0																														
7	10.8																														
8	12.6																														
9	14.4																														
9	16.2																														
251	967	985	*002	*019	*037	*054	*071	*088	*106	*123																					
252	40 140	157	175	192	209	226	243	261	278	295																					
253	312	329	346	364	381	398	415	432	449	466																					
254	483	500	518	535	552	569	586	603	620	637																					
255	654	671	688	705	722	739	756	773	790	807	<table border="1"> <tr><td>1</td><td>17</td></tr> <tr><td>2</td><td>1.7</td></tr> <tr><td>3</td><td>3.4</td></tr> <tr><td>4</td><td>5.1</td></tr> <tr><td>5</td><td>6.8</td></tr> <tr><td>6</td><td>8.5</td></tr> <tr><td>7</td><td>10.2</td></tr> <tr><td>8</td><td>11.9</td></tr> <tr><td>9</td><td>13.6</td></tr> <tr><td>9</td><td>15.3</td></tr> </table>	1	17	2	1.7	3	3.4	4	5.1	5	6.8	6	8.5	7	10.2	8	11.9	9	13.6	9	15.3
1	17																														
2	1.7																														
3	3.4																														
4	5.1																														
5	6.8																														
6	8.5																														
7	10.2																														
8	11.9																														
9	13.6																														
9	15.3																														
256	824	841	858	875	892	909	926	943	960	976																					
257	983	*010	*027	*044	*061	*078	*095	*111	*128	*145																					
258	41 162	179	196	212	229	246	263	280	296	313																					
259	330	347	363	380	397	414	430	447	464	481																					
260	497	514	531	547	564	581	597	614	631	647																					
261	664	681	697	714	731	747	764	780	797	814																					
262	830	847	863	880	896	913	929	946	963	979																					
263	985	*012	*029	*045	*062	*078	*095	*111	*127	*144																					
264	42 160	177	193	210	226	243	259	275	292	308																					
265	325	341	357	374	390	406	423	439	455	472	<table border="1"> <tr><td>1</td><td>16</td></tr> <tr><td>2</td><td>1.6</td></tr> <tr><td>3</td><td>3.2</td></tr> <tr><td>4</td><td>4.8</td></tr> <tr><td>5</td><td>6.4</td></tr> <tr><td>6</td><td>8.0</td></tr> <tr><td>7</td><td>9.6</td></tr> <tr><td>8</td><td>11.2</td></tr> <tr><td>9</td><td>12.8</td></tr> <tr><td>9</td><td>14.4</td></tr> </table>	1	16	2	1.6	3	3.2	4	4.8	5	6.4	6	8.0	7	9.6	8	11.2	9	12.8	9	14.4
1	16																														
2	1.6																														
3	3.2																														
4	4.8																														
5	6.4																														
6	8.0																														
7	9.6																														
8	11.2																														
9	12.8																														
9	14.4																														
266	488	504	521	537	553	570	586	602	619	635																					
267	651	667	684	700	716	732	749	765	781	797																					
268	813	830	846	862	878	894	911	927	943	959																					
269	975	991	*008	*024	*040	*056	*072	*088	*104	*120																					
270	43 136	152	169	185	201	217	233	249	265	281																					
271	267	313	329	345	361	377	393	409	425	441																					
272	457	473	489	505	521	537	553	569	584	600																					
273	616	632	648	664	680	695	712	727	743	759																					
274	775	791	807	823	838	854	870	886	902	917																					
275	933	949	965	981	996	*012	*028	*044	*059	*075																					
276	44 091	107	122	138	154	170	185	201	217	232																					
277	248	264	279	295	311	326	342	358	373	389																					
278	404	420	436	451	467	483	498	514	529	545																					
279	560	576	592	607	623	638	654	669	685	700																					
280	716	731	747	762	778	793	809	824	840	855	<table border="1"> <tr><td>1</td><td>15</td></tr> <tr><td>2</td><td>1.5</td></tr> <tr><td>3</td><td>3.0</td></tr> <tr><td>4</td><td>4.5</td></tr> <tr><td>5</td><td>6.0</td></tr> <tr><td>6</td><td>7.5</td></tr> <tr><td>7</td><td>9.0</td></tr> <tr><td>8</td><td>10.5</td></tr> <tr><td>9</td><td>12.0</td></tr> <tr><td>9</td><td>13.5</td></tr> </table>	1	15	2	1.5	3	3.0	4	4.5	5	6.0	6	7.5	7	9.0	8	10.5	9	12.0	9	13.5
1	15																														
2	1.5																														
3	3.0																														
4	4.5																														
5	6.0																														
6	7.5																														
7	9.0																														
8	10.5																														
9	12.0																														
9	13.5																														
281	871	886	902	917	932	948	963	979	994	*010																					
282	45 026	040	056	071	086	102	117	133	148	163																					
283	179	194	209	225	240	255	271	286	301	317																					
284	332	347	362	378	393	408	423	439	454	469																					
285	484	500	515	530	545	561	576	591	606	621																					
286	637	652	667	682	697	712	728	743	758	773																					
287	788	803	818	834	849	864	879	894	909	924																					
288	939	954	969	984	*000	*015	*030	*045	*060	*075																					
289	46 090	105	120	135	150	165	180	195	210	225																					
290	240	255	270	285	300	315	330	345	360	374																					
291	389	404	418	434	449	464	479	494	509	523																					
292	538	553	568	583	598	613	627	642	657	672																					
293	687	702	716	731	746	761	776	790	805	820																					
294	835	850	864	879	894	909	923	938	953	967																					
295	982	997	*012	*026	*041	*056	*070	*085	*100	*114																					
296	47 129	144	158	173	188	202	217	232	246	261																					
297	276	290	306	319	334	349	363	378	392	407																					
298	422	436	451	465	480	494	509	524	538	553																					
299	567	582	596	611	625	640	654	669	683	698																					
300	712	727	741	756	770	784	799	813	828	842																					
N.	0	1	2	3	4	5	6	7	8	9	log e = 0.43429																				
											Proportional parts																				

Table 1 FIVE-PLACE COMMON LOGARITHMS OF NUMBERS (continued)

300-350

N.	0	1	2	3	4	5	6	7	8	9	Proportional parts
300	47 712	727	741	755	770	784	799	813	828	842	15 1 1.5 2 2.5 3 3.5 4 4.5 5 5.5 6 6.5 7 7.5 8 8.5 9 9.5
301	357	871	885	900	914	929	943	958	972	986	
302	48 001	015	029	044	058	073	087	101	116	130	
303	144	159	173	187	202	216	230	244	259	273	
304	287	302	316	330	344	359	373	387	401	415	
305	430	444	458	473	487	501	515	530	544	558	14 1 1.4 2 2.8 3 3.2 4 4.4 5 5.6 6 6.2 7 7.4 8 8.6 9 9.8
306	572	586	601	615	629	643	657	671	686	700	
307	714	728	742	756	770	785	799	813	827	841	
308	855	869	883	897	911	926	940	954	968	982	
309	996	*010	*024	*038	*052	*066	*080	*094	*108	*122	
310	49 136	150	164	178	192	206	220	234	248	262	13 1 1.3 2 2.6 3 3.4 4 4.8 5 5.2 6 6.6 7 7.2 8 8.4 9 9.6
311	276	290	304	318	332	346	360	374	388	402	
312	415	429	443	457	471	485	499	513	527	541	
313	554	568	582	596	610	624	638	651	665	679	
314	693	707	721	734	748	762	776	790	803	817	
315	831	845	859	872	886	900	914	927	941	955	12 1 1.2 2 2.4 3 3.6 4 4.8 5 5.6 6 6.4 7 7.6 8 8.8 9 9.2
316	969	982	996	*010	*024	*037	*051	*065	*079	*092	
317	50 106	120	133	147	161	174	188	202	215	229	
318	243	256	270	284	297	311	325	338	352	365	
319	379	393	406	420	433	447	461	474	488	501	
320	515	529	542	556	569	583	596	610	623	637	11 1 1.1 2 2.2 3 3.3 4 4.4 5 5.5 6 6.6 7 7.7 8 8.8 9 9.9
321	651	664	678	691	705	718	732	745	759	772	
322	796	799	813	826	840	853	866	880	893	907	
323	920	934	947	961	974	987	*001	*014	*028	*041	
324	51 055	068	081	095	108	121	135	148	162	175	
325	188	202	215	228	242	255	268	282	295	308	10 1 1.0 2 2.0 3 3.0 4 4.0 5 5.0 6 6.0 7 7.0 8 8.0 9 9.0
326	322	335	348	362	375	388	402	415	428	441	
327	455	468	481	495	508	521	534	548	561	574	
328	587	601	614	627	640	654	667	680	693	706	
329	720	733	746	759	772	786	799	812	825	838	
330	851	865	878	891	904	917	930	943	957	970	9 1 1.1 2 2.1 3 3.1 4 4.1 5 5.1 6 6.1 7 7.1 8 8.1 9 9.1
331	983	996	*009	*022	*035	*048	*061	*075	*088	*101	
332	52 114	127	140	153	166	179	192	205	218	231	
333	244	257	270	284	297	310	323	336	349	362	
334	375	388	401	414	427	440	453	466	479	492	
335	504	517	530	543	556	569	582	595	608	621	8 1 1.1 2 2.1 3 3.1 4 4.1 5 5.1 6 6.1 7 7.1 8 8.1 9 9.1
336	634	647	660	673	686	699	711	724	737	750	
337	783	776	789	802	815	827	840	853	866	879	
338	852	905	917	930	943	956	969	982	994	*007	
339	53 020	033	046	058	071	084	097	110	122	135	
340	148	161	173	186	199	212	224	237	250	263	7 1 1.1 2 2.1 3 3.1 4 4.1 5 5.1 6 6.1 7 7.1 8 8.1 9 9.1
341	275	288	301	314	326	339	352	364	377	390	
342	403	415	428	441	453	466	479	491	504	517	
343	529	542	555	567	580	593	606	618	631	643	
344	656	668	681	694	706	719	732	744	757	769	
345	782	794	807	820	832	845	857	870	882	895	6 1 1.1 2 2.1 3 3.1 4 4.1 5 5.1 6 6.1 7 7.1 8 8.1 9 9.1
346	908	920	933	945	958	970	983	995	*008	*020	
347	54 033	045	058	070	083	095	108	120	133	145	
348	158	170	183	195	208	220	233	245	258	270	
349	283	295	307	320	332	345	357	370	382	394	
350	407	419	432	444	456	469	481	494	506	518	log x = 0.49716
N.	0	1	2	3	4	5	6	7	8	9	Proportional parts

.47 712-.54 518

Table 1 FIVE-PLACE COMMON LOGARITHMS OF NUMBERS (continued)

350-400

N.	0	1	2	3	4	5	6	7	8	9	Proportional parts																				
350	54 407	419	432	444	456	469	481	494	506	518	<table border="1"> <tr><td colspan="2">13</td></tr> <tr><td>1</td><td>1.3</td></tr> <tr><td>2</td><td>2.6</td></tr> <tr><td>3</td><td>3.9</td></tr> <tr><td>4</td><td>5.2</td></tr> <tr><td>5</td><td>6.5</td></tr> <tr><td>6</td><td>7.8</td></tr> <tr><td>7</td><td>9.1</td></tr> <tr><td>8</td><td>10.4</td></tr> <tr><td>9</td><td>11.7</td></tr> </table>	13		1	1.3	2	2.6	3	3.9	4	5.2	5	6.5	6	7.8	7	9.1	8	10.4	9	11.7
13																															
1	1.3																														
2	2.6																														
3	3.9																														
4	5.2																														
5	6.5																														
6	7.8																														
7	9.1																														
8	10.4																														
9	11.7																														
351	531	543	555	568	580	593	605	617	630	642																					
352	654	667	679	691	704	716	728	741	753	765																					
353	777	790	802	814	827	839	851	864	876	888																					
354	900	913	925	937	949	962	974	986	998	*011																					
355	55 023	035	047	060	072	084	096	108	121	133	<table border="1"> <tr><td colspan="2">12</td></tr> <tr><td>1</td><td>1.2</td></tr> <tr><td>2</td><td>2.4</td></tr> <tr><td>3</td><td>3.6</td></tr> <tr><td>4</td><td>4.8</td></tr> <tr><td>5</td><td>6.0</td></tr> <tr><td>6</td><td>7.2</td></tr> <tr><td>7</td><td>8.4</td></tr> <tr><td>8</td><td>9.6</td></tr> <tr><td>9</td><td>10.8</td></tr> </table>	12		1	1.2	2	2.4	3	3.6	4	4.8	5	6.0	6	7.2	7	8.4	8	9.6	9	10.8
12																															
1	1.2																														
2	2.4																														
3	3.6																														
4	4.8																														
5	6.0																														
6	7.2																														
7	8.4																														
8	9.6																														
9	10.8																														
356	145	157	169	182	194	206	218	230	242	255																					
357	267	279	291	303	315	328	340	352	364	376																					
358	388	400	413	425	437	449	461	473	485	497																					
359	509	522	534	546	558	570	582	594	606	618																					
360	630	642	654	666	678	691	703	715	727	739																					
361	751	763	775	787	799	811	823	835	847	859																					
362	871	883	895	907	919	931	943	955	967	979																					
363	991	*003	*015	*027	*038	*050	*062	*074	*086	*098																					
364	56 110	122	134	146	158	170	182	194	206	217																					
365	229	241	253	265	277	289	301	312	324	336																					
366	348	350	372	384	396	407	419	431	443	455																					
367	467	478	490	502	514	526	538	549	561	573																					
368	585	597	608	620	632	644	655	667	679	691																					
369	708	714	726	738	750	761	773	785	797	808																					
370	820	832	844	855	867	879	891	902	914	925																					
371	937	949	961	972	984	996	*008	*019	*031	*043																					
372	57 054	066	078	089	101	113	124	135	146	159																					
373	171	182	194	206	217	229	241	252	264	276																					
374	287	299	310	322	334	345	357	368	380	392																					
375	403	415	426	438	449	461	473	484	496	507																					
376	519	530	542	553	565	576	588	600	611	623																					
377	634	646	657	669	680	692	703	715	726	738																					
378	749	761	772	784	795	807	818	830	841	852																					
379	864	875	887	898	910	921	933	944	955	967																					
380	978	990	*001	*013	*024	*035	*047	*058	*070	*081																					
381	58 092	104	115	127	138	149	161	172	184	195																					
382	206	218	228	240	252	263	274	286	297	309																					
383	320	331	343	354	365	377	388	399	410	422																					
384	433	444	456	467	478	490	501	512	524	535																					
385	546	557	569	580	591	602	614	625	636	647																					
386	659	670	681	692	704	715	726	737	749	760																					
387	771	782	794	805	816	827	838	850	861	872																					
388	883	894	906	917	928	939	950	961	973	984																					
389	995	*006	*017	*028	*040	*051	*062	*073	*084	*095																					
390	59 106	118	129	140	151	162	173	184	195	207																					
391	218	229	240	251	262	273	284	295	306	318																					
392	328	340	351	362	373	384	395	406	417	428																					
393	438	450	461	472	483	494	506	517	528	539																					
394	550	561	572	583	594	605	616	627	638	649																					
395	660	671	682	693	704	715	726	737	748	759																					
396	770	780	791	802	813	824	835	846	857	868																					
397	879	890	901	912	923	934	945	956	967	977																					
398	988	999	*010	*021	*032	*043	*054	*065	*076	*086																					
399	60 097	108	119	130	141	152	163	173	184	195																					
400	206	217	228	239	249	260	271	282	293	304																					
N.	0	1	2	3	4	5	6	7	8	9	Proportional parts																				

.54 407-.60 304

Table 1 FIVE-PLACE COMMON LOGARITHMS OF NUMBERS (continued)

400-450

N.	0	1	2	3	4	5	6	7	8	9	Proportional parts
400	60 206	217	228	239	249	260	271	282	293	304	
401	314	325	336	347	358	369	379	390	401	412	
402	423	433	444	455	466	477	487	498	509	520	
403	531	541	552	563	574	584	595	606	617	627	
404	638	649	660	670	681	692	703	713	724	735	
405	746	756	767	778	788	799	810	821	831	842	
406	853	863	874	885	895	906	917	927	938	949	
407	950	970	981	991	*002	*013	*023	*034	*045	*055	
408	61 066	077	087	098	109	119	130	140	151	162	
409	172	183	194	204	215	226	236	247	257	268	
410	278	289	300	310	321	331	342	352	363	374	
411	384	395	405	416	426	437	448	458	469	479	
412	490	500	511	521	532	542	553	563	574	584	
413	595	606	616	627	637	648	658	669	679	690	
414	700	711	721	731	742	752	763	773	784	794	
415	805	815	826	836	847	857	868	878	888	899	
416	909	920	930	941	951	962	972	982	993	*003	
417	62 014	024	034	045	055	066	076	086	097	107	
418	118	128	138	149	159	170	180	190	201	211	
419	221	232	242	252	263	273	284	294	304	315	
420	325	335	346	356	366	377	387	397	408	418	
421	428	439	449	459	469	480	490	500	511	521	
422	531	542	552	562	572	583	593	603	613	624	
423	634	644	655	665	675	685	695	706	716	726	
424	737	747	757	767	778	788	798	808	818	829	
425	839	849	859	870	880	890	900	910	921	931	
426	941	951	961	972	982	992	*002	*012	*022	*033	
427	63 043	053	063	073	083	094	104	114	124	134	
428	144	155	165	175	185	195	205	215	225	235	
429	246	256	266	276	286	296	306	317	327	337	
430	347	357	367	377	387	397	407	417	428	438	
431	448	458	468	478	488	498	508	518	528	538	
432	548	558	568	579	589	599	609	619	629	639	
433	649	659	669	679	689	699	709	719	729	739	
434	749	759	769	779	789	799	809	819	829	839	
435	849	859	869	879	889	899	909	919	929	939	
436	949	959	969	979	989	999	*008	*018	*028	*038	
437	64 048	058	068	078	088	098	108	118	128	137	
438	147	157	167	177	187	197	207	217	227	237	
439	246	256	266	276	286	296	306	316	326	335	
440	345	355	365	375	385	395	404	414	424	434	
441	444	454	464	473	483	493	503	513	523	532	
442	542	552	562	572	582	591	601	611	621	631	
443	640	650	660	670	680	689	699	709	719	729	
444	738	748	758	768	777	787	797	807	816	826	
445	836	846	856	865	875	885	895	904	914	924	
446	933	943	953	963	972	982	992	*002	*011	*021	
447	65 031	040	050	060	070	079	089	099	108	118	
448	128	137	147	157	167	176	186	196	205	215	
449	225	234	244	254	263	273	283	292	302	312	
450	321	331	341	350	360	369	379	389	398	408	
N.	0	1	2	3	4	5	6	7	8	9	Proportional parts

11

1	1.1
2	2.2
3	3.3
4	4.4
5	5.5
6	6.6
7	7.7
8	8.8
9	9.9

10

1	1.0
2	2.0
3	3.0
4	4.0
5	5.0
6	6.0
7	7.0
8	8.0
9	9.0

9

1	0.9
2	1.8
3	2.7
4	3.6
5	4.5
6	5.4
7	6.3
8	7.2
9	8.1

Table I FIVE-PLACE COMMON LOGARITHMS OF NUMBERS (continued)

450-500

N.	0	1	2	3	4	5	6	7	8	9	Proportional parts
450	05 321	331	341	350	350	369	379	389	398	408	
451	418	427	437	447	456	466	475	485	495	504	
452	514	523	533	543	552	562	571	581	591	600	
453	610	619	629	639	648	658	667	677	686	696	
454	705	715	725	734	744	753	763	772	782	792	
455	801	811	820	830	839	849	858	868	877	887	
456	896	906	916	925	935	944	954	963	973	982	
457	992	*001	*011	*020	*030	*039	*049	*058	*068	*077	
458	66 087	095	105	115	124	134	143	153	162	172	
459	181	191	200	210	219	229	238	247	257	266	
460	276	285	295	304	314	323	332	342	351	361	
461	370	380	389	398	408	417	427	436	445	455	
462	464	474	483	492	502	511	521	530	539	549	
463	558	567	577	586	595	605	614	624	633	642	
464	652	661	671	680	689	699	708	717	727	736	
465	745	755	764	773	782	792	801	811	820	829	
466	839	849	857	867	876	885	894	904	913	922	
467	932	941	950	959	969	978	987	997	*005	*015	
468	67 025	034	043	052	062	071	080	089	099	108	
469	117	127	136	145	154	164	173	182	191	201	
470	210	219	228	237	247	256	265	274	284	293	
471	302	311	321	330	339	348	357	367	376	385	
472	394	403	413	422	431	440	449	459	468	477	
473	486	495	504	514	523	532	541	550	560	569	
474	578	587	596	605	614	624	633	642	651	660	
475	669	679	688	697	706	715	724	733	742	752	
476	761	770	779	788	797	806	815	825	834	843	
477	852	861	870	879	888	897	906	915	925	934	
478	943	952	961	970	979	988	997	*006	*015	*024	
479	68 034	043	052	061	070	079	088	097	106	115	
480	124	133	142	151	160	169	178	187	196	205	
481	215	224	233	242	251	260	269	278	287	296	
482	306	314	323	332	341	350	358	368	377	386	
483	395	404	413	422	431	440	449	458	467	476	
484	485	494	502	511	520	529	538	547	556	565	
485	574	583	592	601	610	619	628	637	646	655	
486	664	673	681	690	699	708	717	726	735	744	
487	753	762	771	780	789	797	806	815	824	833	
488	842	851	860	869	878	886	895	904	913	922	
489	931	940	949	958	966	975	984	993	*002	*011	
490	69 020	028	037	046	055	064	073	082	090	099	
491	108	117	126	135	144	152	161	170	179	188	
492	197	205	214	223	232	241	249	258	267	276	
493	285	294	302	311	320	329	338	346	355	364	
494	373	381	390	399	408	417	425	434	443	452	
495	461	469	478	487	496	504	513	522	531	539	
496	548	557	566	574	583	592	601	609	618	627	
497	638	644	653	662	671	679	688	697	705	714	
498	723	732	740	749	758	767	775	784	793	801	
499	810	819	827	836	845	854	862	871	880	888	
500	897	906	914	923	932	940	948	958	966	975	
N.	0	1	2	3	4	5	6	7	8	9	Proportional parts

10	
1	1.0
2	2.0
3	3.0
4	4.0
5	5.0
6	6.0
7	7.0
8	8.0
9	9.0

9	
1	0.9
2	1.8
3	2.7
4	3.6
5	4.5
6	5.4
7	6.3
8	7.2
9	8.1

8	
1	0.8
2	1.6
3	2.4
4	3.2
5	4.0
6	4.8
7	5.6
8	6.4
9	7.2

Table 1 FIVE-PLACE COMMON LOGARITHMS OF NUMBERS (continued)

500-550

N.	0	1	2	3	4	5	6	7	8	9	Proportional parts																		
500	59 897	906	914	923	932	940	949	958	965	975	<table border="1"> <tr><td>0</td><td>0.9</td></tr> <tr><td>1</td><td>1.8</td></tr> <tr><td>2</td><td>2.7</td></tr> <tr><td>3</td><td>3.6</td></tr> <tr><td>4</td><td>4.5</td></tr> <tr><td>5</td><td>5.4</td></tr> <tr><td>6</td><td>6.3</td></tr> <tr><td>7</td><td>7.2</td></tr> <tr><td>8</td><td>8.1</td></tr> </table>	0	0.9	1	1.8	2	2.7	3	3.6	4	4.5	5	5.4	6	6.3	7	7.2	8	8.1
0	0.9																												
1	1.8																												
2	2.7																												
3	3.6																												
4	4.5																												
5	5.4																												
6	6.3																												
7	7.2																												
8	8.1																												
501	884	992	*001	*010	*018	*027	*036	*044	*053	*062																			
502	70 070	079	088	096	105	114	122	131	140	148																			
503	157	165	174	183	191	200	209	217	226	234																			
504	243	252	260	269	278	286	295	303	312	321																			
505	329	338	346	355	364	372	381	389	398	406																			
506	415	424	432	441	449	458	467	475	484	492																			
507	501	509	518	526	535	544	552	561	569	578																			
508	586	595	603	612	621	629	638	646	655	663																			
509	672	680	689	697	706	714	723	731	740	749																			
510	757	766	774	783	791	800	808	817	825	834																			
511	842	851	859	868	876	885	893	902	910	919																			
512	927	935	944	952	961	969	978	986	995	*003																			
513	71 012	020	029	037	046	054	063	071	079	088																			
514	096	105	113	122	130	139	147	155	164	172																			
515	181	189	198	206	214	223	231	240	248	257																			
516	265	273	282	290	299	307	315	324	332	341																			
517	349	357	366	374	383	391	399	408	416	425																			
518	433	441	450	458	466	475	483	492	500	508																			
519	517	525	533	542	550	558	567	575	584	592																			
520	600	609	617	625	634	642	650	659	667	675																			
521	684	692	700	709	717	725	734	742	750	759																			
522	767	775	784	792	800	809	817	825	834	842																			
523	850	858	867	875	883	892	900	908	917	925																			
524	933	941	950	958	966	975	983	991	999	*008																			
525	72 016	024	032	041	049	057	066	074	082	090																			
526	099	107	115	123	132	140	148	156	165	173																			
527	181	189	198	206	214	222	230	239	247	255																			
528	263	272	280	288	296	304	313	321	329	337																			
529	346	354	362	370	378	387	395	403	411	419																			
530	428	436	444	452	460	469	477	485	493	501																			
531	509	518	526	534	542	550	558	567	575	583																			
532	591	599	607	616	624	632	640	648	656	665																			
533	673	681	689	697	705	713	722	730	738	746																			
534	754	762	770	779	787	795	803	811	819	827																			
535	835	843	852	860	868	876	884	892	900	908																			
536	916	925	933	941	949	957	965	973	981	989																			
537	997	*006	*014	*022	*030	*038	*046	*054	*062	*070																			
538	73 078	086	094	102	111	119	127	135	143	151																			
539	159	167	175	183	191	199	207	215	223	231																			
540	239	247	255	263	272	280	288	296	304	312																			
541	320	328	336	344	352	360	368	376	384	392																			
542	400	408	416	424	432	440	448	456	464	472																			
543	480	488	496	504	512	520	528	536	544	552																			
544	560	568	576	584	592	600	608	616	624	632																			
545	640	648	656	664	672	679	687	695	703	711																			
546	719	727	735	743	751	759	767	775	783	791																			
547	799	807	815	823	830	838	846	854	862	870																			
548	878	886	894	902	910	918	926	933	941	949																			
549	957	965	973	981	989	997	*005	*013	*020	*028																			
550	74 036	044	052	060	068	076	084	092	099	107																			
N.	0	1	2	3	4	5	6	7	8	9	Proportional parts																		

0	0.9
1	1.8
2	2.7
3	3.6
4	4.5
5	5.4
6	6.3
7	7.2
8	8.1

8	0.8
1	1.6
2	2.4
3	3.2
4	4.0
5	4.8
6	5.6
7	6.4
8	7.2

7	0.7
1	1.4
2	2.1
3	2.8
4	3.5
5	4.2
6	4.9
7	5.6
8	6.3

Table 1 FIVE-PLACE COMMON LOGARITHMS OF NUMBERS (continued)

550-600

N.	0	1	2	3	4	5	6	7	8	9	Proportional parts
550	74 036	044	052	060	068	076	084	092	099	107	
551	115	123	131	139	147	155	162	170	178	186	
552	194	202	210	218	226	233	241	249	257	265	
553	273	280	288	296	304	312	320	327	335	343	
554	351	359	367	374	382	390	398	406	414	421	
556	429	437	445	453	461	468	476	484	492	500	
556	507	515	523	531	539	547	554	562	570	578	
557	586	593	601	609	617	624	632	640	648	656	
558	663	671	679	687	695	702	710	718	726	733	
559	741	749	757	764	772	780	788	796	803	811	
560	819	827	834	842	850	859	865	873	881	889	
561	896	904	912	920	927	935	943	950	958	966	
562	974	981	989	997	*005	*012	*020	*028	*035	*043	
563	75 051	059	066	074	082	089	097	105	113	120	8
564	128	136	143	151	159	168	174	182	189	197	1 0.6
565	205	213	220	228	236	243	251	259	266	274	2 1.6
566	282	289	297	305	312	320	328	335	343	351	3 2.4
567	358	366	374	381	389	397	404	412	420	427	4 3.2
568	435	442	450	458	465	473	481	488	496	504	5 4.0
569	511	519	526	534	542	549	557	565	572	580	6 5.6
570	587	595	603	610	618	626	633	641	648	656	7 6.4
571	664	671	679	686	694	702	709	717	724	732	8 7.2
572	740	747	755	762	770	776	785	793	800	808	9 8.0
573	815	823	831	838	846	853	861	868	876	884	
574	891	899	906	914	921	929	937	944	952	959	
575	967	974	982	989	997	*006	*012	*020	*027	*035	
576	76 042	050	057	065	072	080	087	095	103	110	
577	118	125	133	140	148	155	163	170	178	185	
578	193	200	208	215	223	230	238	245	253	260	
579	268	275	283	290	298	305	313	320	328	335	
580	343	350	358	365	373	380	388	395	403	410	
581	418	425	433	440	448	455	462	470	477	485	
582	492	500	507	515	522	530	537	545	552	559	
583	567	574	582	589	597	604	612	619	626	634	
584	641	649	656	664	671	678	686	693	701	708	
585	716	723	730	738	745	753	760	768	775	782	
586	790	797	805	812	819	827	834	842	849	856	
587	864	871	879	886	893	901	908	916	923	930	
588	938	945	953	960	967	975	982	989	997	*004	
589	77 012	019	026	034	041	048	055	063	070	078	
590	085	093	100	107	115	122	129	137	144	151	
591	159	166	173	181	188	195	203	210	217	225	
592	232	240	247	254	262	268	276	283	291	298	
593	305	313	320	327	335	342	349	357	364	371	
594	379	386	393	401	408	415	422	430	437	444	
595	452	459	466	474	481	486	495	503	510	517	
596	525	532	539	546	554	561	568	576	583	590	
597	597	605	612	619	627	634	641	648	656	663	
598	670	677	685	692	699	706	714	721	728	735	
599	743	750	757	764	772	779	786	793	801	808	
600	815	822	830	837	844	851	859	866	873	880	
N.	0	1	2	3	4	5	6	7	8	9	Proportional parts

.74 036-.77 880

Table I FIVE-PLACE COMMON LOGARITHMS OF NUMBERS (continued)

600-650

N.	0	1	2	3	4	5	6	7	8	9	Proportional parts	
600	77	815	822	830	837	844	851	859	866	873	880	
601		887	895	902	909	916	924	931	938	945	952	
602		960	967	974	981	988	996	*003	*010	*017	*025	
603	78	032	039	046	053	061	068	075	082	089	097	
604		104	111	118	126	132	140	147	154	161	168	
605		176	183	190	197	204	211	219	226	233	240	
606		247	254	262	269	276	283	290	297	305	312	
607		319	326	333	340	347	355	362	369	376	383	
608		390	398	405	412	419	426	433	440	447	455	
609		462	469	476	483	490	497	504	512	519	526	
610		532	540	547	554	561	569	576	583	590	597	
611		604	611	618	625	633	640	647	654	661	668	
612		675	682	689	696	704	711	718	725	732	739	
613		746	753	760	767	774	781	789	796	803	810	
614		817	824	831	838	845	852	859	866	873	880	
615		888	895	902	909	916	923	930	937	944	951	
616		958	965	972	979	986	993	*000	*007	*014	*021	
617	79	029	036	043	050	057	064	071	078	085	092	
618		099	106	113	120	127	134	141	148	155	162	
619		169	176	183	190	197	204	211	218	225	232	
620		239	246	253	260	267	274	281	288	295	302	
621		309	316	323	330	337	344	351	358	365	372	
622		379	386	393	400	407	414	421	428	435	442	
623		449	456	463	470	477	484	491	498	505	511	
624		518	525	532	539	546	553	560	567	574	581	
625		588	595	602	609	616	623	630	637	644	650	
626		657	664	671	678	685	692	699	706	713	720	
627		727	734	741	748	754	761	768	775	782	789	
628		796	803	810	817	824	831	837	844	851	858	
629		865	872	879	886	893	900	906	913	920	927	
630		934	941	948	955	962	969	975	982	989	995	
631	80	003	010	017	024	030	037	044	051	058	065	
632		072	079	086	092	099	106	113	120	127	134	
633		140	147	154	161	168	175	182	188	195	202	
634		209	216	223	229	236	243	250	257	264	271	
635		277	284	291	298	305	312	318	325	332	339	
636		346	353	359	366	373	380	387	393	400	407	
637		414	421	428	434	441	448	455	462	468	475	
638		482	489	496	502	509	516	523	530	536	543	
639		550	557	564	570	577	584	591	598	604	611	
640		618	625	632	638	645	652	659	665	672	679	
641		686	692	699	706	713	720	726	733	740	747	
642		754	760	767	774	781	787	794	801	806	814	
643		821	828	835	841	848	855	862	868	875	882	
644		899	895	902	909	916	922	929	936	943	949	
645		956	963	969	976	983	990	996	*003	*010	*017	
646	81	023	030	037	043	050	057	064	070	077	084	
647		090	097	104	111	117	124	131	137	144	151	
648		158	164	171	178	184	191	198	204	211	218	
649		224	231	236	245	251	258	265	271	278	285	
650		291	298	305	311	318	325	331	338	345	351	
N.	0	1	2	3	4	5	6	7	8	9	Proportional parts	

8	
1	0.8
2	1.6
3	2.4
4	3.2
5	4.0
6	4.8
7	5.6
8	6.4
9	7.2

7	
1	0.7
2	1.4
3	2.1
4	2.8
5	3.5
6	4.2
7	4.9
8	5.6
9	6.3

6	
1	0.6
2	1.2
3	1.8
4	2.4
5	3.0
6	3.6
7	4.2
8	4.8
9	5.4

Table 1 FIVE-PLACE COMMON LOGARITHMS OF NUMBERS (continued)

650-700

N.	0	1	2	3	4	5	6	7	8	9	Proportional parts																				
650	81 291	298	305	311	318	325	331	338	345	351	<table border="1"> <tr><td>7</td><td>0.7</td></tr> <tr><td>1</td><td>1.4</td></tr> <tr><td>2</td><td>2.1</td></tr> <tr><td>3</td><td>2.8</td></tr> <tr><td>4</td><td>3.5</td></tr> <tr><td>5</td><td>4.2</td></tr> <tr><td>6</td><td>4.9</td></tr> <tr><td>7</td><td>5.6</td></tr> <tr><td>8</td><td>6.3</td></tr> <tr><td>9</td><td></td></tr> </table>	7	0.7	1	1.4	2	2.1	3	2.8	4	3.5	5	4.2	6	4.9	7	5.6	8	6.3	9	
7	0.7																														
1	1.4																														
2	2.1																														
3	2.8																														
4	3.5																														
5	4.2																														
6	4.9																														
7	5.6																														
8	6.3																														
9																															
651		298	305	311	318	325	331	338	345	351																					
652		425	431	438	445	451	458	465	471	478																					
653		491	498	505	511	518	525	531	538	544																					
654		558	564	571	578	584	591	598	604	611																					
655		624	631	637	644	651	657	664	671	677																					
656		690	697	704	710	717	723	730	737	743																					
657		757	763	770	776	783	790	796	803	809																					
658		823	829	836	842	849	856	862	869	875																					
659		889	895	902	908	915	921	928	935	941																					
660		954	961	968	974	981	987	994	*000	*007																					
661	82 020	027	033	040	046	053	060	066	073	079																					
662		088	092	099	105	112	119	125	132	138																					
663		151	158	164	171	178	184	191	197	204																					
664		217	223	230	236	243	249	255	262	268																					
665		282	286	290	297	300	305	311	317	321																					
666		347	354	360	367	373	380	387	393	400																					
667		413	419	426	432	438	445	452	458	465																					
668		478	484	491	497	504	510	517	523	530																					
669		543	549	556	562	569	575	582	588	595																					
670		607	614	620	627	633	640	646	653	659																					
671		672	679	685	692	698	705	711	718	724																					
672		737	743	750	756	763	769	776	782	789																					
673		802	808	814	821	827	834	840	847	853																					
674		866	872	879	885	892	898	905	911	918																					
675		930	937	943	950	956	963	969	975	982																					
676		995	*001	*008	*014	*020	*027	*033	*040	*046																					
677	83 059	065	072	078	085	091	097	104	110	117																					
678		123	129	136	142	149	155	161	168	174																					
679		187	193	200	206	213	219	225	232	238																					
680		251	257	264	270	276	283	289	296	302																					
681		315	321	327	334	340	347	353	359	366																					
682		378	385	391	398	404	410	417	423	429																					
683		442	448	455	461	467	474	480	487	493																					
684		505	512	518	525	531	537	544	550	556																					
685		569	575	582	588	594	601	607	613	620																					
686		632	639	645	651	658	664	670	677	683																					
687		696	702	708	715	721	727	734	740	746																					
688		759	765	771	778	784	790	797	803	809																					
689		822	828	835	841	847	853	860	866	872																					
690		885	891	897	904	910	916	923	929	935																					
691		948	954	960	967	973	979	985	992	998																					
692	84 011	017	023	029	036	042	048	055	061	067																					
693		073	080	086	092	098	105	111	117	123																					
694		136	142	148	155	161	167	173	180	186																					
695		198	205	211	217	223	230	236	242	248																					
696		261	267	273	280	286	292	298	305	311																					
697		323	330	336	342	348	354	361	367	373																					
698		386	392	398	404	410	417	423	429	435																					
699		448	454	460	466	473	479	485	491	497																					
700		510	516	522	528	535	541	547	553	559																					
N.	0	1	2	3	4	5	6	7	8	9	Proportional parts																				

700-750

N.	0	1	2	3	4	5	6	7	8	9	Proportional parts																				
700	84 510	516	522	528	535	541	547	553	559	566																					
701	572	578	584	590	597	603	609	616	621	628																					
702	634	640	646	652	658	665	671	677	683	689																					
703	696	702	708	714	720	726	733	739	745	751																					
704	757	763	770	776	782	788	794	800	807	813	<table border="1"> <tr><td>7</td><td></td></tr> <tr><td>1</td><td>0.7</td></tr> <tr><td>2</td><td>1.4</td></tr> <tr><td>3</td><td>2.1</td></tr> <tr><td>4</td><td>2.8</td></tr> <tr><td>5</td><td>3.5</td></tr> <tr><td>6</td><td>4.2</td></tr> <tr><td>7</td><td>4.9</td></tr> <tr><td>8</td><td>5.6</td></tr> <tr><td>9</td><td>6.3</td></tr> </table>	7		1	0.7	2	1.4	3	2.1	4	2.8	5	3.5	6	4.2	7	4.9	8	5.6	9	6.3
7																															
1	0.7																														
2	1.4																														
3	2.1																														
4	2.8																														
5	3.5																														
6	4.2																														
7	4.9																														
8	5.6																														
9	6.3																														
705	819	825	831	837	844	850	856	862	868	874																					
706	880	887	893	899	905	911	917	924	930	936																					
707	942	948	954	960	967	973	979	985	991	997																					
708	95 003	009	016	022	028	034	040	046	052	058																					
709	065	071	077	083	089	095	101	107	114	120																					
710	126	132	138	144	150	156	163	169	175	181																					
711	187	193	199	205	211	217	224	230	236	242																					
712	248	254	260	266	272	278	285	291	297	303																					
713	309	315	321	327	333	339	345	352	358	364																					
714	370	376	382	388	394	400	406	412	418	425																					
715	431	437	443	449	455	461	467	473	479	485																					
716	491	497	503	509	515	522	528	534	540	546																					
717	552	558	564	570	576	582	588	594	600	606																					
718	612	618	625	631	637	643	649	655	661	667																					
719	673	679	685	691	697	703	709	715	721	727																					
720	733	739	745	751	757	763	769	775	781	788																					
721	794	800	806	812	818	824	830	836	842	848																					
722	854	860	866	872	878	884	890	896	902	908																					
723	914	920	926	932	938	944	950	956	962	968																					
724	974	980	986	992	998	*004	*010	*016	*022	*028																					
725	88 034	040	046	052	058	064	070	076	082	088																					
726	094	100	106	112	118	124	130	136	141	147																					
727	153	159	165	171	177	183	189	195	201	207																					
728	213	219	225	231	237	243	249	255	261	267																					
729	273	278	285	291	297	303	308	314	320	326																					
730	332	338	344	350	356	362	368	374	380	386																					
731	392	398	404	410	415	421	427	433	439	445																					
732	451	457	463	469	475	481	487	493	499	504																					
733	510	516	522	528	534	540	546	552	558	564																					
734	570	576	581	587	593	599	605	611	617	623																					
735	629	635	641	646	652	658	664	670	676	682																					
736	688	694	700	705	711	717	723	729	735	741																					
737	747	753	759	764	770	776	782	788	794	800																					
738	806	812	817	823	829	835	841	847	853	859																					
739	864	870	876	882	888	894	900	906	911	917																					
740	928	929	935	941	947	953	958	964	970	976																					
741	982	988	994	999	*005	*011	*017	*023	*029	*035																					
742	87 040	046	052	058	064	070	075	081	087	093																					
743	099	105	111	116	122	128	134	140	146	151																					
744	157	163	169	175	181	186	192	198	204	210																					
745	216	221	227	233	239	245	251	256	262	268																					
746	274	280	286	291	297	303	309	315	320	326																					
747	332	338	344	349	355	361	367	373	379	384																					
748	390	396	402	408	413	419	425	431	437	442																					
749	448	454	460	466	471	477	483	489	495	500																					
750	506	512	518	523	529	535	541	547	552	558																					

.84 510-.87 558

Table 1 FIVE-PLACE COMMON LOGARITHMS OF NUMBERS (continued)

750-800

N.	0	1	2	3	4	5	6	7	8	9	Proportional parts																				
750	87 506	512	518	523	529	535	541	547	552	558	<table border="1"> <tr><td colspan="2">6</td></tr> <tr><td>1</td><td>0.6</td></tr> <tr><td>2</td><td>1.2</td></tr> <tr><td>3</td><td>1.8</td></tr> <tr><td>4</td><td>2.4</td></tr> <tr><td>5</td><td>3.0</td></tr> <tr><td>6</td><td>3.6</td></tr> <tr><td>7</td><td>4.2</td></tr> <tr><td>8</td><td>4.8</td></tr> <tr><td>9</td><td>5.4</td></tr> </table>	6		1	0.6	2	1.2	3	1.8	4	2.4	5	3.0	6	3.6	7	4.2	8	4.8	9	5.4
6																															
1	0.6																														
2	1.2																														
3	1.8																														
4	2.4																														
5	3.0																														
6	3.6																														
7	4.2																														
8	4.8																														
9	5.4																														
751	564	570	576	581	587	593	599	604	610	616																					
752	622	628	633	639	645	651	656	662	668	674																					
753	679	685	691	697	703	708	714	720	726	731																					
754	737	743	749	754	760	766	772	777	783	789																					
755	795	800	806	812	818	823	829	835	841	846																					
756	852	858	864	869	875	881	887	892	898	904																					
757	910	915	921	927	933	938	944	950	955	961																					
758	967	973	978	984	990	996	*001	*007	*013	*018																					
759	88 024	030	036	041	047	053	059	064	070	076																					
760	081	087	093	098	104	110	116	121	127	133																					
761	138	144	150	156	161	167	173	178	184	190																					
762	195	201	207	213	218	224	230	235	241	247																					
763	252	258	264	270	275	281	287	292	298	304																					
764	309	315	321	326	332	338	343	349	355	360																					
765	366	372	377	383	389	395	400	406	412	417																					
766	423	429	434	440	446	451	457	463	468	474																					
767	480	485	491	497	502	508	513	519	525	530																					
768	536	542	547	553	559	564	570	576	581	587																					
769	593	598	604	610	616	621	627	632	638	643																					
770	649	655	660	666	672	677	683	689	694	700																					
771	705	711	717	722	728	734	739	745	750	756																					
772	762	767	773	779	784	790	795	801	807	812																					
773	818	824	829	835	840	846	852	857	863	868																					
774	874	880	885	891	897	902	908	913	919	925																					
775	930	936	941	947	953	958	964	969	975	981																					
776	986	992	997	*003	*009	*014	*020	*025	*031	*037																					
777	89 042	048	053	059	064	070	076	081	087	092																					
778	098	104	109	115	120	126	131	137	143	148																					
779	154	159	165	170	176	182	187	193	198	204																					
780	209	215	221	226	232	237	243	248	254	260																					
781	265	271	276	282	287	293	298	304	310	316																					
782	321	326	332	337	343	348	354	360	365	371																					
783	376	382	387	393	398	404	409	415	421	426																					
784	432	437	443	448	454	459	465	470	476	481																					
785	487	492	498	504	509	515	520	526	531	537																					
786	542	548	553	559	564	570	575	581	586	592																					
787	597	603	609	614	620	625	631	636	642	647																					
788	653	658	664	669	675	680	686	691	697	702																					
789	708	713	719	724	730	735	741	746	752	757																					
790	763	768	774	779	785	790	796	801	807	812																					
791	818	823	829	834	840	845	851	856	862	867																					
792	873	878	883	889	894	900	905	911	916	922																					
793	927	933	938	944	949	955	960	966	971	977																					
794	982	988	993	998	*004	*009	*015	*020	*026	*031																					
795	90 037	042	048	053	059	064	069	075	080	086																					
796	081	087	092	098	103	109	114	120	125	130																					
797	136	141	147	152	158	163	169	174	179	185																					
798	190	196	201	207	212	217	223	228	233	239																					
799	244	250	255	260	266	271	276	282	287	293																					
800	309	314	320	325	331	336	342	347	352	358																					
N.	0	1	2	3	4	5	6	7	8	9	Proportional parts																				

Table I FIVE-PLACE COMMON LOGARITHMS OF NUMBERS (continued)

800-850

N.	0	1	2	3	4	5	6	7	8	9	Proportional parts
800	90	309	314	320	325	331	336	342	347	352	358
801		363	369	374	380	385	390	396	401	407	412
802		417	423	428	434	439	445	450	455	461	466
803		472	477	482	488	493	499	504	509	515	520
804		525	531	536	542	547	553	558	563	569	574
805		580	585	590	596	601	607	612	617	623	628
806		634	639	644	650	655	660	666	671	677	682
807		687	693	698	703	709	714	720	725	730	735
808		741	747	752	757	763	768	773	779	784	789
809		795	800	806	811	816	822	827	832	838	843
810		849	854	859	865	870	875	881	886	891	897
811		902	907	913	918	924	929	934	940	945	950
812		956	961	966	972	977	982	988	993	998	*004
813	91	009	014	020	025	030	036	041	046	052	057
814		062	068	073	078	084	089	094	100	105	110
815		115	121	126	132	137	142	148	153	158	164
816		169	174	180	185	190	196	201	206	212	217
817		222	228	233	238	243	249	254	259	265	270
818		275	281	286	291	297	302	307	312	318	323
819		328	334	339	344	350	355	360	365	371	376
820		381	387	392	397	403	408	413	418	424	429
821		434	440	445	450	456	461	466	471	477	482
822		487	492	498	503	508	514	519	524	529	535
823		540	545	551	556	561	566	572	577	582	587
824		593	598	603	609	614	619	624	630	636	640
825		645	651	656	661	666	672	677	682	687	693
826		698	703	709	714	719	724	730	735	740	745
827		751	756	761	766	772	777	782	787	793	798
828		803	808	814	819	824	829	834	840	845	850
829		855	861	866	871	876	882	887	892	897	903
830		908	913	918	924	929	934	939	944	950	955
831		960	965	971	976	981	986	991	997	*002	*007
832	92	012	018	023	028	033	038	044	049	054	059
833		065	070	076	080	085	091	096	101	106	111
834		117	122	127	132	137	143	148	153	158	163
835		169	174	179	184	189	195	200	205	210	215
836		221	226	231	236	241	247	252	257	262	267
837		273	278	283	288	293	298	304	309	314	319
838		324	330	335	340	345	350	355	361	366	371
839		376	381	387	392	397	402	407	412	418	423
840		428	433	438	443	449	454	459	464	469	474
841		480	485	490	495	500	505	511	516	521	526
842		531	535	542	547	552	557	562	567	572	578
843		583	588	593	598	603	609	614	619	624	629
844		634	639	645	650	655	660	665	670	675	681
845		686	691	696	701	706	711	716	722	727	732
846		737	742	747	752	758	763	768	773	778	783
847		788	793	799	804	809	814	819	824	829	834
848		840	845	850	855	860	865	870	875	881	886
849		891	896	901	906	911	916	921	927	932	937
850		942	947	952	957	962	967	973	978	983	988
N.	0	1	2	3	4	5	6	7	8	9	Proportional parts

Table I FIVE-PLACE COMMON LOGARITHMS OF NUMBERS (continued)

850-900

N.	0	1	2	3	4	5	6	7	8	9	Proportional parts	
850	92	942	947	952	957	962	967	973	978	983	988	
851		993	999	*003	*008	*013	*018	*024	*029	*034	*038	
852	93	044	049	054	059	064	069	075	080	085	090	
853		095	100	105	110	115	120	125	131	136	141	
854		146	151	156	161	166	171	176	181	186	192	
855		197	202	207	212	217	222	227	232	237	242	
856		247	252	256	263	268	273	278	283	288	293	
857		298	303	308	313	318	323	328	334	339	344	
858		349	354	359	364	369	374	379	384	389	394	
859		399	404	409	414	420	426	430	435	440	445	
860		450	455	460	465	470	475	480	485	490	495	
861		500	505	510	515	520	526	531	536	541	546	
862		551	556	561	566	571	576	581	586	591	596	
863		601	606	611	616	621	626	631	636	641	646	
864		651	656	661	666	671	676	682	687	692	697	
865		702	707	712	717	722	727	732	737	742	747	
866		752	757	762	767	772	777	782	787	792	797	
867		802	807	812	817	822	827	832	837	842	847	
868		852	857	862	867	872	877	882	887	892	897	
869		902	907	912	917	922	927	932	937	942	947	
870		952	957	962	967	972	977	982	987	992	997	
871	94	002	007	012	017	022	027	032	037	042	047	
872		052	057	062	067	072	077	082	086	091	096	
873		101	105	111	116	121	126	131	136	141	146	
874		151	156	161	166	171	176	181	186	191	196	
875		201	206	211	216	221	226	231	236	240	245	
876		250	255	260	265	270	275	280	285	290	295	
877		300	305	310	315	320	325	330	335	340	345	
878		349	354	359	364	369	374	379	384	389	394	
879		399	404	409	414	419	424	429	433	438	443	
880		448	453	458	463	468	473	478	483	488	493	
881		498	503	507	512	517	522	527	532	537	542	
882		547	552	557	562	567	571	576	581	586	591	
883		596	601	606	611	616	621	626	630	635	640	
884		645	650	655	660	665	670	675	680	685	689	
885		694	699	704	709	714	719	724	729	734	738	
886		743	748	753	758	763	768	773	778	783	787	
887		792	797	802	807	812	817	822	827	832	836	
888		841	846	851	856	861	865	871	876	880	885	
889		890	895	900	905	910	915	919	924	929	934	
890		938	944	949	954	959	963	968	973	978	983	
891		988	993	998	*002	*007	*012	*017	*022	*027	*032	
892	98	036	041	046	051	056	061	066	071	075	080	
893		085	090	095	100	105	109	114	119	124	129	
894		134	139	143	148	153	158	163	168	173	177	
895		182	187	192	197	202	207	211	216	221	226	
896		231	236	240	245	250	255	260	265	270	274	
897		279	284	289	294	299	303	308	313	318	323	
898		328	332	337	342	347	352	357	361	366	371	
899		376	381	385	390	395	400	405	410	415	419	
900		424	429	434	439	444	448	453	458	463	468	
N.	0	1	2	3	4	5	6	7	8	9	Proportional parts	

1	0.8
2	1.2
3	1.6
4	2.4
5	3.0
6	3.6
7	4.2
8	4.8
9	5.4

1	0.5
2	1.0
3	1.5
4	2.0
5	2.5
6	3.0
7	3.5
8	4.0
9	4.5

1	0.4
2	0.8
3	1.2
4	1.6
5	2.0
6	2.4
7	2.8
8	3.2
9	3.6

Table I FIVE-PLACE COMMON LOGARITHMS OF NUMBERS (continued)

900-950

N.	0	1	2	3	4	5	6	7	8	9	Proportional parts	
900	95	424	429	434	439	444	448	453	458	463	468	
901		472	477	482	487	492	497	501	506	511	516	
902		521	525	530	535	540	545	550	554	559	564	
903		589	594	598	603	608	613	618	622	627	632	
904		617	622	626	631	636	641	646	650	655	660	
905		665	670	674	679	684	689	694	698	703	708	
906		713	718	722	727	732	737	742	746	751	756	
907		761	766	770	775	780	785	789	794	799	804	
908		809	813	818	823	828	832	837	842	847	852	
909		856	861	865	871	875	880	885	890	895	899	
910		904	909	914	918	923	928	933	938	942	947	
911		952	957	961	966	971	976	980	985	990	995	
912		999	*004	*009	*014	*019	*023	*028	*033	*038	*042	
913	86	047	052	057	061	066	071	076	080	085	090	
914		095	099	104	109	114	118	123	128	133	137	
915		142	147	152	156	161	166	171	175	180	185	
916		190	194	199	204	209	213	219	223	227	232	
917		237	242	246	251	256	261	265	270	275	280	
918		284	289	294	298	303	308	313	317	322	327	
919		332	336	341	346	350	355	360	365	369	374	
920		379	384	388	393	398	402	407	412	417	421	
921		426	431	435	440	445	450	454	459	464	468	
922		473	478	483	487	492	497	501	506	511	515	
923		520	525	530	534	539	544	548	553	558	562	
924		567	572	577	581	586	591	595	600	605	609	
925		614	619	624	628	633	638	642	647	652	656	
926		661	666	670	675	680	685	689	694	699	703	
927		708	713	717	722	727	731	736	741	745	750	
928		755	759	764	769	774	778	783	788	792	797	
929		802	806	811	816	820	825	830	834	839	844	
930		848	853	858	862	867	872	876	881	886	890	
931		895	900	904	909	914	918	923	928	932	937	
932		942	946	951	956	960	965	970	974	979	984	
933		988	993	997	*002	*007	*011	*016	*021	*025	*030	
934	97	035	039	044	049	053	058	063	067	072	077	
935		081	086	090	095	100	104	109	114	118	123	
936		128	132	137	142	146	151	156	160	165	169	
937		174	179	183	188	192	197	202	206	211	216	
938		220	225	230	234	239	243	248	253	257	262	
939		267	271	276	280	285	290	294	299	304	308	
940		313	317	322	327	331	336	340	345	350	354	
941		359	364	368	373	377	382	387	391	396	400	
942		405	410	414	419	424	428	433	437	442	447	
943		451	456	460	465	470	474	479	483	488	493	
944		497	502	506	511	516	520	525	529	534	539	
945		543	548	552	557	562	566	571	575	580	585	
946		589	594	598	603	607	612	617	621	626	630	
947		635	640	644	649	653	658	663	667	672	676	
948		681	685	690	695	699	704	708	713	717	722	
949		727	731	736	740	745	749	754	759	763	768	
950		772	777	782	786	791	795	800	804	809	813	
N.	0	1	2	3	4	5	6	7	8	9	Proportional parts	

5

1	0.5
2	1.0
3	1.5
4	2.0
5	2.5
6	3.0
7	3.5
8	4.0
9	4.5

4

1	0.4
2	0.8
3	1.2
4	1.6
5	2.0
6	2.4
7	2.8
8	3.2
9	3.6

Table I FIVE-PLACE COMMON LOGARITHMS OF NUMBERS (continued)

950-1000

N.	0	1	2	3	4	5	6	7	8	9	Proportional parts	
950	97	772	777	782	786	791	795	800	804	809	813	5 1 0.5 2 1.0 3 1.5 4 2.0 5 2.5 6 3.0 7 3.5 8 4.0 9 4.5
951		818	823	827	832	836	841	845	850	855	859	
952		864	868	873	877	882	886	891	896	900	905	
953		909	914	918	923	928	932	937	941	946	950	
954		955	959	964	968	973	978	982	987	991	996	
955	98	000	006	009	014	019	023	028	032	037	041	
956		046	050	055	059	064	068	073	078	082	087	
957		091	096	100	105	109	114	118	123	127	132	
958		137	141	146	150	155	159	164	168	173	177	
959		182	186	191	195	200	204	209	214	218	223	
960		227	232	236	241	245	250	254	259	263	268	
961		272	277	281	286	290	295	299	304	308	313	
962		318	322	327	331	336	340	345	349	354	358	
963		363	367	372	376	381	385	390	394	399	403	
964		408	412	417	421	426	430	435	439	444	448	
965		453	457	462	466	471	475	480	484	489	493	
966		496	502	507	511	516	520	525	529	534	538	
967		543	547	552	556	561	565	570	574	579	583	
968		588	592	597	601	605	610	614	619	623	628	
969		632	637	641	646	650	655	659	664	668	673	
970		677	682	686	691	695	700	704	709	713	717	
971		722	726	731	735	740	744	749	753	758	762	
972		767	771	776	780	784	789	793	798	802	807	
973		811	816	820	825	829	834	838	843	847	851	
974		856	860	865	869	874	878	883	887	892	896	
975		900	905	909	914	918	923	927	932	936	941	
976		945	949	954	958	963	967	972	976	981	985	
977		989	994	998	*003	*007	*012	*016	*021	*025	*029	
978	99	034	038	043	047	052	056	061	065	069	074	
979		078	083	087	092	096	100	105	109	114	118	
980		123	127	131	136	140	145	149	154	158	162	
981		167	171	176	180	185	189	193	198	202	207	
982		211	216	220	224	229	233	238	242	247	251	
983		255	260	264	269	273	277	282	286	291	295	
984		300	304	308	313	317	322	326	330	335	339	
985		344	348	352	357	361	366	370	374	379	383	
986		388	392	396	401	405	410	414	419	423	427	
987		432	436	441	445	449	454	458	463	467	471	
988		476	480	484	489	493	498	502	506	511	515	
989		520	524	528	533	537	542	546	550	555	559	
990		564	568	572	577	581	585	590	594	599	603	
991		607	612	616	621	625	629	634	638	642	647	
992		651	656	660	664	669	673	677	682	686	691	
993		695	699	704	708	712	717	721	726	730	734	
994		739	743	747	752	756	760	765	769	774	778	
995		782	787	791	795	800	804	808	813	817	822	
996		826	830	835	839	843	848	852	856	861	865	
997		870	874	878	883	887	891	896	900	904	909	
998		913	917	922	926	930	935	939	944	948	952	
999		957	961	965	970	974	978	983	987	991	996	
1000	00	009	004	009	013	017	022	026	030	035	039	
N.	0	1	2	3	4	5	6	7	8	9	Proportional parts	

Table 2 SEVEN-PLACE COMMON LOGARITHMS OF NUMBERS 209

1000-1050

N.	0	1	2	3	4	5	6	7	8	9	d.	
1000	000	0000	0434	0869	1303	1737	2171	2605	3039	3473	3907	434
1001		4341	4776	5208	5642	6076	6510	6943	7377	7810	8244	434
1002		8677	9111	9544	9977	*0411	*0844	*1277	*1710	*2143	*2576	433
1003	001	3009	3442	3875	4308	4741	5174	5607	6039	6472	6905	433
1004		7337	7770	8202	8635	9067	9499	9932	*0364	*0796	*1228	432
1005	002	1661	2093	2525	2957	3389	3821	4253	4685	5116	5548	432
1006		5980	6411	6843	7275	7706	8138	8569	9001	9432	9863	431
1007	003	0295	0726	1157	1588	2019	2451	2882	3313	3744	4174	431
1008		4605	5036	5467	5898	6328	6759	7190	7620	8051	8481	431
1009		8912	9342	9772	*0203	*0633	*1063	*1493	*1924	*2354	*2784	430
1010	004	3214	3644	4074	4504	4933	5363	5793	6223	6652	7082	430
1011		7512	7941	8371	8800	9229	9659	*0088	*0517	*0947	*1376	429
1012	005	1805	2234	2663	3092	3521	3950	4379	4808	5237	5666	429
1013		6094	6523	6952	7380	7809	8238	8666	9094	9523	9951	429
1014	006	0360	0808	1236	1664	2092	2521	2949	3377	3805	4233	428
1015		4660	5088	5516	5944	6372	6799	7227	7655	8082	8510	428
1016		8937	9365	9792	*0219	*0647	*1074	*1501	*1928	*2355	*2782	427
1017	007	3210	3637	4064	4490	4917	5344	5771	6198	6624	7051	427
1018		7478	7904	8331	8757	9184	9610	*0037	*0463	*0889	*1316	426
1019	008	1742	2168	2594	3020	3446	3872	4298	4724	5150	5576	426
1020		6002	6427	6853	7279	7704	8130	8556	8981	9407	9832	426
1021	009	0257	0683	1108	1533	1959	2384	2809	3234	3659	4084	425
1022		4609	4934	5359	5784	6208	6633	7058	7483	7907	8332	425
1023		8786	9181	9605	*0030	*0454	*0878	*1303	*1727	*2151	*2575	424
1024	010	3000	3424	3848	4272	4696	5120	5544	5967	6391	6815	424
1025		7239	7662	8086	8510	8933	9357	9780	*0204	*0627	*1050	424
1026	011	1474	1897	2320	2743	3166	3590	4013	4436	4859	5282	423
1027		5704	6127	6550	6973	7396	7818	8241	8664	9086	9509	423
1028		9931	*0354	*0776	*1198	*1621	*2043	*2465	*2887	*3310	*3732	422
1029	012	4154	4576	4998	5420	5842	6264	6685	7107	7529	7951	422
1030		8372	8794	9215	9637	*0059	*0480	*0901	*1323	*1744	*2165	422
1031	013	2587	3008	3429	3850	4271	4692	5113	5534	5955	6376	421
1032		6797	7218	7639	8059	8480	8901	9321	9742	*0162	*0583	421
1033	014	1003	1424	1844	2264	2685	3105	3525	3945	4365	4785	420
1034		5206	5626	6045	6465	6885	7305	7725	8144	8564	8984	420
1035		9403	9823	*0243	*0662	*1082	*1501	*1920	*2340	*2759	*3178	420
1036	015	3598	4017	4436	4855	5274	5693	6112	6531	6950	7369	419
1037		7798	8206	8625	9044	9462	9881	*0300	*0718	*1137	*1555	419
1038	016	1974	2392	2810	3229	3647	4065	4483	4901	5319	5737	418
1039		6156	6573	6991	7409	7827	8245	8663	9080	9498	9916	418
1040	017	0333	0751	1168	1585	2003	2421	2838	3256	3673	4090	417
1041		4507	4924	5342	5759	6176	6593	7010	7427	7844	8260	417
1042		8677	9094	9511	9927	*0344	*0761	*1177	*1594	*2010	*2427	417
1043	018	2843	3259	3676	4092	4508	4925	5341	5757	6173	6589	416
1044		7005	7421	7837	8253	8669	9084	9500	9916	*0332	*0747	416
1045	019	1163	1578	1994	2410	2825	3240	3656	4071	4486	4902	415
1046		5317	5732	6147	6562	6977	7392	7807	8222	8637	9052	415
1047		9467	9882	*0296	*0711	*1126	*1540	*1955	*2369	*2784	*3198	415
1048	020	3613	4027	4442	4856	5270	5684	6099	6513	6927	7341	414
1049		7756	8169	8583	8997	9411	9824	*0238	*0652	*1066	*1479	414
1050	021	1893	2307	2720	3134	3547	3961	4374	4787	5201	5614	413
N.	0	1	2	3	4	5	6	7	8	9	d.	

.000 0000-.021 5614

Table 2 SEVEN-PLACE COMMON LOGARITHMS OF NUMBERS (continued)

1050-1100

N.	0	1	2	3	4	5	6	7	8	9	d.	
1050	021	1893	2307	2720	3134	3547	3961	4374	4787	5201	5614	413
1051		5027	6440	6854	7267	7680	8093	8506	8919	9332	9745	413
1052	022	0157	0570	0983	1396	1808	2221	2634	3046	3459	3871	413
1053		4284	4696	5109	5521	5933	6345	6758	7170	7582	7994	412
1054		8406	8818	9230	9642	*0054	*0466	*0878	*1289	*1701	*2113	412
1055	023	2825	2936	3348	3759	4171	4582	4994	5405	5817	6228	411
1056		6639	7050	7462	7873	8284	8695	9106	9517	9928	*0339	411
1057	024	0750	1161	1572	1982	2393	2804	3214	3625	4036	4446	411
1058		4857	5267	5678	6088	6498	6909	7319	7729	8139	8549	410
1059		8960	9370	9780	*0190	*0600	*1010	*1419	*1829	*2239	*2649	410
1060	025	3059	3468	3878	4288	4697	5107	5516	5926	6335	6744	410
1061		7154	7563	7972	8382	8791	9200	9609	*0018	*0427	*0836	409
1062	026	1245	1654	2063	2472	2881	3289	3698	4107	4515	4924	409
1063		5333	5741	6150	6558	6967	7375	7783	8192	8600	9008	408
1064		9416	9824	*0233	*0641	*1049	*1457	*1865	*2273	*2680	*3088	408
1065	027	3496	3904	4312	4719	5127	5535	5942	6350	6757	7165	408
1066		7572	7979	8387	8794	9201	9608	*0016	*0423	*0830	*1237	407
1067	028	1644	2051	2458	2865	3272	3679	4086	4492	4899	5306	407
1068		5713	6119	6526	6932	7339	7746	8152	8558	8964	9371	406
1069		9777	*0183	*0590	*0996	*1402	*1808	*2214	*2620	*3026	*3432	406
1070	029	3838	4244	4649	5055	5461	5867	6272	6678	7084	7490	406
1071		7895	8300	8706	9111	9516	9922	*0327	*0732	*1136	*1543	405
1072	030	1848	2253	2758	3163	3568	3973	4378	4783	5188	5592	405
1073		5997	6402	6807	7211	7616	8020	8425	8830	9234	9638	405
1074	031	0043	0447	0851	1256	1660	2064	2468	2872	3277	3681	404
1075		4085	4489	4893	5296	5700	6104	6508	6912	7315	7719	404
1076		8123	8526	8930	9333	9737	*0140	*0544	*0947	*1350	*1754	403
1077	032	2167	2569	2963	3367	3770	4173	4576	4979	5382	5785	403
1078		6188	6590	6993	7396	7799	8201	8604	9007	9409	9812	403
1079	033	0214	0617	1019	1422	1824	2226	2629	3031	3433	3835	402
1080		4238	4640	5042	5444	5846	6248	6650	7052	7453	7855	402
1081		8257	8659	9060	9462	9864	*0266	*0667	*1068	*1470	*1871	402
1082	034	2273	2674	3075	3477	3878	4279	4680	5081	5482	5884	401
1083		6285	6686	7087	7487	7888	8289	8690	9091	9491	9892	401
1084	035	0293	0693	1094	1495	1895	2296	2696	3096	3497	3897	400
1085		4297	4698	5098	5498	5898	6298	6698	7098	7498	7898	400
1086		8298	8698	9098	9498	9898	*0297	*0697	*1097	*1496	*1896	400
1087	036	2295	2695	3094	3494	3893	4293	4692	5091	5491	5890	399
1088		6289	6688	7087	7486	7885	8284	8683	9082	9481	9880	399
1089	037	0270	0678	1076	1475	1874	2272	2671	3070	3468	3867	398
1090		4265	4663	5062	5460	5858	6257	6655	7053	7451	7849	398
1091		8248	8646	9044	9442	9839	*0287	*0685	*1083	*1481	*1879	398
1092	038	2226	2624	3022	3419	3817	4214	4612	5009	5407	5804	398
1093		6202	6599	6996	7393	7791	8188	8585	8982	9379	9776	397
1094	039	0173	0570	0967	1364	1761	2158	2554	2951	3348	3745	397
1095		4141	4538	4934	5331	5727	6124	6520	6917	7313	7709	397
1096		8106	8502	8898	9294	9690	*0088	*0482	*0878	*1274	*1670	396
1097	040	2066	2462	2858	3254	3650	4045	4441	4837	5232	5628	396
1098		6023	6419	6814	7210	7606	8001	8396	8791	9187	9582	395
1099		9977	*0372	*0767	*1162	*1557	*1952	*2347	*2742	*3137	*3532	395
1100	041	3927	4322	4716	5111	5506	5900	6295	6690	7084	7479	395
N.	0	1	2	3	4	5	6	7	8	9	d.	

Table 2 SEVEN-PLACE COMMON LOGARITHMS OF NUMBERS (continued)

1100-1150

N.	0	1	2	3	4	5	6	7	8	9	d.	
1100	041 3927	4322	4716	5111	5505	5900	6295	6690	7084	7479	385	
1101		7873	8268	8662	9056	9451	9845	*0239	*0633	*1028	*1422	384
1102	042	1816	2210	2604	2998	3392	3786	4180	4574	4968	5361	384
1103		5755	6149	6543	6936	7330	7723	8117	8510	8904	9297	384
1104		9691	*0084	*0477	*0871	*1264	*1657	*2050	*2444	*2837	*3230	383
1106	043	3823	4016	4409	4802	5195	5587	5980	6373	6766	7159	383
1106		7551	7944	8337	8729	9122	9514	9907	*0299	*0692	*1084	383
1107	044	1476	1869	2261	2653	3045	3437	3829	4222	4614	5005	382
1108		5398	5790	6181	6573	6965	7357	7749	8140	8532	8924	382
1109		9315	9707	*0099	*0490	*0882	*1273	*1664	*2056	*2447	*2839	382
1110	045	3230	3621	4012	4403	4795	5186	5577	5968	6359	6750	381
1111		7141	7531	7922	8313	8704	9095	9485	9876	*0267	*0657	381
1112	046	1048	1438	1829	2219	2610	3000	3391	3781	4171	4561	380
1113		4952	5342	5732	6122	6512	6902	7292	7682	8072	8462	380
1114		8852	9242	9632	*0021	*0411	*0801	*1190	*1580	*1970	*2359	380
1116	047	2749	3138	3528	3917	4306	4696	5085	5474	5864	6253	380
1116		6642	7031	7420	7809	8198	8587	8976	9365	9754	*0143	380
1117	048	0532	0921	1309	1698	2087	2475	2864	3253	3641	4030	380
1118		4418	4806	5195	5583	5972	6360	6748	7136	7525	7913	380
1119		8301	8689	9077	9465	9853	*0241	*0629	*1017	*1405	*1792	380
1120	049	2130	2568	2956	3343	3731	4119	4506	4894	5281	5669	380
1121		6056	6444	6831	7218	7606	7993	8380	8767	9154	9541	380
1122		9929	*0316	*0703	*1090	*1477	*1863	*2250	*2637	*3024	*3411	380
1123	050	3798	4184	4571	4958	5344	5731	6117	6504	6890	7277	380
1124		7653	8049	8436	8822	9208	9595	9981	*0367	*0753	*1139	380
1126	051	1625	1911	2297	2683	3059	3455	3841	4227	4612	4998	380
1126		5384	5770	6155	6541	6926	7312	7697	8083	8468	8854	380
1127		9239	9624	*0010	*0395	*0780	*1166	*1551	*1936	*2321	*2706	380
1128	052	3091	3476	3861	4246	4631	5016	5400	5785	6170	6555	380
1129		6939	7324	7709	8093	8478	8862	9247	9631	*0016	*0400	380
1130	053	0784	1169	1553	1937	2321	2706	3090	3474	3858	4242	380
1131		4626	5010	5394	5778	6162	6546	6929	7313	7697	8081	380
1132		8454	8848	9232	9615	9999	*0382	*0766	*1149	*1532	*1916	380
1133	054	2299	2682	3066	3449	3832	4215	4598	4981	5365	5748	380
1134		6131	6514	6896	7279	7662	8045	8428	8811	9193	9576	380
1135		9959	*0341	*0724	*1108	*1489	*1871	*2254	*2636	*3019	*3401	380
1136	055	3783	4166	4548	4930	5312	5694	6077	6459	6841	7223	380
1137		7608	7987	8369	8750	9132	9514	9896	*0278	*0659	*1041	380
1138	056	1423	1804	2185	2567	2949	3330	3712	4093	4475	4856	380
1139		5237	5619	6000	6381	6762	7143	7524	7906	8287	8668	380
1140		9049	9429	9810	*0191	*0572	*0953	*1334	*1714	*2095	*2476	380
1141	057	2858	3237	3618	3998	4379	4759	5140	5520	5900	6281	380
1142		5661	7041	7422	7802	8182	8562	8942	9322	9702	*0082	380
1143	058	0462	0842	1222	1602	1982	2362	2741	3121	3501	3881	380
1144		4260	4640	5019	5399	5778	6158	6537	6917	7296	7676	380
1145		8055	8434	8813	9193	9572	9951	*0330	*0709	*1088	*1467	379
1146	059	1846	2225	2604	2983	3362	3741	4119	4498	4877	5256	379
1147		5634	6013	6391	6770	7148	7527	7905	8284	8662	9041	379
1148		9419	9797	*0175	*0554	*0932	*1310	*1688	*2066	*2444	*2822	379
1149	060	3200	3578	3956	4334	4712	5090	5468	5846	6223	6601	379
1150		6978	7356	7734	8111	8489	8866	9244	9621	9999	*0376	378

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Table 2 SEVEN-PLACE COMMON LOGARITHMS OF NUMBERS (continued)

1150-1200

N.	0	1	2	3	4	5	6	7	8	9	d.	
1180	060	6978	7356	7734	8111	8489	8866	9244	9621	9999	*0376	378
1181	061	0753	1131	1508	1885	2262	2639	3017	3394	3771	4148	377
1182		4323	4802	5279	5656	6032	6409	6786	7163	7540	7916	377
1183		8293	8670	9046	9422	9799	*0176	*0552	*0928	*1305	*1682	377
1184	062	2053	2434	2811	3187	3563	3939	4316	4692	5068	5444	378
1185		5820	6196	6572	6948	7324	7699	8075	8451	8827	9203	378
1186		9576	9954	*0330	*0705	*1081	*1456	*1832	*2207	*2583	*2958	378
1187	063	3334	3709	4084	4460	4835	5210	5585	5960	6335	6711	378
1188		7086	7461	7836	8211	8585	8960	9335	9710	*0085	*0460	378
1189	064	0834	1209	1584	1958	2333	2708	3082	3457	3831	4205	378
1190		4580	4954	5329	5703	6077	6451	6825	7200	7574	7948	374
1191		8322	8696	9070	9444	9818	*0192	*0566	*0940	*1314	*1688	374
1192	065	2061	2435	2809	3182	3556	3930	4303	4677	5050	5424	374
1193		5797	6171	6544	6917	7291	7664	8037	8410	8784	9157	373
1194		9530	9903	*0276	*0649	*1022	*1395	*1768	*2141	*2514	*2886	373
1195	066	3259	3632	4005	4377	4750	5123	5495	5868	6241	6613	373
1196		6986	7358	7730	8103	8475	8847	9220	9592	9964	*0336	372
1197	067	0709	1081	1453	1825	2197	2569	2941	3313	3685	4057	372
1198		4423	4800	5172	5544	5915	6287	6659	7030	7402	7774	372
1199		8145	8517	8888	9259	9631	*0002	*0374	*0745	*1116	*1487	371
1190	068	1859	2230	2601	2972	3343	3714	4085	4456	4827	5198	371
1191		5569	5940	6311	6681	7052	7423	7794	8164	8535	8906	371
1192		9276	9647	*0017	*0388	*0758	*1129	*1499	*1869	*2240	*2610	370
1193	069	2980	3350	3721	4091	4461	4831	5201	5571	5941	6311	370
1194		6681	7051	7421	7791	8160	8530	8900	9270	9639	*0009	370
1195	070	0379	0749	1118	1487	1857	2226	2596	2965	3335	3704	369
1196		4073	4442	4812	5181	5550	5919	6288	6658	7027	7396	369
1197		7765	8134	8503	8871	9240	9609	9978	*0347	*0715	*1084	369
1198	071	1463	1822	2190	2559	2927	3296	3664	4033	4401	4770	369
1199		5138	5506	5875	6243	6611	6979	7346	7714	8084	8452	368
1190		8820	9188	9556	9924	*0292	*0660	*1028	*1396	*1763	*2131	368
1191	072	2499	2867	3234	3602	3970	4337	4705	5072	5440	5807	368
1192		6175	6542	6910	7277	7644	8011	8379	8746	9113	9480	367
1193		9847	*0215	*0582	*0949	*1316	*1683	*2050	*2418	*2783	*3150	367
1194	073	3517	3884	4251	4617	4984	5351	5717	6084	6450	6817	367
1195		7184	7550	7916	8283	8649	9016	9382	9748	*0114	*0481	366
1196	074	0847	1213	1579	1945	2311	2677	3043	3409	3775	4141	366
1197		4507	4873	5239	5605	5970	6336	6702	7068	7433	7799	366
1198		8164	8530	8895	9261	9626	9992	*0357	*0723	*1088	*1453	365
1199	075	1819	2184	2549	2914	3279	3644	4010	4375	4740	5105	365
1190		5470	5835	6199	6564	6929	7294	7659	8024	8388	8753	365
1191		9118	9482	9847	*0211	*0576	*0940	*1305	*1669	*2034	*2398	364
1192	076	2763	3127	3491	3855	4220	4584	4948	5312	5676	6040	364
1193		6404	6768	7132	7496	7860	8224	8588	8952	9316	9680	364
1194	077	0043	0407	0771	1134	1498	1862	2225	2589	2952	3316	364
1195		3679	4042	4406	4769	5133	5496	5859	6222	6585	6949	363
1196		7312	7675	8038	8401	8764	9127	9490	9853	*0216	*0579	363
1197	078	0942	1304	1667	2030	2393	2756	3118	3480	3843	4208	363
1198		4568	4931	5293	5656	6018	6380	6743	7105	7467	7830	362
1199		8192	8554	8916	9278	9640	*0003	*0365	*0727	*1089	*1451	362
1200	079	1812	2174	2536	2898	3260	3622	3983	4345	4707	5068	362
N.	0	1	2	3	4	5	6	7	8	9	d.	

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Table 3 COMMON LOGARITHMS OF PRIMES

2-997

N	Log N		N	Log N		N	Log N				
2	0.301	029	9957	269	2.429	752	2890	617	2.790	285	1640
3	.477	121	2547	271	.432	989	2909	619	.791	690	6490
6	.692	870	0043	277	.442	479	7691	631	.800	029	3692
7	.845	098	0400	281	.448	706	3199	641	.806	858	0295
11	1.041	392	6852	283	.451	786	4355	643	.808	210	9729
13	1.113	943	3523	293	2.466	897	6204	647	2.810	904	2807
17	.230	458	9214	307	.487	138	3755	653	.814	913	1813
19	.278	753	6010	311	.492	760	3890	659	.818	885	4146
23	.361	727	8360	313	.495	544	3375	661	.820	201	4695
29	.462	397	9979	317	.501	059	2622	673	.828	015	0642
31	1.491	351	6938	331	2.519	827	9938	677	2.830	589	6687
37	.568	201	7241	337	.527	629	9009	683	.834	420	7037
41	.612	783	8557	347	.540	329	4748	691	.839	478	0474
43	.633	468	4556	349	.542	825	4270	701	.845	719	0180
47	.672	697	8579	353	.547	774	7054	709	.850	646	2362
53	1.724	275	8696	359	2.555	094	4486	719	2.856	723	8904
59	.770	852	0116	367	.564	666	0643	727	.861	534	4109
61	.785	329	8390	373	.571	708	8318	733	.865	103	9746
67	.820	074	8027	379	.578	639	2100	739	.868	644	4384
71	.851	258	3487	383	.583	196	7740	743	.870	082	8138
73	1.863	322	8601	389	2.589	949	6013	751	2.875	639	9370
79	.397	627	0913	397	.598	790	5068	757	.879	095	8795
83	.919	078	0924	401	.603	144	3726	761	.881	384	6568
89	.949	390	0686	409	.611	723	3080	769	.885	926	3388
97	.986	771	7343	419	.622	214	0230	773	.888	179	4939
101	2.004	321	3736	421	2.624	282	0958	787	2.895	974	7324
103	.012	837	2247	431	.634	477	2792	797	.901	456	3214
107	.029	383	7777	433	.638	487	8984	809	.907	948	5216
109	.037	425	4979	439	.642	464	5202	811	.909	020	8642
113	.053	078	4435	443	.646	403	7262	821	.914	343	1671
127	2.103	803	7210	449	2.652	246	3410	823	2.915	399	8352
131	.117	271	2957	457	.659	916	2001	827	.917	505	5096
137	.136	720	5672	461	.663	700	9254	829	.918	554	6306
139	.143	014	8003	463	.665	680	9910	839	.923	761	9608
149	.173	188	2884	487	.689	318	8806	853	.930	948	0312
151	2.178	976	9473	479	2.680	335	5134	857	2.932	590	8219
157	.195	899	6524	487	.687	528	9612	859	.933	993	1638
163	.212	167	6044	491	.691	081	4921	863	.936	010	7957
167	.222	716	4711	499	.698	100	5456	877	.942	999	5934
173	.238	045	1031	503	.701	567	9851	881	.944	976	9084
179	2.252	858	0310	509	2.706	717	7823	883	2.945	960	7035
181	.257	578	5749	521	.716	837	7233	887	.947	923	0198
191	.281	033	3572	523	.718	501	6889	907	.957	607	2871
193	.285	557	3990	541	.733	197	2651	911	.958	518	3770
197	.294	466	2262	547	.737	987	3263	919	.963	315	5114
199	2.298	853	0764	557	2.745	855	1952	929	2.968	015	7140
211	.324	282	4583	563	.750	508	3949	937	.971	739	5909
223	.348	304	8530	569	.755	112	2064	941	.973	589	6234
227	.356	025	8572	571	.756	636	1082	947	.976	349	9790
229	.359	835	4823	577	.761	175	8132	953	.979	092	9606
233	2.367	355	9210	587	2.768	638	1012	967	2.985	426	4741
239	.378	387	9009	603	.773	054	6934	971	.987	219	2299
241	.382	017	0426	599	.777	426	8224	977	.988	894	5627
251	.399	673	7215	601	.778	874	4720	983	.992	553	5178
257	.409	933	1233	607	.783	188	6911	991	.996	073	6545
263	2.419	955	7485	613	2.787	460	4745	997	2.998	595	1583

Table 4* COMMON LOGARITHMS OF GAMMA FUNCTION $\Gamma(n)$

$$\Gamma(n) = \int_0^\infty x^{n-1} e^{-x} dx = \int_0^1 \left[\log_e \left(\frac{1}{x} \right) \right]^{n-1} dx$$

n	log ₁₀ Γ(n) + 10	n	log ₁₀ Γ(n) + 10	n	log ₁₀ Γ(n) + 10	n	log ₁₀ Γ(n) + 10	n	log ₁₀ Γ(n) + 10
1.01	9.9975	1.21	9.9617	1.41	9.9478	1.61	9.9517	1.81	9.9704
1.02	9.9951	1.22	9.9605	1.42	9.9476	1.62	9.9523	1.82	9.9717
1.03	9.9928	1.23	9.9594	1.43	9.9475	1.63	9.9529	1.83	9.9730
1.04	9.9905	1.24	9.9583	1.44	9.9473	1.64	9.9536	1.84	9.9743
1.05	9.9883	1.25	9.9573	1.45	9.9473	1.65	9.9543	1.85	9.9757
1.06	9.9862	1.26	9.9564	1.46	9.9472	1.66	9.9550	1.86	9.9771
1.07	9.9841	1.27	9.9554	1.47	9.9473	1.67	9.9558	1.87	9.9786
1.08	9.9821	1.28	9.9546	1.48	9.9473	1.68	9.9566	1.88	9.9800
1.09	9.9802	1.29	9.9538	1.49	9.9474	1.69	9.9575	1.89	9.9815
1.10	9.9783	1.30	9.9530	1.50	9.9475	1.70	9.9584	1.90	9.9831
1.11	9.9765	1.31	9.9523	1.51	9.9477	1.71	9.9593	1.91	9.9846
1.12	9.9748	1.32	9.9516	1.52	9.9479	1.72	9.9603	1.92	9.9862
1.13	9.9731	1.33	9.9510	1.53	9.9482	1.73	9.9613	1.93	9.9878
1.14	9.9715	1.34	9.9505	1.54	9.9485	1.74	9.9623	1.94	9.9895
1.15	9.9699	1.35	9.9500	1.55	9.9488	1.75	9.9633	1.95	9.9912
1.16	9.9684	1.36	9.9495	1.56	9.9492	1.76	9.9644	1.96	9.9929
1.17	9.9669	1.37	9.9491	1.57	9.9496	1.77	9.9656	1.97	9.9946
1.18	9.9655	1.38	9.9487	1.58	9.9501	1.78	9.9667	1.98	9.9964
1.19	9.9642	1.39	9.9483	1.59	9.9506	1.79	9.9679	1.99	9.9982
1.20	9.9629	1.40	9.9481	1.60	9.9511	1.80	9.9691	2.00	10.0000

$\Gamma(n + 1) = n \Gamma(n), n > 0. \quad \Gamma(2) = \Gamma(1) = 1.$

* See page 103.

Interpolation

Let $f(x)$ be an analytic function of x . If the values of $f(x)$ are given in a table for a set of values of x separated from one another consecutively by the constant small interval h , the differences between the successive values of the function as tabulated are called *first tabular differences*, the differences of these first differences, *second tabular differences*, etc. The first, second, and third tabular differences corresponding to $x = a$ and the tabulated value of $f(a)$ are

$$\begin{aligned} \Delta_1 &\equiv f(a + h) - f(a) \\ \Delta_2 &\equiv f(a + 2h) - 2f(a + h) + f(a) \\ \Delta_3 &\equiv f(a + 3h) - 3f(a + 2h) + 3f(a + h) - f(a) \end{aligned}$$

The value of $f(x)$ for $x = a + \delta$, where $\delta = kh, 0 < k < 1$, is

$$f(a + \delta) = f(a) + k \Delta_1 + \frac{k(k-1)}{2!} \Delta_2 + \frac{k(k-1)(k-2)}{3!} \Delta_3 + \dots$$

This is known as the *Gregory-Newton formula of interpolation* (see paragraph on interpolation of tables, page 181).

TRIGONOMETRIC FUNCTIONS



- TABLE 5. *Auxiliary Table of S and T for A in Minutes, 216*
- TABLE 6. *Common Logarithms of the Trigonometric Functions from 0° to 90°, 216*
- TABLE 7. *Natural Trigonometric Functions, 262*
- TABLE 8. *Natural Secants and Cosecants, 285*
- TABLE 9. *Minutes and Seconds to Decimal Parts of a Degree, 300*
- TABLE 10. *Natural Trigonometric Functions for Decimal Fractions of a Degree, 301*
- TABLE 11. *Common Logarithms of Trigonometric Functions in Radian Measure, 305*
- TABLE 12. *Degrees, Minutes, and Seconds to Radians, 306*
- TABLE 13. *Trigonometric Functions in Radian Measure, 307*
- TABLE 14. *Radians to Degrees, Minutes, and Seconds, 308*

Table 5 AUXILIARY TABLE OF *S* AND *T* FOR *A* IN 216 MINUTES (COMMON LOGARITHMS)

$S = \log \sin A - \log A'$ and $T = \log \tan A - \log A'$
 A' = number of minutes in A and $(90^\circ - A)'$ = number of minutes in $90^\circ - A$

<i>A'</i>	<i>S</i>	<i>A'</i>	<i>T</i>	<i>A'</i>	<i>T</i>
0 to 13	6.46373 - 10	0 to 26	6.46373 - 10	131 to 133	6.46394 - 10
14 to 42	72 - 10	27 to 39	74 - 10	134 to 136	95 - 10
43 to 58	71 - 10	40 to 48	75 - 10	137 to 139	96 - 10
59 to 71	6.46370 - 10	49 to 56	6.46376 - 10	140 to 142	6.46397 - 10
72 to 81	69 - 10	57 to 63	77 - 10	143 to 145	98 - 10
82 to 91	68 - 10	64 to 69	78 - 10	146 to 148	99 - 10
92 to 99	6.46367 - 10	70 to 74	6.46379 - 10	149 to 150	6.46400 - 10
100 to 107	66 - 10	75 to 80	80 - 10	151 to 153	01 - 10
108 to 115	65 - 10	81 to 85	81 - 10	154 to 156	02 - 10
116 to 121	6.46364 - 10	86 to 89	6.46382 - 10	157 to 158	6.46403 - 10
122 to 128	63 - 10	90 to 94	83 - 10	159 to 151	04 - 10
129 to 134	62 - 10	95 to 98	84 - 10	162 to 163	05 - 10
135 to 149	6.46361 - 10	99 to 102	6.46385 - 10	164 to 166	6.46406 - 10
141 to 148	60 - 10	103 to 106	86 - 10	167 to 168	07 - 10
147 to 151	59 - 10	107 to 110	87 - 10	169 to 171	08 - 10
152 to 157	6.46358 - 10	111 to 116	6.46388 - 10	172 to 173	6.46409 - 10
158 to 162	57 - 10	114 to 117	89 - 10	174 to 175	10 - 10
163 to 167	56 - 10	118 to 120	90 - 10	176 to 178	11 - 10
168 to 171	6.46355 - 10	121 to 124	6.46391 - 10	179 to 180	6.46412 - 10
172 to 176	54 - 10	125 to 127	92 - 10	181 to 182	13 - 10
177 to 181	53 - 10	128 to 130	93 - 10	183 to 184	14 - 10

For small angles, $\log \sin A = \log A' + S$ and $\log \tan A = \log A' + T$; for angles near 90° , $\log \cos A = \log (90^\circ - A)' + S$ and $\log \cot A = \log (90^\circ - A)' + T$.

Table 6 COMMON LOGARITHMS OF THE TRIGONOMETRIC FUNCTIONS FROM 0° TO 90°

At Intervals of One Minute to Five Decimal Places

For degrees indicated at the top of the page use the column headings at the top. For degrees indicated at the bottom of the page use the column headings at the bottom.

With degrees at the left of each block (at the top or bottom of the page) use the minute column at the left. With degrees at the right of each block (at the top or bottom of the page) use the minute column at the right.

In order to conserve space, the -10 portion of the characteristic of the logarithm is not printed in Table 6 but must be written down whenever such a logarithm is used.

Table 6 COMMON LOGARITHMS OF TRIGONOMETRIC FUNCTIONS (continued) 217

The -- 10 portion of the characteristic of the logarithm is not printed but must be written down whenever such a logarithm is used.

0° (180°)		(59°) 179°					
'	L Sin	d	L Tan	c d	L Ctn	L Cos	'
0						10.00 000	60
1	6.46 373	30103	6.46 373	30103	13.53 627	10.00 000	59
2	6.76 476	17609	6.76 476	17609	13.23 524	10.00 000	58
3	6.94 085	12494	6.94 085	12494	13.05 915	10.00 000	57
4	7.05 579	9691	7.05 579	9691	12.93 421	10.00 000	56
5	7.16 270	7318	7.16 270	7318	12.83 730	10.00 000	55
6	7.24 183	6494	7.24 183	6494	12.75 812	10.00 000	54
7	7.30 882	5800	7.30 882	5800	12.69 118	10.00 000	53
8	7.36 682	5115	7.36 682	5115	12.63 318	10.00 000	52
9	7.41 797	4576	7.41 797	4576	12.58 203	10.00 000	51
10	7.46 373	4139	7.46 373	4139	12.53 627	10.00 000	50
11	7.50 512	3779	7.50 512	3779	12.49 488	10.00 000	49
12	7.54 291	3476	7.54 291	3476	12.45 709	10.00 000	48
13	7.57 767	3218	7.57 767	3218	12.42 233	10.00 000	47
14	7.60 986	2997	7.60 986	2996	12.39 014	10.00 000	46
15	7.63 982	2802	7.63 982	2803	12.36 018	10.00 000	45
16	7.66 784	2633	7.66 785	2633	12.33 215	10.00 000	44
17	7.69 417	2483	7.69 418	2482	12.30 582	9.99 999	43
18	7.71 900	2346	7.71 900	2348	12.28 100	9.99 999	42
19	7.74 248	2227	7.74 248	2228	12.25 752	9.99 999	41
20	7.76 475	2119	7.76 476	2119	12.23 524	9.99 999	40
21	7.78 594	2021	7.78 595	2020	12.21 406	9.99 999	39
22	7.80 615	1930	7.80 615	1931	12.19 383	9.99 999	38
23	7.82 546	1848	7.82 546	1848	12.17 454	9.99 999	37
24	7.84 393	1773	7.84 394	1773	12.15 606	9.99 999	36
25	7.86 166	1704	7.86 167	1704	12.13 833	9.99 999	35
26	7.87 870	1639	7.87 871	1639	12.12 129	9.99 999	34
27	7.89 509	1579	7.89 510	1579	12.10 490	9.99 999	33
28	7.91 088	1524	7.91 088	1524	12.08 911	9.99 999	32
29	7.92 612	1472	7.92 613	1473	12.07 387	9.99 998	31
30	7.94 084	1424	7.94 086	1424	12.05 914	9.99 998	30
31	7.95 508	1379	7.95 510	1376	12.04 490	9.99 998	29
32	7.96 887	1336	7.96 888	1336	12.03 111	9.99 998	28
33	7.98 223	1297	7.98 225	1297	12.01 775	9.99 998	27
34	7.99 526	1259	7.99 522	1259	12.00 476	9.99 998	26
35	8.00 779	1223	8.00 781	1223	11.99 216	9.99 998	25
36	8.02 002	1190	8.02 004	1190	11.97 996	9.99 998	24
37	8.03 192	1158	8.03 194	1158	11.96 806	9.99 997	23
38	8.04 356	1128	8.04 353	1128	11.95 647	9.99 997	22
39	8.05 479	1100	8.05 481	1100	11.94 519	9.99 997	21
40	8.06 578	1072	8.06 581	1072	11.93 419	9.99 997	20
41	8.07 650	1046	8.07 653	1047	11.92 347	9.99 997	19
42	8.08 696	1022	8.08 700	1022	11.91 300	9.99 997	18
43	8.09 718	999	8.09 722	998	11.90 278	9.99 997	17
44	8.10 717	976	8.10 720	976	11.89 280	9.99 996	16
45	8.11 693	956	8.11 696	955	11.88 304	9.99 996	15
46	8.12 647	934	8.12 651	934	11.87 349	9.99 996	14
47	8.13 583	914	8.13 585	915	11.86 415	9.99 996	13
48	8.14 495	896	8.14 500	895	11.85 500	9.99 996	12
49	8.15 391	877	8.15 395	878	11.84 605	9.99 996	11
50	8.16 268	860	8.16 273	860	11.83 727	9.99 995	10
51	8.17 128	843	8.17 133	843	11.82 867	9.99 995	9
52	8.17 971	827	8.17 976	826	11.82 024	9.99 995	8
53	8.18 798	812	8.18 804	812	11.81 196	9.99 995	7
54	8.19 610	797	8.19 616	797	11.80 384	9.99 995	6
55	8.20 407	782	8.20 413	782	11.79 587	9.99 994	5
56	8.21 189	769	8.21 195	769	11.78 805	9.99 994	4
57	8.21 958	755	8.21 964	756	11.78 036	9.99 994	3
58	8.22 713	743	8.22 720	742	11.77 280	9.99 994	2
59	8.23 456	730	8.23 462	730	11.76 538	9.99 994	1
60	8.24 186		8.24 192		11.75 808	9.99 993	0
'	L Cos	d	L Ctn	c d	L Tan	L Sin	'

90° (270°)

(269°) 89°

For more accurate values of L sin and L tan for interpolated values of angles less than 3° (or L cos or L ctn of angles greater than 87°) use Table 5, page 216.

Table 6 COMMON LOGARITHMS OF TRIGONOMETRIC FUNCTIONS (continued) 218

The —10 portion of the characteristic of the logarithm is not printed but must be written down whenever such a logarithm is used.

1° (181°) (358°) 178°

°	L Sin	d	L Tan	c d	L Ctn	L Cos	'
0	8.24 186	717	8.24 192	718	11.75 808	9.99 993	60
1	8.24 503	706	8.24 910	706	11.75 090	9.99 993	59
2	8.25 609	695	8.25 616	695	11.74 384	9.99 993	58
3	8.26 304	684	8.26 312	684	11.73 688	9.99 993	57
4	8.26 988	673	8.26 996	673	11.73 004	9.99 992	56
5	8.27 661	663	8.27 669	663	11.72 331	9.99 992	55
6	8.28 324	653	8.28 332	654	11.71 668	9.99 992	54
7	8.28 977	644	8.28 986	643	11.71 014	9.99 992	53
8	8.29 621	634	8.29 629	634	11.70 371	9.99 992	52
9	8.30 255	624	8.30 263	623	11.69 737	9.99 991	51
10	8.30 879	616	8.30 888	617	11.69 112	9.99 991	50
11	8.31 495	608	8.31 505	607	11.68 495	9.99 991	49
12	8.32 103	599	8.32 112	599	11.67 888	9.99 990	48
13	8.32 702	590	8.32 711	591	11.67 289	9.99 990	47
14	8.33 292	581	8.33 302	584	11.66 698	9.99 990	46
15	8.33 876	573	8.33 886	573	11.66 114	9.99 989	45
16	8.34 450	565	8.34 461	566	11.65 539	9.99 989	44
17	8.35 018	556	8.35 029	561	11.64 971	9.99 989	43
18	8.35 578	548	8.35 590	553	11.64 410	9.99 989	42
19	8.36 131	547	8.36 143	546	11.63 857	9.99 989	41
20	8.36 678	539	8.36 689	540	11.63 311	9.99 988	40
21	8.37 217	531	8.37 229	533	11.62 771	9.99 988	39
22	8.37 750	526	8.37 762	527	11.62 236	9.99 988	38
23	8.38 276	520	8.38 289	520	11.61 711	9.99 987	37
24	8.38 796	514	8.38 809	514	11.61 191	9.99 987	36
25	8.39 310	508	8.39 323	509	11.60 677	9.99 987	35
26	8.39 818	502	8.39 832	502	11.60 168	9.99 986	34
27	8.40 320	496	8.40 334	496	11.59 666	9.99 986	33
28	8.40 816	491	8.40 830	491	11.59 170	9.99 986	32
29	8.41 307	485	8.41 321	486	11.58 679	9.99 985	31
30	8.41 792	480	8.41 807	480	11.58 193	9.99 985	30
31	8.42 272	474	8.42 287	473	11.57 713	9.99 985	29
32	8.42 746	470	8.42 762	470	11.57 238	9.99 984	28
33	8.43 216	464	8.43 232	464	11.56 768	9.99 984	27
34	8.43 680	459	8.43 696	460	11.56 304	9.99 984	26
35	8.44 139	455	8.44 156	455	11.55 844	9.99 983	25
36	8.44 594	450	8.44 611	450	11.55 389	9.99 983	24
37	8.45 044	443	8.45 061	446	11.54 939	9.99 983	23
38	8.45 489	441	8.45 507	441	11.54 493	9.99 982	22
39	8.45 930	436	8.45 948	437	11.54 052	9.99 982	21
40	8.46 366	431	8.46 385	432	11.53 615	9.99 982	20
41	8.46 799	427	8.46 817	428	11.53 188	9.99 981	19
42	8.47 226	424	8.47 245	424	11.52 755	9.99 981	18
43	8.47 650	419	8.47 669	420	11.52 331	9.99 981	17
44	8.48 069	416	8.48 089	416	11.51 911	9.99 980	16
45	8.48 485	411	8.48 505	412	11.51 495	9.99 980	15
46	8.48 896	408	8.48 917	408	11.51 083	9.99 979	14
47	8.49 304	404	8.49 325	404	11.50 675	9.99 979	13
48	8.49 708	400	8.49 729	401	11.50 271	9.99 979	12
49	8.50 108	396	8.50 130	397	11.49 870	9.99 978	11
50	8.50 504	391	8.50 527	393	11.49 473	9.99 978	10
51	8.50 897	380	8.50 920	390	11.49 080	9.99 977	9
52	8.51 287	385	8.51 310	386	11.48 690	9.99 977	8
53	8.51 673	382	8.51 696	383	11.48 304	9.99 977	7
54	8.52 055	379	8.52 079	380	11.47 921	9.99 976	6
55	8.52 434	376	8.52 459	376	11.47 541	9.99 976	5
56	8.52 810	373	8.52 835	373	11.47 165	9.99 975	4
57	8.53 183	369	8.53 208	370	11.46 792	9.99 975	3
58	8.53 552	367	8.53 578	367	11.46 422	9.99 974	2
59	8.53 919	363	8.53 945	363	11.46 055	9.99 974	1
60	8.54 282		8.54 308		11.45 692	9.99 974	0
'	L Cos	d	L Ctn	c d	L Tan	L Sin	'

For more accurate values of L sin and L tan for interpolated values of angles less than 3° (or L cos or L ctn of angles greater than 87°) use Table 5, page 216.

91° (271°)

(358°) 88°

Table 6 COMMON LOGARITHMS OF TRIGONOMETRIC FUNCTIONS (continued) 219

The -10 portion of the characteristic of the logarithm is not printed but must be written down whenever such a logarithm is used.

2° (182°)		(357°) 177°					
'	L Sin	d	L Tan	c d	L Ctn	L Cos	'
0	8.54 282	389	8.54 308	361	11.45 692	9.99 974	60
1	8.54 642	357	8.54 569	339	11.45 331	9.99 973	59
2	8.54 999	325	8.55 027	317	11.44 973	9.99 973	58
3	8.55 354	294	8.55 382	295	11.44 618	9.99 972	57
4	8.55 708	262	8.55 734	273	11.44 266	9.99 972	56
5	8.56 054	230	8.56 083	246	11.43 917	9.99 971	55
6	8.56 400	198	8.56 429	214	11.43 571	9.99 971	54
7	8.56 743	166	8.56 772	181	11.43 227	9.99 970	53
8	8.57 084	134	8.57 114	149	11.42 886	9.99 970	52
9	8.57 421	102	8.57 452	116	11.42 548	9.99 969	51
10	8.57 757	70	8.57 788	83	11.42 212	9.99 969	50
11	8.58 089	38	8.58 121	51	11.41 879	9.99 968	49
12	8.58 419	6	8.58 451	19	11.41 549	9.99 968	48
13	8.58 747	26	8.58 779	38	11.41 221	9.99 967	47
14	8.59 072	54	8.59 105	66	11.40 895	9.99 967	46
15	8.59 395	82	8.59 428	94	11.40 572	9.99 967	45
16	8.59 715	110	8.59 749	122	11.40 251	9.99 966	44
17	8.60 033	138	8.60 068	150	11.39 932	9.99 966	43
18	8.60 349	166	8.60 384	178	11.39 615	9.99 965	42
19	8.60 662	194	8.60 698	206	11.39 302	9.99 964	41
20	8.60 973	222	8.61 009	234	11.38 991	9.99 964	40
21	8.61 282	250	8.61 319	262	11.38 681	9.99 963	39
22	8.61 589	278	8.61 626	290	11.38 374	9.99 963	38
23	8.61 894	306	8.61 931	318	11.38 069	9.99 962	37
24	8.62 198	334	8.62 234	346	11.37 766	9.99 962	36
25	8.62 497	362	8.62 535	374	11.37 465	9.99 961	35
26	8.62 795	390	8.62 834	402	11.37 166	9.99 961	34
27	8.63 091	418	8.63 131	430	11.36 869	9.99 960	33
28	8.63 385	446	8.63 426	458	11.36 574	9.99 960	32
29	8.63 678	474	8.63 718	486	11.36 282	9.99 959	31
30	8.63 968	502	8.64 009	514	11.35 991	9.99 959	30
31	8.64 256	530	8.64 298	542	11.35 702	9.99 958	29
32	8.64 543	558	8.64 585	570	11.35 415	9.99 958	28
33	8.64 827	586	8.64 870	598	11.35 130	9.99 957	27
34	8.65 110	614	8.65 154	626	11.34 846	9.99 956	26
35	8.65 391	642	8.65 435	654	11.34 565	9.99 955	25
36	8.65 670	670	8.65 715	682	11.34 285	9.99 955	24
37	8.65 947	698	8.65 993	710	11.34 007	9.99 955	23
38	8.66 223	726	8.66 269	738	11.33 731	9.99 954	22
39	8.66 497	754	8.66 543	766	11.33 457	9.99 954	21
40	8.66 769	782	8.66 816	794	11.33 184	9.99 953	20
41	8.67 039	810	8.67 087	822	11.32 913	9.99 952	19
42	8.67 308	838	8.67 356	850	11.32 644	9.99 952	18
43	8.67 575	866	8.67 624	878	11.32 376	9.99 951	17
44	8.67 841	894	8.67 890	906	11.32 110	9.99 951	16
45	8.68 104	922	8.68 154	934	11.31 846	9.99 950	15
46	8.68 367	950	8.68 417	962	11.31 583	9.99 949	14
47	8.68 627	978	8.68 678	990	11.31 322	9.99 949	13
48	8.68 885	1006	8.68 938	1018	11.31 062	9.99 948	12
49	8.69 141	1034	8.69 196	1046	11.30 804	9.99 948	11
50	8.69 400	1062	8.69 453	1074	11.30 547	9.99 947	10
51	8.69 654	1090	8.69 708	1102	11.30 292	9.99 946	9
52	8.69 907	1118	8.69 962	1130	11.30 038	9.99 946	8
53	8.70 159	1146	8.70 214	1158	11.29 786	9.99 945	7
54	8.70 409	1174	8.70 465	1186	11.29 536	9.99 944	6
55	8.70 658	1202	8.70 714	1214	11.29 286	9.99 944	5
56	8.70 905	1230	8.70 962	1242	11.29 038	9.99 943	4
57	8.71 151	1258	8.71 208	1270	11.28 792	9.99 942	3
58	8.71 395	1286	8.71 453	1298	11.28 547	9.99 942	2
59	8.71 638	1314	8.71 697	1326	11.28 303	9.99 941	1
60	8.71 880		8.71 940		11.28 060	9.99 940	0
'	L Cos	d	L Ctn	c d	L Tan	L Sin	'

For more accurate values of L sin and L tan for interpolated values of angles less than 3° (or L cos or L ctn of angles greater than 87°) use Table 5, page 216.

Table 6 COMMON LOGARITHMS OF TRIGONOMETRIC FUNCTIONS (continued) 220

The --10 portion of the characteristic of the logarithm is not printed but must be written down whenever such a logarithm is used.

3° (183°)					(356°) 176°								
	L Sin	d	L Tan	c d	L Ctn	L Cos		Proportional parts					
0	8.71 880	240	8.71 940	241	11.28 060	9.99 940	60	"	241	239	237	235	234
1	8.72 120	239	8.72 181	239	11.27 819	9.99 940	59	1	4.0	4.0	4.0	3.9	3.9
2	8.72 359	238	8.72 420	239	11.27 580	9.99 939	58	2	8.0	8.0	7.9	7.8	7.8
3	8.72 597	237	8.72 659	237	11.27 341	9.99 938	57	3	12.0	12.0	11.8	11.8	11.7
4	8.72 834	235	8.72 896	236	11.27 104	9.99 938	56	4	16.1	15.9	15.8	15.7	15.6
5	8.73 069	234	8.73 132	234	11.26 868	9.99 937	55	5	20.1	19.9	19.8	19.6	19.5
6	8.73 303	232	8.73 366	234	11.26 634	9.99 936	54	6	24.1	23.9	23.7	23.5	23.4
7	8.73 535	232	8.73 600	232	11.26 400	9.99 936	53	7	28.1	27.9	27.6	27.4	27.3
8	8.73 767	230	8.73 832	231	11.26 168	9.99 935	52	8	32.1	31.9	31.6	31.3	31.2
9	8.73 997	229	8.74 063	229	11.25 937	9.99 934	51	9	36.2	35.8	35.6	35.2	35.1
10	8.74 226	228	8.74 292	229	11.25 708	9.99 934	50	"	232	229	227	225	223
11	8.74 454	226	8.74 521	227	11.25 479	9.99 933	49	1	3.9	3.8	3.8	3.7	3.7
12	8.74 680	226	8.74 748	226	11.25 252	9.99 932	48	2	7.7	7.6	7.6	7.5	7.4
13	8.74 906	224	8.74 974	225	11.25 026	9.99 932	47	3	11.6	11.4	11.4	11.2	11.2
14	8.75 130	223	8.75 199	224	11.24 801	9.99 931	46	4	15.5	15.3	15.1	15.0	14.9
15	8.75 353	222	8.75 423	222	11.24 577	9.99 930	45	5	19.3	19.1	19.0	18.8	18.7
16	8.75 575	220	8.75 645	222	11.24 355	9.99 929	44	6	23.2	22.9	22.7	22.5	22.4
17	8.75 795	220	8.75 867	220	11.24 135	9.99 929	43	7	27.1	26.7	26.5	26.2	26.0
18	8.76 015	219	8.76 087	219	11.23 913	9.99 928	42	8	30.9	30.5	30.3	30.0	29.7
19	8.76 234	217	8.76 306	219	11.23 694	9.99 927	41	9	34.8	34.4	34.0	33.6	33.4
20	8.76 451	216	8.76 525	217	11.23 475	9.99 926	40	"	222	220	217	215	213
21	8.76 667	216	8.76 742	216	11.23 258	9.99 926	39	1	3.7	3.7	3.6	3.6	3.5
22	8.76 883	214	8.76 958	215	11.23 042	9.99 925	38	2	7.4	7.3	7.2	7.2	7.1
23	8.77 097	213	8.77 173	214	11.22 827	9.99 924	37	3	11.1	11.0	10.8	10.6	10.6
24	8.77 310	212	8.77 387	213	11.22 613	9.99 923	36	4	14.8	14.7	14.5	14.3	14.2
25	8.77 522	211	8.77 600	211	11.22 400	9.99 923	35	5	18.5	18.3	18.1	17.9	17.8
26	8.77 733	210	8.77 811	211	11.22 189	9.99 922	34	6	22.2	22.0	21.7	21.5	21.4
27	8.77 943	209	8.78 022	210	11.21 978	9.99 921	33	7	25.9	25.7	25.3	25.1	24.8
28	8.78 152	208	8.78 232	209	11.21 768	9.99 920	32	8	29.6	29.3	28.9	28.7	28.4
29	8.78 360	208	8.78 441	208	11.21 559	9.99 920	31	9	33.3	33.0	32.6	32.2	32.0
30	8.78 568	206	8.78 649	206	11.21 351	9.99 919	30	"	211	208	206	203	201
31	8.78 774	205	8.78 855	206	11.21 145	9.99 918	29	1	3.5	3.5	3.4	3.4	3.4
32	8.78 979	204	8.79 061	205	11.20 939	9.99 917	28	2	7.0	6.9	6.9	6.8	6.7
33	8.79 183	203	8.79 266	204	11.20 734	9.99 917	27	3	10.6	10.4	10.3	10.2	10.0
34	8.79 386	202	8.79 470	203	11.20 529	9.99 916	26	4	14.1	13.9	13.7	13.5	13.4
35	8.79 588	201	8.79 673	202	11.20 327	9.99 915	25	5	17.6	17.3	17.2	16.9	16.8
36	8.79 789	201	8.79 875	201	11.20 125	9.99 914	24	6	21.1	20.8	20.6	20.3	20.1
37	8.79 990	199	8.80 076	201	11.19 924	9.99 913	23	7	24.6	24.3	24.0	23.7	23.4
38	8.80 189	199	8.80 277	199	11.19 723	9.99 913	22	8	28.1	27.7	27.3	27.1	26.8
39	8.80 388	197	8.80 476	198	11.19 524	9.99 912	21	9	31.6	31.2	30.9	30.4	30.2
40	8.80 585	197	8.80 674	198	11.19 326	9.99 911	20	"	199	197	195	193	192
41	8.80 782	196	8.80 872	196	11.19 128	9.99 910	19	1	3.3	3.3	3.2	3.2	3.2
42	8.80 978	195	8.81 068	196	11.18 932	9.99 909	18	2	6.6	6.6	6.5	6.4	6.4
43	8.81 173	194	8.81 264	195	11.18 736	9.99 908	17	3	10.0	9.8	9.8	9.6	9.6
44	8.81 367	193	8.81 459	194	11.18 541	9.99 908	16	4	13.3	13.1	13.0	12.9	12.8
45	8.81 560	192	8.81 653	193	11.18 347	9.99 907	15	5	16.6	16.4	16.2	16.1	16.0
46	8.81 752	192	8.81 846	192	11.18 154	9.99 906	14	6	19.9	19.7	19.5	19.3	19.2
47	8.81 944	190	8.82 038	192	11.17 962	9.99 905	13	7	23.2	23.0	22.8	22.5	22.4
48	8.82 134	190	8.82 230	190	11.17 770	9.99 904	12	8	26.5	26.3	26.0	25.7	25.5
49	8.82 324	189	8.82 420	190	11.17 580	9.99 904	11	9	29.8	29.6	29.3	28.9	28.8
50	8.82 513	188	8.82 610	189	11.17 390	9.99 903	10	"	189	187	185	183	181
51	8.82 701	187	8.82 799	188	11.17 201	9.99 902	9	1	3.2	3.1	3.1	3.0	3.0
52	8.82 888	187	8.82 987	188	11.17 013	9.99 901	8	2	6.3	6.2	6.2	6.1	6.0
53	8.83 075	186	8.83 175	186	11.16 825	9.99 900	7	3	9.4	9.4	9.2	9.2	9.0
54	8.83 261	185	8.83 361	186	11.16 639	9.99 899	6	4	12.6	12.5	12.3	12.2	12.1
55	8.83 446	184	8.83 547	185	11.16 453	9.99 898	5	5	15.8	15.6	15.5	15.2	15.1
56	8.83 630	183	8.83 732	184	11.16 268	9.99 898	4	6	18.9	18.7	18.5	18.3	18.1
57	8.83 813	183	8.83 916	184	11.16 084	9.99 897	3	7	22.0	21.8	21.6	21.4	21.1
58	8.83 995	181	8.84 100	182	11.15 900	9.99 896	2	8	25.2	24.9	24.7	24.4	24.1
59	8.84 177	181	8.84 282	182	11.15 718	9.99 895	1	9	28.4	28.0	27.8	27.4	27.2
60	8.84 358		8.84 464		11.15 536	9.99 894	0	"	222	220	217	215	213
	L Cos	d	L Ctn	c d	L Tan	L Sin		Proportional parts					

93° (273°)

(356°) 86°

Table 6 COMMON LOGARITHMS OF TRIGONOMETRIC FUNCTIONS (continued)

The -10 portion of the characteristic of the logarithm is not printed but must be written down whenever such a logarithm is used.

4° (184°)

(355°) 175°

4° (184°)					(355°) 175°								
	L Sin	d	L Tan	c d	L Ctn	L Cos	Proportional parts						
0	8.84 358	181	8.84 464	182	11.15 536	9.99 894	60	"	182	181	179	178	177
1	8.84 539	179	8.84 646	180	11.15 354	9.99 893	59	1	3.0	3.0	3.0	3.0	3.0
2	8.84 718	179	8.84 826	180	11.15 174	9.99 892	58	2	6.1	6.0	6.0	5.9	5.9
3	8.84 897	178	8.85 006	179	11.14 994	9.99 891	57	3	9.1	9.0	9.0	8.9	8.8
4	8.85 075	177	8.85 185	178	11.14 815	9.99 891	56	4	12.1	12.1	11.9	11.9	11.8
5	8.85 252	177	8.85 363	177	11.14 637	9.99 890	55	5	15.2	15.1	14.9	14.8	14.8
6	8.85 429	176	8.85 540	177	11.14 460	9.99 889	54	6	18.2	18.1	17.9	17.8	17.7
7	8.85 605	175	8.85 717	176	11.14 283	9.99 888	53	7	21.2	21.1	20.9	20.8	20.8
8	8.85 780	175	8.85 893	176	11.14 107	9.99 887	52	8	24.2	24.1	23.9	23.7	23.6
9	8.85 955	174	8.86 069	174	11.13 931	9.99 886	51	9	27.3	27.2	26.9	26.7	26.6
10	8.86 128	173	8.86 243	174	11.13 757	9.99 885	50	"	178	176	174	173	172
11	8.86 301	173	8.86 417	174	11.13 583	9.99 884	49	1	3.0	3.0	2.9	2.9	2.9
12	8.86 474	171	8.86 591	172	11.13 409	9.99 883	48	2	5.9	5.8	5.8	5.8	5.7
13	8.86 645	171	8.86 763	172	11.13 237	9.99 882	47	3	8.8	8.8	8.7	8.6	8.6
14	8.86 816	171	8.86 935	171	11.13 065	9.99 881	46	4	11.7	11.7	11.6	11.5	11.5
15	8.86 987	169	8.87 106	171	11.12 894	9.99 880	45	5	14.7	14.6	14.5	14.4	14.3
16	8.87 156	169	8.87 277	170	11.12 723	9.99 879	44	6	17.6	17.5	17.4	17.3	17.2
17	8.87 325	169	8.87 447	169	11.12 553	9.99 879	43	7	20.5	20.4	20.3	20.2	20.1
18	8.87 494	167	8.87 618	169	11.12 384	9.99 878	42	8	23.5	23.3	23.2	23.1	23.0
19	8.87 661	168	8.87 785	168	11.12 215	9.99 877	41	9	26.4	26.2	26.1	26.0	25.9
20	8.87 829	166	8.87 953	167	11.12 047	9.99 876	40	"	171	170	169	168	167
21	8.87 995	166	8.88 120	167	11.11 880	9.99 875	39	1	2.8	2.8	2.8	2.8	2.8
22	8.88 161	165	8.88 287	166	11.11 713	9.99 874	38	2	5.7	5.7	5.6	5.6	5.6
23	8.88 326	164	8.88 453	165	11.11 547	9.99 873	37	3	8.6	8.5	8.4	8.4	8.4
24	8.88 490	164	8.88 618	165	11.11 382	9.99 872	36	4	11.4	11.3	11.3	11.2	11.1
25	8.88 654	163	8.88 783	165	11.11 217	9.99 871	35	5	14.2	14.2	14.1	14.0	13.9
26	8.88 817	163	8.88 948	163	11.11 052	9.99 870	34	6	17.1	17.0	16.9	16.8	16.7
27	8.88 980	162	8.89 111	163	11.10 889	9.99 869	33	7	20.0	19.8	19.7	19.6	19.5
28	8.89 142	162	8.89 274	163	11.10 726	9.99 868	32	8	22.8	22.7	22.5	22.4	22.3
29	8.89 304	160	8.89 437	161	11.10 563	9.99 867	31	9	25.6	25.5	25.4	25.2	25.0
30	8.89 464	161	8.89 598	162	11.10 402	9.99 866	30	"	168	165	164	163	162
31	8.89 625	159	8.89 760	160	11.10 240	9.99 865	29	1	2.8	2.8	2.7	2.7	2.7
32	8.89 784	159	8.89 920	160	11.10 080	9.99 864	28	2	5.5	5.5	5.5	5.4	5.4
33	8.89 943	159	8.90 080	160	11.09 920	9.99 863	27	3	8.3	8.2	8.2	8.2	8.1
34	8.90 102	158	8.90 240	159	11.09 760	9.99 862	26	4	11.1	11.0	10.9	10.9	10.8
35	8.90 260	157	8.90 399	158	11.09 601	9.99 861	25	5	13.8	13.8	13.7	13.6	13.5
36	8.90 417	157	8.90 557	158	11.09 443	9.99 860	24	6	16.6	16.5	16.4	16.3	16.2
37	8.90 574	155	8.90 715	157	11.09 285	9.99 859	23	7	19.4	19.2	19.1	19.0	18.9
38	8.90 730	155	8.90 872	157	11.09 128	9.99 858	22	8	22.1	22.0	21.9	21.7	21.6
39	8.90 886	155	8.91 029	155	11.08 971	9.99 857	21	9	24.9	24.8	24.6	24.4	24.3
40	8.91 040	155	8.91 185	155	11.08 815	9.99 856	20	"	161	160	159	158	157
41	8.91 195	154	8.91 340	155	11.08 660	9.99 855	19	1	2.7	2.7	2.6	2.6	2.6
42	8.91 349	153	8.91 495	155	11.08 505	9.99 854	18	2	5.4	5.3	5.3	5.3	5.2
43	8.91 502	153	8.91 659	153	11.08 350	9.99 853	17	3	8.0	8.0	8.0	7.9	7.8
44	8.91 655	152	8.91 803	154	11.08 197	9.99 852	16	4	10.7	10.7	10.5	10.5	10.5
45	8.91 807	152	8.91 957	152	11.08 043	9.99 851	15	5	13.4	13.3	13.2	13.2	13.1
46	8.91 959	151	8.92 110	152	11.07 890	9.99 850	14	6	16.1	16.0	15.9	15.8	15.7
47	8.92 110	151	8.92 262	152	11.07 738	9.99 849	13	7	18.8	18.7	18.6	18.4	18.3
48	8.92 261	150	8.92 414	151	11.07 586	9.99 847	12	8	21.5	21.3	21.3	21.1	20.9
49	8.92 411	150	8.92 565	151	11.07 435	9.99 846	11	9	24.2	24.0	23.8	23.7	23.6
50	8.92 561	149	8.92 716	150	11.07 284	9.99 845	10	"	156	155	154	153	152
51	8.92 710	149	8.92 866	150	11.07 134	9.99 844	9	1	2.6	2.6	2.6	2.6	2.5
52	8.92 859	148	8.93 016	149	11.06 984	9.99 843	8	2	5.2	5.2	5.1	5.1	5.1
53	8.93 007	147	8.93 165	148	11.06 835	9.99 842	7	3	7.8	7.8	7.7	7.6	7.6
54	8.93 154	147	8.93 313	149	11.06 687	9.99 841	6	4	10.4	10.3	10.3	10.2	10.1
55	8.93 301	147	8.93 462	147	11.06 538	9.99 840	5	5	13.0	12.9	12.8	12.8	12.7
56	8.93 448	145	8.93 609	147	11.06 391	9.99 839	4	6	15.6	15.5	15.4	15.3	15.2
57	8.93 594	146	8.93 756	147	11.06 244	9.99 838	3	7	18.2	18.1	18.0	17.8	17.7
58	8.93 740	145	8.93 903	146	11.06 097	9.99 837	2	8	20.8	20.7	20.5	20.4	20.3
59	8.93 885	145	8.94 049	146	11.05 951	9.99 836	1	9	23.3	23.2	23.1	23.0	23.0
60	8.94 039		8.94 195		11.05 805	9.99 834	0	"					
	L Cos	d	L Ctn	c d	L Tan	L Sin		Proportional parts					

94° (274°)

(265°) 85°

Table 6 COMMON LOGARITHMS OF TRIGONOMETRIC FUNCTIONS (continued) 222

The -10 portion of the characteristic of the logarithm is not printed but must be written down whenever such a logarithm is used.

5° (185°)					(354°) 174°								
<i>n</i>	L Sin	<i>d</i>	L Tan	<i>cd</i>	L Ctn	L Cos	Proportional parts						
0	8.94 030	144	8.94 195	145	11.05 805	9.99 824	60	"	151	149	148	147	146
1	8.94 174	143	8.94 340	148	11.05 660	9.99 833	59	1	2.5	2.5	2.5	2.4	2.4
2	8.94 317	144	8.94 485	148	11.05 515	9.99 832	58	2	5.0	5.0	4.9	4.9	4.9
3	8.94 461	142	8.94 630	143	11.05 370	9.99 831	57	3	7.5	7.4	7.4	7.4	7.3
4	8.94 603	143	8.94 773	144	11.05 227	9.99 830	56	4	10.1	9.9	9.9	9.8	9.7
5	8.94 746	141	8.94 917	143	11.05 083	9.99 829	55	5	12.6	12.4	12.3	12.2	12.2
6	8.94 887	142	8.95 080	142	11.04 940	9.99 828	54	6	15.1	14.9	14.8	14.7	14.6
7	8.95 029	141	8.95 202	142	11.04 798	9.99 827	53	7	17.6	17.4	17.3	17.2	17.0
8	8.95 170	140	8.95 344	142	11.04 656	9.99 825	52	8	20.1	19.9	19.7	19.6	19.5
9	8.95 310	140	8.95 486	141	11.04 514	9.99 824	51	9	22.6	22.4	22.2	22.0	21.9
10	8.95 450	139	8.95 627	140	11.04 373	9.99 823	50	"	145	144	143	142	141
11	8.95 589	139	8.95 767	141	11.04 233	9.99 822	49	1	2.4	2.4	2.4	2.4	2.4
12	8.95 728	138	8.95 908	139	11.04 092	9.99 821	48	2	4.8	4.8	4.8	4.7	4.7
13	8.95 867	138	8.96 047	140	11.03 953	9.99 820	47	3	7.2	7.2	7.2	7.1	7.0
14	8.96 005	138	8.96 187	138	11.03 813	9.99 819	46	4	9.7	9.6	9.5	9.5	9.4
15	8.96 143	137	8.96 325	139	11.03 675	9.99 817	45	5	12.1	12.0	11.9	11.8	11.7
16	8.96 286	137	8.96 464	138	11.03 536	9.99 816	44	6	14.5	14.4	14.3	14.2	14.1
17	8.96 417	136	8.96 602	137	11.03 398	9.99 815	43	7	16.9	16.8	16.7	16.6	16.4
18	8.96 552	136	8.96 739	136	11.03 261	9.99 814	42	8	19.3	19.2	19.1	18.9	18.8
19	8.96 689	136	8.96 877	136	11.03 123	9.99 813	41	9	21.8	21.6	21.4	21.3	21.2
20	8.96 825	135	8.97 013	137	11.02 987	9.99 812	40	"	140	139	138	137	136
21	8.96 960	135	8.97 150	138	11.02 850	9.99 811	39	1	2.3	2.3	2.3	2.3	2.3
22	8.97 095	134	8.97 286	136	11.02 715	9.99 809	38	2	4.7	4.6	4.6	4.6	4.5
23	8.97 228	134	8.97 421	135	11.02 579	9.99 808	37	3	7.0	7.0	6.9	6.9	6.8
24	8.97 363	135	8.97 556	135	11.02 444	9.99 807	36	4	9.3	9.3	9.2	9.1	9.1
25	8.97 496	133	8.97 691	134	11.02 309	9.99 806	35	5	11.7	11.6	11.5	11.4	11.3
26	8.97 629	133	8.97 825	134	11.02 178	9.99 804	34	6	14.0	13.9	13.8	13.7	13.6
27	8.97 762	132	8.97 959	133	11.02 041	9.99 803	33	7	16.3	16.2	16.1	16.0	15.9
28	8.97 894	132	8.98 092	132	11.01 903	9.99 802	32	8	18.7	18.5	18.4	18.3	18.1
29	8.98 026	131	8.98 225	132	11.01 775	9.99 801	31	9	21.0	20.8	20.7	20.5	20.4
30	8.98 157	131	8.98 358	132	11.01 642	9.99 800	30	"	135	134	133	132	131
31	8.98 288	131	8.98 490	132	11.01 510	9.99 798	29	1	2.2	2.2	2.2	2.2	2.2
32	8.98 419	130	8.98 622	131	11.01 378	9.99 797	28	2	4.5	4.5	4.4	4.4	4.4
33	8.98 549	130	8.98 753	131	11.01 247	9.99 796	27	3	6.8	6.7	6.6	6.6	6.6
34	8.98 679	129	8.98 884	131	11.01 113	9.99 795	26	4	9.0	8.9	8.9	8.8	8.7
35	8.98 808	129	8.99 016	130	11.00 985	9.99 793	25	5	11.2	11.2	11.1	11.0	10.9
36	8.98 937	129	8.99 146	130	11.00 855	9.99 792	24	6	13.5	13.4	13.3	13.2	13.1
37	8.99 066	128	8.99 275	129	11.00 725	9.99 791	23	7	15.8	15.6	15.5	15.4	15.3
38	8.99 194	128	8.99 405	129	11.00 595	9.99 790	22	8	18.0	17.9	17.7	17.6	17.5
39	8.99 322	128	8.99 534	128	11.00 468	9.99 788	21	9	20.2	20.1	20.0	19.8	19.6
40	8.99 450	127	8.99 662	128	11.00 338	9.99 787	20	"	130	129	128	127	126
41	8.99 577	127	8.99 791	128	11.00 209	9.99 785	19	1	2.1	2.1	2.1	2.1	2.1
42	8.99 704	126	8.99 919	127	11.00 081	9.99 785	18	2	4.3	4.3	4.3	4.2	4.2
43	8.99 830	126	9.00 046	128	10.99 954	9.99 783	17	3	6.5	6.4	6.4	6.4	6.3
44	8.99 956	126	9.00 174	127	10.99 826	9.99 782	16	4	8.7	8.6	8.5	8.5	8.4
45	9.00 082	125	9.00 301	126	10.99 699	9.99 781	15	5	10.9	10.8	10.7	10.5	10.5
46	9.00 207	125	9.00 427	126	10.99 573	9.99 780	14	6	13.0	12.9	12.8	12.7	12.6
47	9.00 332	124	9.00 553	125	10.99 447	9.99 778	13	7	15.2	15.0	14.9	14.8	14.7
48	9.00 456	125	9.00 679	126	10.99 321	9.99 777	12	8	17.3	17.2	17.1	16.9	16.8
49	9.00 581	123	9.00 805	125	10.99 195	9.99 776	11	9	19.5	19.4	19.2	19.0	18.9
50	9.00 704	124	9.00 930	125	10.99 070	9.99 775	10	"	125	124	123	122	121
51	9.00 828	123	9.01 056	124	10.98 945	9.99 773	9	1	2.1	2.1	2.0	2.0	2.0
52	9.00 951	123	9.01 179	124	10.98 821	9.99 772	8	2	4.2	4.1	4.1	4.1	4.0
53	9.01 074	122	9.01 303	124	10.98 697	9.99 771	7	3	6.2	6.2	6.2	6.1	6.0
54	9.01 196	122	9.01 427	123	10.98 573	9.99 769	6	4	8.2	8.1	8.1	8.1	8.1
55	9.01 318	122	9.01 550	123	10.98 450	9.99 768	5	5	10.4	10.3	10.2	10.2	10.1
56	9.01 440	121	9.01 673	123	10.98 327	9.99 767	4	6	12.5	12.4	12.3	12.2	12.1
57	9.01 561	121	9.01 796	122	10.98 204	9.99 765	3	7	14.6	14.5	14.4	14.2	14.1
58	9.01 682	121	9.01 918	122	10.98 082	9.99 764	2	8	16.7	16.5	16.4	16.3	16.1
59	9.01 803	120	9.02 040	122	10.97 960	9.99 763	1	9	18.8	18.6	18.4	18.3	18.2
60	9.01 923		9.02 162		10.97 838	9.99 761	0						
<i>n</i>	L Cos	<i>d</i>	L Ctn	<i>cd</i>	L Tan	L Sin	<i>n</i>	Proportional parts					

95° (275°)

(264°) 84°

Table 6 COMMON LOGARITHMS OF TRIGONOMETRIC FUNCTIONS (continued) 223

The -10 portion of the characteristic of the logarithm is not printed but must be written down whenever such a logarithm is used.

6° (186°)				(353°) 173°						
<i>n</i>	L. Sin <i>d</i>	L. Tan <i>c d</i>	L. Ctn	L. Cos	<i>n</i>	Proportional parts				
0	9.01 923	9.02 162	10.97 838	9.99 761	60	"	121	120	119	118
1	9.02 043	9.02 283	10.97 717	9.99 760	59	1	2.0	2.0	2.0	2.0
2	9.02 163	9.02 404	10.97 596	9.99 759	58	2	4.0	4.0	4.0	3.9
3	9.02 283	9.02 528	10.97 475	9.99 757	57	3	6.0	6.0	6.0	5.9
4	9.02 402	9.02 645	10.97 355	9.99 756	56	4	8.1	8.0	7.9	7.9
5	9.02 520	9.02 766	10.97 234	9.99 755	55	5	10.1	10.0	9.9	9.8
6	9.02 639	9.02 886	10.97 115	9.99 753	54	6	12.1	12.0	11.9	11.8
7	9.02 757	9.03 008	10.96 995	9.99 752	53	7	14.1	14.0	13.9	13.8
8	9.02 874	9.03 124	10.96 876	9.99 751	52	8	16.1	16.0	15.9	15.7
9	9.02 992	9.03 242	10.96 758	9.99 749	51	9	18.2	18.0	17.8	17.7
10	9.03 109	9.03 361	10.96 639	9.99 748	50	10	20.2	20.0	19.8	19.7
11	9.03 226	9.03 479	10.96 521	9.99 747	49	20	40.3	40.0	39.7	39.3
12	9.03 342	9.03 597	10.96 403	9.99 745	48	30	60.5	60.0	59.5	59.0
13	9.03 458	9.03 714	10.96 286	9.99 744	47	40	80.7	80.0	79.3	78.7
14	9.03 574	9.03 832	10.96 168	9.99 742	46	50	100.8	100.0	99.2	98.3
15	9.03 690	9.03 948	10.96 052	9.99 741	45	"	127	116	115	114
16	9.03 806	9.04 065	10.95 935	9.99 740	44	1	2.0	1.9	1.9	1.9
17	9.03 920	9.04 181	10.95 819	9.99 738	43	2	3.9	3.9	3.8	3.8
18	9.04 034	9.04 297	10.95 703	9.99 737	42	3	5.8	5.8	5.6	5.7
19	9.04 149	9.04 413	10.95 587	9.99 736	41	4	7.8	7.7	7.7	7.5
20	9.04 262	9.04 528	10.95 472	9.99 734	40	5	9.8	9.7	9.6	9.5
21	9.04 376	9.04 543	10.95 357	9.99 733	39	6	11.7	11.6	11.5	11.4
22	9.04 490	9.04 768	10.95 242	9.99 731	38	7	13.6	13.5	13.4	13.3
23	9.04 603	9.04 873	10.95 127	9.99 730	37	8	15.6	15.5	15.3	15.2
24	9.04 716	9.04 987	10.95 013	9.99 728	36	9	17.6	17.4	17.2	17.1
25	9.04 828	9.05 101	10.94 899	9.99 727	35	10	19.5	19.3	19.2	19.0
26	9.04 940	9.05 214	10.94 786	9.99 726	34	20	39.0	38.7	38.3	38.0
27	9.05 052	9.05 328	10.94 672	9.99 724	33	30	58.5	58.0	57.5	57.0
28	9.05 164	9.05 441	10.94 559	9.99 723	32	40	78.0	77.3	76.7	76.0
29	9.05 275	9.05 553	10.94 447	9.99 721	31	50	97.5	96.7	95.8	95.0
30	9.05 386	9.05 666	10.94 334	9.99 720	30	"	113	112	111	110
31	9.05 497	9.05 778	10.94 222	9.99 718	29	1	1.9	1.9	1.8	1.8
32	9.05 607	9.05 890	10.94 110	9.99 717	28	2	3.8	3.7	3.7	3.7
33	9.05 717	9.05 002	10.93 998	9.99 716	27	3	5.6	5.6	5.4	5.5
34	9.05 827	9.05 113	10.93 887	9.99 714	26	4	7.5	7.4	7.4	7.3
35	9.05 937	9.05 224	10.93 776	9.99 713	25	5	9.4	9.3	9.2	9.2
36	9.06 046	9.05 335	10.93 665	9.99 711	24	6	11.3	11.2	11.1	11.0
37	9.06 155	9.05 445	10.93 555	9.99 710	23	7	13.2	13.1	13.0	12.9
38	9.06 264	9.05 556	10.93 444	9.99 708	22	8	15.1	14.9	14.8	14.7
39	9.06 372	9.05 666	10.93 334	9.99 707	21	9	17.0	16.8	16.6	16.5
40	9.06 481	9.06 775	10.93 225	9.99 705	20	10	18.8	18.7	18.5	18.3
41	9.06 589	9.06 885	10.93 115	9.99 704	19	20	37.7	37.3	37.0	36.7
42	9.06 696	9.06 994	10.93 005	9.99 702	18	30	56.5	56.0	55.5	55.0
43	9.06 804	9.07 103	10.92 897	9.99 701	17	40	75.3	74.7	74.0	73.3
44	9.06 911	9.07 211	10.92 789	9.99 699	16	50	94.2	93.3	92.5	91.7
45	9.07 018	9.07 320	10.92 680	9.99 698	15	"	109	108	107	106
46	9.07 124	9.07 428	10.92 572	9.99 696	14	1	1.8	1.8	1.8	1.8
47	9.07 231	9.07 536	10.92 464	9.99 695	13	2	3.6	3.6	3.6	3.5
48	9.07 337	9.07 643	10.92 357	9.99 693	12	3	5.4	5.4	5.4	5.3
49	9.07 442	9.07 751	10.92 249	9.99 692	11	4	7.3	7.2	7.1	7.1
50	9.07 548	9.07 858	10.92 142	9.99 690	10	5	9.1	9.0	8.9	8.8
51	9.07 653	9.07 964	10.92 036	9.99 689	9	6	10.9	10.8	10.7	10.6
52	9.07 758	9.08 071	10.91 929	9.99 687	8	7	12.7	12.6	12.5	12.4
53	9.07 863	9.08 177	10.91 823	9.99 685	7	8	14.5	14.4	14.3	14.1
54	9.07 968	9.08 283	10.91 717	9.99 684	6	9	16.4	16.2	16.0	15.9
55	9.08 072	9.08 389	10.91 611	9.99 683	5	10	18.2	18.0	17.8	17.7
56	9.08 176	9.08 495	10.91 505	9.99 681	4	20	36.3	36.0	35.7	35.3
57	9.08 280	9.08 600	10.91 400	9.99 680	3	30	54.5	54.0	53.5	53.0
58	9.08 383	9.08 705	10.91 295	9.99 678	2	40	72.7	72.0	71.3	70.7
59	9.08 486	9.08 810	10.91 190	9.99 677	1	50	90.8	90.0	89.2	88.3
60	9.08 589	9.08 914	10.91 085	9.99 675	0	"				
<i>n</i>	L. Cos <i>d</i>	L. Ctn <i>c d</i>	L. Tan	L. Sin	<i>n</i>	Proportional parts				

96° (276°)

(263°) 83°

Table 6 COMMON LOGARITHMS OF TRIGONOMETRIC FUNCTIONS (continued) 224

The -10 portion of the characteristic of the logarithm is not printed but must be written down whenever such a logarithm is used.

7° (187°)		(352°) 172°										
L. Sin	d	L. Tan	cd	L. Ctn	L. Cos	Proportional parts						
0	9.08 589	103	9.08 914	105	10.91 086	9.99 675	60	"	105	104	103	102
1	9.08 692	103	9.09 019	104	10.90 981	9.99 674	59	1	1.2	1.7	1.7	1.9
2	9.08 795	102	9.09 123	104	10.90 877	9.99 672	58	2	3.5	3.5	3.4	3.4
3	9.08 897	102	9.09 227	102	10.90 773	9.99 670	57	3	8.2	5.2	8.2	5.1
4	9.08 999	102	9.09 330	104	10.90 670	9.99 669	56	4	7.0	6.9	6.9	6.8
5	9.09 101	101	9.09 434	103	10.90 566	9.99 667	55	5	8.8	8.7	8.6	8.5
6	9.09 202	102	9.09 537	103	10.90 463	9.99 666	54	6	10.5	10.4	10.3	10.2
7	9.09 304	101	9.09 640	102	10.90 360	9.99 664	53	7	12.2	12.1	12.0	11.9
8	9.09 405	101	9.09 742	103	10.90 258	9.99 663	52	8	14.0	13.9	13.7	13.6
9	9.09 506	100	9.09 845	102	10.90 156	9.99 661	51	9	15.8	15.6	15.4	15.3
10	9.09 606	101	9.09 947	102	10.90 053	9.99 659	50	10	17.5	17.3	17.2	17.0
11	9.09 707	100	9.10 049	101	10.89 951	9.99 658	49	20	35.0	34.7	34.3	34.0
12	9.09 807	100	9.10 150	102	10.89 850	9.99 656	48	30	52.5	52.0	51.6	51.0
13	9.09 907	99	9.10 252	101	10.89 746	9.99 655	47	40	70.0	69.3	68.7	68.0
14	9.10 006	100	9.10 353	101	10.89 647	9.99 653	46	50	87.5	86.7	85.8	85.0
15	9.10 106	99	9.10 454	101	10.89 546	9.99 651	45	"	101	100	99	98
16	9.10 205	99	9.10 555	101	10.89 445	9.99 650	44	1	1.7	1.7	1.6	1.6
17	9.10 304	98	9.10 656	100	10.89 344	9.99 648	43	2	3.4	3.3	3.3	3.3
18	9.10 402	99	9.10 756	100	10.89 244	9.99 647	42	3	5.0	5.0	5.0	4.9
19	9.10 501	98	9.10 856	100	10.89 144	9.99 645	41	4	6.7	6.7	6.6	6.5
20	9.10 599	98	9.10 956	100	10.89 044	9.99 643	40	5	8.4	8.3	8.2	8.2
21	9.10 697	98	9.11 056	99	10.88 944	9.99 642	39	6	10.1	10.0	9.9	9.8
22	9.10 795	98	9.11 155	99	10.88 845	9.99 640	38	7	11.8	11.7	11.6	11.4
23	9.10 893	97	9.11 254	99	10.88 746	9.99 638	37	8	13.5	13.3	13.2	13.1
24	9.10 990	97	9.11 353	99	10.88 647	9.99 637	36	9	15.2	15.0	14.8	14.7
25	9.11 087	97	9.11 452	99	10.88 548	9.99 635	35	10	16.8	16.7	16.5	16.3
26	9.11 184	97	9.11 551	98	10.88 449	9.99 633	34	20	32.7	32.3	32.0	32.7
27	9.11 281	96	9.11 649	98	10.88 351	9.99 632	33	30	50.5	50.0	49.5	49.0
28	9.11 377	97	9.11 747	98	10.88 253	9.99 630	32	40	67.3	66.7	66.0	65.3
29	9.11 474	96	9.11 845	98	10.88 156	9.99 629	31	50	84.2	83.3	82.5	81.7
30	9.11 570	96	9.11 943	97	10.88 057	9.99 627	30	"	97	96	95	94
31	9.11 666	95	9.12 040	98	10.87 960	9.99 625	29	1	1.6	1.6	1.6	1.6
32	9.11 761	96	9.12 138	97	10.87 862	9.99 624	28	2	3.2	3.2	3.2	3.1
33	9.11 857	95	9.12 235	97	10.87 765	9.99 622	27	3	4.8	4.8	4.8	4.7
34	9.11 952	95	9.12 332	96	10.87 668	9.99 620	26	4	6.5	6.4	6.3	6.3
35	9.12 047	95	9.12 428	97	10.87 572	9.99 618	25	5	8.1	8.0	7.9	7.8
36	9.12 142	94	9.12 525	96	10.87 475	9.99 617	24	6	9.7	9.6	9.5	9.4
37	9.12 236	94	9.12 621	96	10.87 379	9.99 615	23	7	11.3	11.2	11.1	11.0
38	9.12 331	93	9.12 717	96	10.87 283	9.99 613	22	8	12.9	12.8	12.7	12.5
39	9.12 425	94	9.12 813	96	10.87 187	9.99 612	21	9	14.6	14.4	14.2	14.1
40	9.12 519	93	9.12 909	95	10.87 091	9.99 610	20	10	16.2	16.0	15.8	15.7
41	9.12 612	94	9.13 004	95	10.86 996	9.99 608	19	20	32.3	32.0	31.7	31.3
42	9.12 706	93	9.13 099	95	10.86 901	9.99 607	18	30	48.5	48.0	47.5	47.0
43	9.12 799	93	9.13 194	95	10.86 806	9.99 605	17	40	64.7	64.0	63.3	62.7
44	9.12 892	93	9.13 289	95	10.86 711	9.99 603	16	50	80.6	80.0	79.2	78.5
45	9.12 985	93	9.13 384	94	10.86 616	9.99 601	15	"	93	92	91	90
46	9.13 078	93	9.13 478	95	10.86 522	9.99 600	14	1	1.5	1.5	1.5	1.5
47	9.13 171	92	9.13 573	94	10.86 427	9.99 598	13	2	3.1	3.1	3.0	3.0
48	9.13 263	92	9.13 667	94	10.86 333	9.99 596	12	3	4.4	4.6	4.6	4.6
49	9.13 355	92	9.13 761	93	10.86 239	9.99 595	11	4	6.2	6.1	6.1	6.0
50	9.13 447	92	9.13 854	94	10.86 146	9.99 593	10	5	7.8	7.7	7.6	7.5
51	9.13 539	91	9.13 948	93	10.86 052	9.99 591	9	6	9.3	9.2	9.1	9.0
52	9.13 630	92	9.14 041	93	10.85 959	9.99 589	8	7	10.8	10.7	10.6	10.5
53	9.13 722	91	9.14 134	93	10.85 866	9.99 588	7	8	12.4	12.3	12.1	12.0
54	9.13 813	91	9.14 227	93	10.85 773	9.99 586	6	9	14.4	14.2	13.6	13.5
55	9.13 904	90	9.14 320	92	10.85 680	9.99 584	5	10	15.5	15.3	15.2	15.0
56	9.13 994	91	9.14 412	92	10.85 588	9.99 582	4	20	31.0	30.7	30.3	30.0
57	9.14 088	90	9.14 504	93	10.85 496	9.99 581	3	30	45.5	45.0	44.5	44.0
58	9.14 175	91	9.14 597	91	10.85 403	9.99 579	2	40	62.0	61.3	60.7	60.0
59	9.14 266	90	9.14 688	92	10.85 312	9.99 577	1	50	77.5	76.7	75.8	75.0
60	9.14 356		9.14 780		10.85 220	9.99 575	0					
L. Cos	d	L. Ctn	cd	L. Tan	L. Sin	Proportional parts						

Table 6 COMMON LOGARITHMS OF TRIGONOMETRIC FUNCTIONS (continued) 225

The -10 portion of the characteristic of the logarithm is not printed but must be written down whenever such a logarithm is used.

8° (188°)						(351°) 171°					
d	L Sin	d	L Tan	d	L Ctn	L Cos	d	Proportional parts			
0	9.14 356	89	9.14 780	92	10.85 220	9.99 575	60	92	91	90	
1	9.14 445	90	9.14 872	91	10.85 128	9.99 574	59	1.5	1.5	1.5	
2	9.14 535	89	9.14 963	91	10.85 037	9.99 572	58	3.1	3.0	3.0	
3	9.14 624	90	9.15 054	91	10.84 948	9.99 570	57	4.6	4.6	4.5	
4	9.14 714	89	9.15 145	91	10.84 856	9.99 568	56	6.1	6.1	6.0	
5	9.14 803	88	9.15 236	91	10.84 764	9.99 566	55	7.7	7.4	7.5	
6	9.14 891	89	9.15 327	90	10.84 673	9.99 565	54	9.2	9.1	9.0	
7	9.14 980	88	9.15 417	91	10.84 583	9.99 563	53	10.7	10.6	10.5	
8	9.15 069	89	9.15 508	90	10.84 492	9.99 561	52	12.3	12.1	12.0	
9	9.15 157	88	9.15 598	90	10.84 402	9.99 559	51	13.8	13.6	13.5	
10	9.15 245	88	9.15 688	89	10.84 312	9.99 557	50	15.3	15.2	15.0	
11	9.15 333	88	9.15 777	90	10.84 223	9.99 556	49	16.7	16.3	16.0	
12	9.15 421	87	9.15 867	89	10.84 133	9.99 554	48	18.0	18.5	18.0	
13	9.15 508	88	9.15 956	90	10.84 044	9.99 552	47	19.3	19.7	19.0	
14	9.15 596	87	9.16 046	89	10.83 954	9.99 550	46	20.7	20.8	20.0	
15	9.15 683	87	9.16 135	89	10.83 865	9.99 548	45	22.1	22.1	21.0	
16	9.15 770	87	9.16 224	88	10.83 776	9.99 546	44	23.5	23.5	22.0	
17	9.15 857	87	9.16 312	89	10.83 688	9.99 545	43	24.9	24.9	23.0	
18	9.15 944	86	9.16 401	88	10.83 599	9.99 543	42	26.3	26.3	24.0	
19	9.16 030	86	9.16 489	88	10.83 511	9.99 541	41	27.7	27.7	25.0	
20	9.16 116	87	9.16 577	89	10.83 423	9.99 539	40	29.1	29.1	26.0	
21	9.16 203	86	9.16 665	88	10.83 335	9.99 537	39	30.5	30.5	27.0	
22	9.16 289	85	9.16 753	88	10.83 247	9.99 535	38	31.9	31.9	28.0	
23	9.16 374	86	9.16 841	87	10.83 159	9.99 533	37	33.3	33.3	29.0	
24	9.16 460	85	9.16 928	88	10.83 072	9.99 532	36	34.7	34.7	30.0	
25	9.16 545	86	9.17 016	87	10.82 984	9.99 530	35	36.1	36.1	31.0	
26	9.16 631	85	9.17 103	87	10.82 897	9.99 528	34	37.5	37.5	32.0	
27	9.16 716	85	9.17 190	87	10.82 810	9.99 526	33	38.9	38.9	33.0	
28	9.16 801	85	9.17 277	86	10.82 723	9.99 524	32	40.3	40.3	34.0	
29	9.16 886	84	9.17 363	87	10.82 637	9.99 522	31	41.7	41.7	35.0	
30	9.16 970	85	9.17 450	86	10.82 550	9.99 520	30	43.1	43.1	36.0	
31	9.17 055	84	9.17 536	86	10.82 464	9.99 518	29	44.5	44.5	37.0	
32	9.17 139	84	9.17 622	86	10.82 378	9.99 517	28	45.9	45.9	38.0	
33	9.17 223	84	9.17 708	86	10.82 292	9.99 515	27	47.3	47.3	39.0	
34	9.17 307	84	9.17 794	86	10.82 206	9.99 513	26	48.7	48.7	40.0	
35	9.17 391	83	9.17 880	85	10.82 120	9.99 511	25	50.1	50.1	41.0	
36	9.17 474	84	9.17 965	86	10.82 035	9.99 509	24	51.5	51.5	42.0	
37	9.17 558	83	9.18 051	85	10.81 949	9.99 507	23	52.9	52.9	43.0	
38	9.17 641	83	9.18 136	85	10.81 864	9.99 505	22	54.3	54.3	44.0	
39	9.17 724	83	9.18 221	85	10.81 779	9.99 503	21	55.7	55.7	45.0	
40	9.17 807	83	9.18 306	85	10.81 694	9.99 501	20	57.1	57.1	46.0	
41	9.17 890	83	9.18 391	84	10.81 609	9.99 499	19	58.5	58.5	47.0	
42	9.17 973	82	9.18 475	85	10.81 525	9.99 497	18	59.9	59.9	48.0	
43	9.18 056	82	9.18 560	84	10.81 440	9.99 495	17	61.3	61.3	49.0	
44	9.18 137	83	9.18 644	84	10.81 356	9.99 494	16	62.7	62.7	50.0	
45	9.18 220	82	9.18 728	84	10.81 272	9.99 492	15	64.1	64.1	51.0	
46	9.18 302	81	9.18 812	84	10.81 188	9.99 490	14	65.5	65.5	52.0	
47	9.18 383	82	9.18 895	83	10.81 104	9.99 488	13	66.9	66.9	53.0	
48	9.18 465	82	9.18 979	84	10.81 021	9.99 486	12	68.3	68.3	54.0	
49	9.18 547	81	9.19 063	83	10.80 937	9.99 484	11	69.7	69.7	55.0	
50	9.18 628	81	9.19 146	83	10.80 854	9.99 482	10	71.1	71.1	56.0	
51	9.18 709	81	9.19 229	83	10.80 771	9.99 480	9	72.5	72.5	57.0	
52	9.18 790	81	9.19 312	83	10.80 688	9.99 478	8	73.9	73.9	58.0	
53	9.18 871	81	9.19 395	83	10.80 605	9.99 476	7	75.3	75.3	59.0	
54	9.18 952	81	9.19 478	83	10.80 522	9.99 474	6	76.7	76.7	60.0	
55	9.19 033	80	9.19 561	82	10.80 439	9.99 472	5	78.1	78.1	61.0	
56	9.19 113	80	9.19 643	82	10.80 357	9.99 470	4	79.5	79.5	62.0	
57	9.19 193	80	9.19 725	82	10.80 275	9.99 468	3	80.9	80.9	63.0	
58	9.19 273	80	9.19 807	82	10.80 193	9.99 466	2	82.3	82.3	64.0	
59	9.19 353	80	9.19 889	82	10.80 111	9.99 464	1	83.7	83.7	65.0	
60	9.19 433		9.19 971		10.80 029	9.99 462	0	85.1	85.1	66.0	
	L Cos	d	L Ctn	od	L Tan	L Sin		Proportional parts			

98° (278°)

(261°) 31°

Table 6 COMMON LOGARITHMS OF TRIGONOMETRIC FUNCTIONS (continued) 226

The -10 portion of the characteristic of the logarithm is not printed but must be written down whenever such a logarithm is used.

9° (189°)					(350°) 170°								
	L Sin	d	L Tan	cd	L Ctn	L Cos		Proportional parts					
0	9.19 433	80	9.19 971	82	10.80 029	9.99 462	60						
1	9.19 513	79	9.20 033	81	10.79 947	9.99 460	59						
2	9.19 592	80	9.20 134	82	10.79 866	9.99 458	58						
3	9.19 672	79	9.20 215	81	10.79 784	9.99 456	57						
4	9.19 751	79	9.20 297	81	10.79 703	9.99 454	56						
5	9.19 830	79	9.20 378	81	10.79 622	9.99 452	55						
6	9.19 909	79	9.20 459	81	10.79 541	9.99 450	54						
7	9.19 988	79	9.20 540	81	10.79 460	9.99 448	53						
8	9.20 067	78	9.20 621	80	10.79 379	9.99 446	52						
9	9.20 145	78	9.20 701	81	10.79 299	9.99 444	51						
10	9.20 223	79	9.20 782	80	10.79 218	9.99 442	50						
11	9.20 302	78	9.20 862	80	10.79 138	9.99 440	49						
12	9.20 380	78	9.20 942	80	10.79 058	9.99 438	48						
13	9.20 458	77	9.21 022	80	10.78 978	9.99 436	47						
14	9.20 535	78	9.21 102	80	10.78 898	9.99 434	46						
15	9.20 613	78	9.21 182	79	10.78 818	9.99 432	45						
16	9.20 691	77	9.21 261	80	10.78 738	9.99 429	44						
17	9.20 768	77	9.21 341	79	10.78 658	9.99 427	43						
18	9.20 845	77	9.21 420	79	10.78 578	9.99 425	42						
19	9.20 922	77	9.21 499	79	10.78 501	9.99 423	41						
20	9.20 999	77	9.21 578	79	10.78 422	9.99 421	40						
21	9.21 076	77	9.21 657	79	10.78 343	9.99 419	39						
22	9.21 153	76	9.21 736	78	10.78 264	9.99 417	38						
23	9.21 229	77	9.21 814	79	10.78 186	9.99 415	37						
24	9.21 306	76	9.21 893	78	10.78 107	9.99 413	36						
25	9.21 382	76	9.21 971	78	10.78 029	9.99 411	35						
26	9.21 458	76	9.22 049	78	10.77 951	9.99 409	34						
27	9.21 534	76	9.22 127	78	10.77 873	9.99 407	33						
28	9.21 610	75	9.22 205	78	10.77 795	9.99 404	32						
29	9.21 685	76	9.22 283	78	10.77 717	9.99 402	31						
30	9.21 761	75	9.22 361	77	10.77 639	9.99 400	30						
31	9.21 836	75	9.22 438	77	10.77 562	9.99 398	29						
32	9.21 912	75	9.22 516	77	10.77 484	9.99 396	28						
33	9.21 987	75	9.22 593	77	10.77 407	9.99 394	27						
34	9.22 062	75	9.22 670	77	10.77 330	9.99 392	26						
35	9.22 137	74	9.22 747	77	10.77 253	9.99 390	25						
36	9.22 211	75	9.22 824	77	10.77 176	9.99 388	24						
37	9.22 286	75	9.22 901	76	10.77 099	9.99 385	23						
38	9.22 361	74	9.22 977	77	10.77 023	9.99 383	22						
39	9.22 435	74	9.23 054	76	10.76 946	9.99 381	21						
40	9.22 509	74	9.23 130	76	10.76 870	9.99 379	20						
41	9.22 583	74	9.23 206	77	10.76 794	9.99 377	19						
42	9.22 657	74	9.23 283	77	10.76 717	9.99 375	18						
43	9.22 731	74	9.23 359	78	10.76 641	9.99 372	17						
44	9.22 805	73	9.23 435	78	10.76 565	9.99 370	16						
45	9.22 878	74	9.23 510	76	10.76 490	9.99 368	15						
46	9.22 952	73	9.23 586	75	10.76 414	9.99 366	14						
47	9.23 025	73	9.23 661	75	10.76 339	9.99 364	13						
48	9.23 098	73	9.23 737	75	10.76 263	9.99 362	12						
49	9.23 171	73	9.23 812	75	10.76 188	9.99 359	11						
50	9.23 244	73	9.23 887	74	10.76 113	9.99 357	10						
51	9.23 317	73	9.23 962	74	10.76 038	9.99 355	9						
52	9.23 380	72	9.24 037	75	10.75 963	9.99 353	8						
53	9.23 462	73	9.24 112	74	10.75 888	9.99 351	7						
54	9.23 535	72	9.24 186	75	10.75 814	9.99 348	6						
55	9.23 607	72	9.24 261	74	10.75 739	9.99 346	5						
56	9.23 679	73	9.24 335	74	10.75 665	9.99 344	4						
57	9.23 752	71	9.24 410	74	10.75 590	9.99 342	3						
58	9.23 823	72	9.24 484	74	10.75 516	9.99 340	2						
59	9.23 895	72	9.24 558	74	10.75 442	9.99 337	1						
60	9.23 967		9.24 632		10.75 368	9.99 335	0						
	L Cos	d	L Ctn	cd	L Tan	L Sin		Proportional parts					

90° (279°)

(260°) 86°

Table 6 COMMON LOGARITHMS OF TRIGONOMETRIC FUNCTIONS (continued) 227

The -10 portion of the characteristic of the logarithm is not printed but must be written down whenever such a logarithm is used.

10° (190°)										(149°) 169°									
	L Sin.	d	L Tan	c d	L Ctn	L Cos	d			Proportional parts									
0	9.23 957	72	9.24 632	74	10.75 366	9.99 336	2	60											
1	9.24 039	71	9.24 709	73	10.75 294	9.99 333	2	59											
2	9.24 110	71	9.24 779	74	10.75 221	9.99 331	3	58											
3	9.24 181	72	9.24 853	73	10.75 147	9.99 328	2	57											
4	9.24 253	71	9.24 926	74	10.75 074	9.99 326	2	56											
5	9.24 324	71	9.25 000	73	10.75 000	9.99 324	2	55											
6	9.24 395	71	9.25 073	73	10.74 927	9.99 322	3	54											
7	9.24 466	70	9.25 146	73	10.74 854	9.99 319	2	53											
8	9.24 538	71	9.25 219	73	10.74 781	9.99 317	2	52											
9	9.24 607	70	9.25 292	73	10.74 708	9.99 315	2	51											
10	9.24 677	71	9.25 365	72	10.74 635	9.99 313	3	50											
11	9.24 749	70	9.25 437	73	10.74 563	9.99 310	2	49											
12	9.24 818	70	9.25 510	72	10.74 490	9.99 308	2	48											
13	9.24 888	70	9.25 582	73	10.74 418	9.99 306	2	47											
14	9.24 958	70	9.25 655	72	10.74 345	9.99 304	3	46											
15	9.25 028	70	9.25 727	72	10.74 273	9.99 301	2	45											
16	9.25 098	70	9.25 799	72	10.74 201	9.99 299	2	44											
17	9.25 168	69	9.25 871	72	10.74 129	9.99 297	3	43											
18	9.25 237	70	9.25 943	72	10.74 057	9.99 294	2	42											
19	9.25 307	69	9.26 015	71	10.73 985	9.99 292	2	41											
20	9.25 375	69	9.26 086	72	10.73 914	9.99 290	2	40											
21	9.25 445	69	9.26 158	71	10.73 842	9.99 288	3	39											
22	9.25 514	69	9.26 229	72	10.73 771	9.99 285	2	38											
23	9.25 583	69	9.26 301	71	10.73 699	9.99 283	2	37											
24	9.25 652	69	9.26 372	71	10.73 628	9.99 281	3	36											
25	9.25 721	69	9.26 443	71	10.73 557	9.99 278	2	35											
26	9.25 790	68	9.26 514	71	10.73 486	9.99 276	2	34											
27	9.25 859	69	9.26 585	70	10.73 415	9.99 274	3	33											
28	9.25 927	68	9.26 656	71	10.73 345	9.99 271	2	32											
29	9.25 995	68	9.26 726	71	10.73 274	9.99 269	2	31											
30	9.26 063	68	9.26 797	70	10.73 203	9.99 267	3	30											
31	9.26 131	68	9.26 867	70	10.73 133	9.99 264	2	29											
32	9.26 199	68	9.26 937	71	10.73 063	9.99 262	2	28											
33	9.26 267	68	9.27 008	70	10.72 992	9.99 260	3	27											
34	9.26 335	68	9.27 078	70	10.72 922	9.99 257	2	26											
35	9.26 403	67	9.27 148	70	10.72 852	9.99 255	3	25											
36	9.26 470	68	9.27 218	70	10.72 782	9.99 252	2	24											
37	9.26 538	67	9.27 288	69	10.72 712	9.99 250	2	23											
38	9.26 605	67	9.27 357	70	10.72 643	9.99 248	3	22											
39	9.26 672	67	9.27 427	69	10.72 573	9.99 245	2	21											
40	9.26 739	67	9.27 496	70	10.72 504	9.99 243	2	20											
41	9.26 806	67	9.27 565	69	10.72 434	9.99 241	3	19											
42	9.26 873	67	9.27 635	69	10.72 365	9.99 238	2	18											
43	9.26 940	67	9.27 704	69	10.72 296	9.99 236	3	17											
44	9.27 007	66	9.27 773	69	10.72 227	9.99 233	2	16											
45	9.27 073	67	9.27 842	69	10.72 158	9.99 231	2	15											
46	9.27 140	66	9.27 911	69	10.72 089	9.99 229	3	14											
47	9.27 206	67	9.27 980	69	10.72 020	9.99 226	2	13											
48	9.27 273	66	9.28 049	68	10.71 951	9.99 224	3	12											
49	9.27 339	66	9.28 117	69	10.71 883	9.99 221	2	11											
50	9.27 405	66	9.28 186	68	10.71 814	9.99 219	7	10											
51	9.27 471	66	9.28 254	69	10.71 746	9.99 217	3	9											
52	9.27 537	65	9.28 323	68	10.71 677	9.99 214	2	8											
53	9.27 602	66	9.28 391	68	10.71 609	9.99 212	1	7											
54	9.27 668	66	9.28 459	68	10.71 541	9.99 209	2	6											
55	9.27 734	65	9.28 527	68	10.71 473	9.99 207	1	5											
56	9.27 799	65	9.28 595	67	10.71 405	9.99 204	2	4											
57	9.27 864	66	9.28 662	68	10.71 338	9.99 202	3	3											
58	9.27 930	65	9.28 730	68	10.71 270	9.99 200	2	2											
59	9.27 995	65	9.28 798	67	10.71 202	9.99 197	2	1											
60	9.28 060		9.28 865		10.71 135	9.99 195		0											
	L Cos	d	L Ctn	c d	L Tan	L Sin	d			Proportional parts									

100° (280°)

(149°) 149°

Table 6 COMMON LOGARITHMS OF TRIGONOMETRIC FUNCTIONS (continued) 228

The - 10 portion of the logarithm is not printed but must be written down whenever such a logarithm is used.

11° (212')						(348°) 168°					
'	L Sin	d	L Tan	c d	L Ctn	L Cos	d	'	Proportional parts		
0	9.28 060	65	9.28 865	68	10.71 135	9.99 195	3	60			
1	9.28 125	65	9.28 933	67	10.71 067	9.99 192	2	59			
2	9.28 190	64	9.28 000	67	10.71 000	9.99 190	3	58			
3	9.28 254	65	9.28 067	67	10.70 933	9.99 187	2	57			
4	9.28 319	65	9.28 134	67	10.70 866	9.99 185	3	56			
5	9.28 384	64	9.28 201	67	10.70 799	9.99 182	2	55			
6	9.28 448	64	9.28 268	67	10.70 732	9.99 180	3	54			
7	9.28 512	65	9.28 335	67	10.70 665	9.99 177	2	53			
8	9.28 577	64	9.28 402	66	10.70 598	9.99 175	3	52			
9	9.28 641	64	9.28 468	67	10.70 532	9.99 172	2	51			
10	9.28 705	64	9.28 535	66	10.70 465	9.99 170	3	50			
11	9.28 769	64	9.28 601	67	10.70 399	9.99 167	2	49			
12	9.28 833	65	9.28 668	68	10.70 332	9.99 165	3	48			
13	9.28 898	64	9.28 734	66	10.70 266	9.99 162	2	47			
14	9.28 960	64	9.28 800	66	10.70 200	9.99 160	3	46			
15	9.29 024	65	9.29 866	66	10.70 134	9.99 157	2	45			
16	9.29 087	65	9.29 932	66	10.70 068	9.99 155	3	44			
17	9.29 150	64	9.29 998	65	10.70 002	9.99 152	2	43			
18	9.29 214	64	9.30 064	65	10.69 936	9.99 150	3	42			
19	9.29 277	63	9.30 130	65	10.69 870	9.99 147	2	41			
20	9.29 340	63	9.30 195	65	10.69 805	9.99 145	3	40			
21	9.29 403	63	9.30 261	65	10.69 739	9.99 142	2	39			
22	9.29 466	63	9.30 325	65	10.69 674	9.99 140	3	38			
23	9.29 529	62	9.30 391	65	10.69 608	9.99 137	2	37			
24	9.29 591	62	9.30 457	65	10.69 543	9.99 135	3	36			
25	9.29 654	62	9.30 522	65	10.69 478	9.99 132	2	35			
26	9.29 716	61	9.30 587	65	10.69 413	9.99 130	3	34			
27	9.29 779	62	9.30 652	65	10.69 348	9.99 127	2	33			
28	9.29 841	62	9.30 717	65	10.69 283	9.99 124	3	32			
29	9.29 903	61	9.30 782	64	10.69 218	9.99 122	2	31			
30	9.29 966	62	9.30 846	65	10.69 154	9.99 119	3	30			
31	9.30 028	62	9.30 911	64	10.69 089	9.99 117	2	29			
32	9.30 090	61	9.30 975	65	10.69 025	9.99 114	3	28			
33	9.30 151	62	9.31 040	64	10.68 960	9.99 112	2	27			
34	9.30 213	62	9.31 104	64	10.68 896	9.99 109	3	26			
35	9.30 275	61	9.31 168	65	10.68 832	9.99 106	2	25			
36	9.30 336	62	9.31 233	64	10.68 767	9.99 104	3	24			
37	9.30 398	61	9.31 297	64	10.68 703	9.99 101	2	23			
38	9.30 460	61	9.31 361	64	10.68 639	9.99 099	3	22			
39	9.30 521	61	9.31 425	64	10.68 575	9.99 096	2	21			
40	9.30 582	61	9.31 489	63	10.68 511	9.99 093	3	20			
41	9.30 643	61	9.31 552	64	10.68 448	9.99 091	2	19			
42	9.30 704	61	9.31 615	63	10.68 384	9.99 088	3	18			
43	9.30 765	61	9.31 679	64	10.68 321	9.99 086	2	17			
44	9.30 826	61	9.31 743	63	10.68 257	9.99 083	3	16			
45	9.30 887	60	9.31 806	64	10.68 194	9.99 080	2	15			
46	9.30 947	61	9.31 870	63	10.68 130	9.99 078	3	14			
47	9.31 008	60	9.31 933	63	10.68 067	9.99 076	2	13			
48	9.31 068	60	9.31 996	63	10.68 004	9.99 072	3	12			
49	9.31 129	60	9.32 059	63	10.67 941	9.99 070	2	11			
50	9.31 189	61	9.32 122	63	10.67 878	9.99 067	3	10			
51	9.31 250	60	9.32 185	63	10.67 815	9.99 064	2	9			
52	9.31 310	60	9.32 248	63	10.67 752	9.99 062	3	8			
53	9.31 370	60	9.32 311	62	10.67 689	9.99 059	2	7			
54	9.31 430	60	9.32 373	63	10.67 627	9.99 056	3	6			
55	9.31 490	59	9.32 436	62	10.67 564	9.99 054	2	5			
56	9.31 549	60	9.32 498	63	10.67 502	9.99 051	3	4			
57	9.31 608	60	9.32 561	62	10.67 439	9.99 048	2	3			
58	9.31 668	59	9.32 623	62	10.67 377	9.99 046	3	2			
59	9.31 726	60	9.32 685	62	10.67 315	9.99 043	2	1			
60	9.31 788		9.32 747		10.67 253	9.99 040		0			
'	L Cos	d	L Ctn	c d	L Tan	L Sin	d	'	Proportional parts		

101° (281')

(258°) 76°

Table 6 COMMON LOGARITHMS OF TRIGONOMETRIC FUNCTIONS (continued) 229

The -10 portion of the characteristic of the logarithm is not printed but must be written down whenever such a logarithm is used.

12° (192°)										(347°) 167°									
°	L Sin			L Tan			L Ctn			L Cos			Proportional parts						
	d			d			d			d									
0	9.31	788	59	9.32	747	62	10.67	253	9.98	040	2	60							
1	9.31	847	60	9.32	810	62	10.67	190	9.98	038	3	59							
2	9.31	907	59	9.32	872	61	10.67	128	9.98	035	3	58							
3	9.31	966	58	9.32	933	62	10.67	067	9.98	032	2	57							
4	9.32	025	59	9.32	995	62	10.67	005	9.98	030	3	56							
5	9.32	084	59	9.33	067	62	10.66	943	9.98	027	3	55							
6	9.32	143	58	9.33	119	61	10.66	881	9.98	024	2	54							
7	9.32	202	58	9.33	180	62	10.66	820	9.98	022	3	53							
8	9.32	261	58	9.33	242	61	10.66	758	9.98	019	3	52							
9	9.32	319	59	9.33	303	62	10.66	697	9.98	016	3	51							
10	9.32	378	59	9.33	365	61	10.66	635	9.98	013	2	50							
11	9.32	437	58	9.33	426	61	10.66	574	9.98	011	3	49							
12	9.32	495	58	9.33	487	61	10.66	513	9.98	008	3	49							
13	9.32	553	58	9.33	548	61	10.66	452	9.98	005	3	47							
14	9.32	612	58	9.33	609	61	10.66	391	9.98	002	2	48							
15	9.32	670	58	9.33	670	61	10.66	330	9.98	000	3	45							
16	9.32	728	58	9.33	731	61	10.66	269	9.98	997	3	44							
17	9.32	786	58	9.33	792	61	10.66	208	9.98	994	3	43							
18	9.32	844	58	9.33	853	60	10.66	147	9.98	991	2	42							
19	9.32	902	58	9.33	913	61	10.66	087	9.98	989	3	41							
20	9.32	960	58	9.33	974	60	10.66	026	9.98	986	3	40							
21	9.33	018	57	9.34	034	61	10.65	966	9.98	983	3	39							
22	9.33	075	58	9.34	095	60	10.65	905	9.98	980	2	38							
23	9.33	133	57	9.34	155	60	10.65	845	9.98	978	3	37							
24	9.33	190	58	9.34	215	61	10.65	785	9.98	975	3	36							
25	9.33	248	57	9.34	276	60	10.65	724	9.98	972	3	35							
26	9.33	305	57	9.34	336	60	10.65	664	9.98	969	2	34							
27	9.33	362	58	9.34	396	60	10.65	604	9.98	967	3	33							
28	9.33	420	57	9.34	456	60	10.65	544	9.98	964	3	32							
29	9.33	477	57	9.34	516	60	10.65	484	9.98	961	3	31							
30	9.33	534	57	9.34	576	59	10.65	424	9.98	958	3	30							
31	9.33	591	56	9.34	635	60	10.65	365	9.98	955	2	29							
32	9.33	647	57	9.34	695	60	10.65	305	9.98	953	3	28							
33	9.33	704	57	9.34	755	59	10.65	245	9.98	950	3	27							
34	9.33	761	57	9.34	814	60	10.65	186	9.98	947	3	26							
35	9.33	818	56	9.34	874	59	10.65	126	9.98	944	3	25							
36	9.33	874	57	9.34	933	59	10.65	067	9.98	941	2	24							
37	9.33	931	56	9.34	992	59	10.65	008	9.98	938	2	23							
38	9.33	987	56	9.35	051	60	10.64	949	9.98	936	3	22							
39	9.34	043	57	9.35	111	59	10.64	889	9.98	933	3	21							
40	9.34	100	56	9.35	170	59	10.64	830	9.98	930	3	20							
41	9.34	156	56	9.35	229	59	10.64	771	9.98	927	3	19							
42	9.34	212	56	9.35	288	59	10.64	712	9.98	924	3	18							
43	9.34	268	56	9.35	347	58	10.64	653	9.98	921	2	17							
44	9.34	324	56	9.35	405	59	10.64	595	9.98	919	3	16							
45	9.34	380	56	9.35	464	59	10.64	536	9.98	916	3	15							
46	9.34	436	55	9.35	523	59	10.64	477	9.98	913	3	14							
47	9.34	491	56	9.35	581	59	10.64	419	9.98	910	3	13							
48	9.34	547	55	9.35	640	58	10.64	360	9.98	907	3	12							
49	9.34	602	56	9.35	698	59	10.64	302	9.98	904	3	11							
50	9.34	658	55	9.35	757	58	10.64	243	9.98	901	3	10							
51	9.34	713	56	9.35	815	58	10.64	185	9.98	898	2	9							
52	9.34	769	55	9.35	873	58	10.64	127	9.98	896	3	8							
53	9.34	824	55	9.35	931	58	10.64	069	9.98	893	3	7							
54	9.34	879	55	9.35	989	58	10.64	011	9.98	890	3	6							
55	9.34	934	55	9.36	047	58	10.63	953	9.98	887	3	5							
56	9.34	989	54	9.36	105	58	10.63	895	9.98	884	3	4							
57	9.35	044	55	9.36	163	58	10.63	837	9.98	881	3	3							
58	9.35	098	55	9.36	221	58	10.63	779	9.98	878	3	2							
59	9.35	154	55	9.36	279	57	10.63	721	9.98	875	3	1							
60	9.35	209		9.36	338		10.63	664	9.98	872		0							
°	L Cos			L Ctn			L Tan			L Sin			Proportional parts						
d				d			d			d									

102° (282°)

(257°) 77°

Table 6 COMMON LOGARITHMS OF TRIGONOMETRIC FUNCTIONS (continued) 230

The -10 portion of the characteristic of the logarithm is not printed but must be written down whenever such a logarithm is used.

13° (193°)				(346°) 166°							
'	L Sin	d	L Tan	c d	L Ctn	L Cos	d	Proportional parts			
0	9.35 209	54	9.36 336	58	10.63 664	9.98 872	3	60			
1	9.35 263	55	9.36 394	59	10.63 606	9.98 889	2	58			
2	9.35 318	55	9.36 452	57	10.63 548	9.98 897	3	58			
3	9.35 373	54	9.36 509	57	10.63 491	9.98 894	2	57			
4	9.35 427	54	9.36 566	58	10.63 434	9.98 881	2	56			
5	9.35 481	55	9.36 624	57	10.63 376	9.98 886	1	55			
6	9.35 536	54	9.36 681	57	10.63 319	9.98 885	3	54			
7	9.35 590	54	9.36 738	57	10.63 262	9.98 882	3	53			
8	9.35 644	54	9.36 795	57	10.63 205	9.98 848	3	52			
9	9.35 698	54	9.36 852	57	10.63 148	9.98 846	3	51			
10	9.35 752	54	9.36 909	57	10.63 091	9.98 843	3	50			
11	9.35 805	54	9.36 966	57	10.63 034	9.98 840	3	49			
12	9.35 860	54	9.37 023	57	10.62 977	9.98 837	3	48			
13	9.35 914	54	9.37 080	57	10.62 920	9.98 834	3	47			
14	9.35 968	54	9.37 137	56	10.62 863	9.98 831	3	46			
15	9.36 022	53	9.37 193	57	10.62 807	9.98 828	3	45			
16	9.36 075	53	9.37 250	56	10.62 750	9.98 825	3	44			
17	9.36 129	53	9.37 306	57	10.62 694	9.98 822	3	43			
18	9.36 182	53	9.37 363	56	10.62 637	9.98 819	3	42			
19	9.36 236	53	9.37 419	57	10.62 581	9.98 816	3	41			
20	9.36 289	53	9.37 476	56	10.62 524	9.98 813	3	40			
21	9.36 342	53	9.37 532	56	10.62 468	9.98 810	3	39			
22	9.36 395	53	9.37 588	56	10.62 412	9.98 807	3	38			
23	9.36 449	53	9.37 644	56	10.62 356	9.98 804	3	37			
24	9.36 502	53	9.37 700	56	10.62 300	9.98 801	3	36			
25	9.36 555	53	9.37 756	56	10.62 244	9.98 798	3	35			
26	9.36 608	53	9.37 812	56	10.62 188	9.98 795	3	34			
27	9.36 660	53	9.37 868	56	10.62 132	9.98 792	3	33			
28	9.36 713	53	9.37 924	56	10.62 076	9.98 789	3	32			
29	9.36 766	53	9.37 980	55	10.62 020	9.98 786	3	31			
30	9.36 819	52	9.38 035	56	10.61 965	9.98 783	3	30			
31	9.36 871	52	9.38 091	56	10.61 909	9.98 780	3	29			
32	9.36 924	52	9.38 147	55	10.61 853	9.98 777	3	28			
33	9.36 976	52	9.38 202	55	10.61 798	9.98 774	3	27			
34	9.37 028	52	9.38 257	56	10.61 743	9.98 771	3	26			
35	9.37 081	52	9.38 313	55	10.61 687	9.98 768	3	25			
36	9.37 133	52	9.38 368	55	10.61 632	9.98 765	3	24			
37	9.37 185	52	9.38 423	55	10.61 577	9.98 762	3	23			
38	9.37 237	52	9.38 479	55	10.61 521	9.98 759	3	22			
39	9.37 289	52	9.38 534	55	10.61 466	9.98 756	3	21			
40	9.37 341	52	9.38 589	55	10.61 411	9.98 753	3	20			
41	9.37 393	52	9.38 644	55	10.61 356	9.98 750	3	19			
42	9.37 445	52	9.38 699	55	10.61 301	9.98 746	3	18			
43	9.37 497	52	9.38 754	54	10.61 246	9.98 743	3	17			
44	9.37 549	51	9.38 808	55	10.61 192	9.98 740	3	16			
45	9.37 600	52	9.38 863	55	10.61 137	9.98 737	3	15			
46	9.37 652	51	9.38 918	54	10.61 082	9.98 734	3	14			
47	9.37 703	52	9.38 972	55	10.61 028	9.98 731	3	13			
48	9.37 755	51	9.39 027	55	10.60 973	9.98 728	3	12			
49	9.37 806	52	9.39 082	54	10.60 918	9.98 725	3	11			
50	9.37 858	51	9.39 136	54	10.60 864	9.98 722	3	10			
51	9.37 909	51	9.39 190	53	10.60 810	9.98 719	4	9			
52	9.37 960	51	9.39 245	54	10.60 755	9.98 715	3	8			
53	9.38 011	51	9.39 299	54	10.60 701	9.98 712	3	7			
54	9.38 062	51	9.39 353	54	10.60 647	9.98 709	3	6			
55	9.38 113	51	9.39 407	54	10.60 593	9.98 706	3	5			
56	9.38 164	51	9.39 461	54	10.60 539	9.98 703	3	4			
57	9.38 215	51	9.39 515	54	10.60 485	9.98 700	3	3			
58	9.38 266	51	9.39 569	54	10.60 431	9.98 697	3	2			
59	9.38 317	51	9.39 623	54	10.60 377	9.98 694	4	1			
60	9.38 368		9.39 677		10.60 323	9.98 690		0			
'	L Cos	d	L Ctn	c d	L Tan	L Sin	d	Proportional parts			

198° (283°)

(256°) 76°

Table 6 COMMON LOGARITHMS OF TRIGONOMETRIC FUNCTIONS (continued) 231

The — 10 portion of the characteristic of the logarithm is not printed but must be written down whenever such a logarithm is used.

14° (194°)				(345°) 165°				Proportional parts				
d	L Sin	d	L Tan	c d	L Ctn	L Cos	d	d	d	d	d	d
0	9.38 368	50	9.39 677	54	10.60 323	9.98 690	3	60				
1	9.38 418	51	9.39 731	54	10.60 269	9.98 687	3	59				
2	9.38 469	50	9.39 785	53	10.60 215	9.98 684	3	58				
3	9.38 519	51	9.39 838	54	10.60 162	9.98 681	3	57				
4	9.38 570	50	9.39 892	53	10.60 108	9.98 678	3	56				
5	9.38 620	50	9.39 945	54	10.60 055	9.98 675	4	55				
6	9.38 670	51	9.39 999	53	10.60 001	9.98 671	3	54				
7	9.38 721	50	9.40 052	54	10.59 946	9.98 668	3	53				
8	9.38 771	50	9.40 106	53	10.59 894	9.98 665	3	52				
9	9.38 821	50	9.40 159	53	10.59 841	9.98 662	3	51				
10	9.38 871	50	9.40 212	54	10.59 788	9.98 659	3	50				
11	9.38 921	50	9.40 266	53	10.59 734	9.98 656	4	49				
12	9.38 971	50	9.40 319	53	10.59 681	9.98 652	3	48				
13	9.39 021	50	9.40 372	53	10.59 628	9.98 649	3	47				
14	9.39 071	50	9.40 425	53	10.59 575	9.98 646	3	46				
15	9.39 121	49	9.40 478	53	10.59 522	9.98 643	3	45				
16	9.39 170	50	9.40 531	53	10.59 469	9.98 640	4	44				
17	9.39 220	50	9.40 584	52	10.59 416	9.98 636	3	43				
18	9.39 270	49	9.40 636	52	10.59 364	9.98 633	3	42				
19	9.39 319	50	9.40 689	52	10.59 311	9.98 630	3	41				
20	9.39 369	49	9.40 742	52	10.59 258	9.98 627	4	40				
21	9.39 418	49	9.40 795	52	10.59 205	9.98 623	3	39				
22	9.39 467	50	9.40 847	52	10.59 153	9.98 620	3	38				
23	9.39 517	49	9.40 899	52	10.59 100	9.98 617	3	37				
24	9.39 566	49	9.40 952	52	10.59 048	9.98 614	4	36				
25	9.39 615	49	9.41 005	52	10.58 995	9.98 610	3	35				
26	9.39 664	49	9.41 057	52	10.58 943	9.98 607	3	34				
27	9.39 713	49	9.41 109	52	10.58 891	9.98 604	3	33				
28	9.39 762	48	9.41 161	52	10.58 839	9.98 601	4	32				
29	9.39 811	49	9.41 214	52	10.58 786	9.98 597	3	31				
30	9.39 860	49	9.41 266	52	10.58 734	9.98 594	3	30				
31	9.39 909	49	9.41 318	52	10.58 682	9.98 591	3	29				
32	9.39 958	48	9.41 370	52	10.58 630	9.98 588	4	28				
33	9.40 008	48	9.41 422	52	10.58 578	9.98 584	3	27				
34	9.40 056	48	9.41 474	52	10.58 526	9.98 581	3	26				
35	9.40 103	48	9.41 526	52	10.58 474	9.98 578	4	25				
36	9.40 152	48	9.41 578	51	10.58 422	9.98 574	3	24				
37	9.40 200	48	9.41 629	52	10.58 371	9.98 571	3	23				
38	9.40 249	48	9.41 681	52	10.58 319	9.98 568	3	22				
39	9.40 297	49	9.41 733	51	10.58 267	9.98 565	4	21				
40	9.40 346	48	9.41 784	52	10.58 216	9.98 561	3	20				
41	9.40 394	48	9.41 836	51	10.58 164	9.98 558	3	19				
42	9.40 442	48	9.41 887	52	10.58 113	9.98 555	4	18				
43	9.40 490	48	9.41 939	51	10.58 061	9.98 551	3	17				
44	9.40 538	48	9.41 990	51	10.58 010	9.98 548	3	16				
45	9.40 586	48	9.42 041	52	10.57 959	9.98 545	4	15				
46	9.40 634	48	9.42 093	51	10.57 907	9.98 541	3	14				
47	9.40 682	48	9.42 144	51	10.57 856	9.98 538	3	13				
48	9.40 730	48	9.42 195	51	10.57 805	9.98 535	4	12				
49	9.40 778	47	9.42 246	51	10.57 754	9.98 531	3	11				
50	9.40 825	48	9.42 297	51	10.57 703	9.98 528	3	10				
51	9.40 873	48	9.42 348	51	10.57 652	9.98 525	4	9				
52	9.40 921	47	9.42 399	51	10.57 601	9.98 521	3	8				
53	9.40 968	48	9.42 450	51	10.57 550	9.98 518	3	7				
54	9.41 016	47	9.42 501	51	10.57 499	9.98 515	4	6				
55	9.41 063	48	9.42 552	51	10.57 448	9.98 511	3	5				
56	9.41 111	47	9.42 603	50	10.57 397	9.98 508	3	4				
57	9.41 158	47	9.42 653	51	10.57 347	9.98 505	4	3				
58	9.41 206	47	9.42 704	51	10.57 296	9.98 501	3	2				
59	9.41 252	46	9.42 755	50	10.57 245	9.98 498	4	1				
60	9.41 300		9.42 805		10.57 195	9.98 494		0				
	L Cos	d	L Ctn	c d	L Tan	L Sin	d					

Proportional parts				
d	d	d	d	
1	0.9	0.9	0.9	
2	1.8	1.8	1.7	
3	2.7	2.6	2.6	
4	3.6	3.5	3.5	
5	4.5	4.4	4.3	
6	5.4	5.3	5.2	
7	6.3	6.2	6.1	
8	7.2	7.1	6.9	
9	8.1	8.0	7.8	
10	9.0	8.8	8.7	
20	18.0	17.7	17.3	
30	27.0	26.5	26.0	
40	36.0	35.3	34.7	
50	45.0	44.2	43.3	
	51	50	49	
1	0.8	0.8	0.8	
2	1.7	1.7	1.6	
3	2.5	2.5	2.4	
4	3.4	3.3	3.3	
5	4.2	4.2	4.1	
6	5.1	5.0	4.9	
7	6.0	5.8	5.7	
8	6.8	6.7	6.5	
9	7.6	7.5	7.4	
10	8.5	8.3	8.2	
20	17.0	16.7	16.3	
30	25.5	25.0	24.5	
40	34.0	33.3	32.7	
50	42.5	41.7	40.8	
	48	47	4	3
1	0.8	0.8	0.1	0.0
2	1.6	1.6	0.1	0.1
3	2.4	2.4	0.2	0.2
4	3.2	3.1	0.3	0.2
5	4.0	3.9	0.3	0.2
6	4.8	4.7	0.4	0.3
7	5.6	5.5	0.5	0.4
8	6.4	6.3	0.5	0.4
9	7.2	7.0	0.6	0.4
10	8.0	7.8	0.7	0.5
20	16.0	15.7	1.3	1.0
30	24.0	23.5	2.0	1.5
40	32.0	31.3	2.7	2.0
50	40.0	39.2	3.3	2.5

Table 6 COMMON LOGARITHMS OF TRIGONOMETRIC FUNCTIONS (continued) 232

The —10 portion of the characteristic of the logarithm is not printed but must be written down whenever such a logarithm is used.

15° (195°)				(344°) 164°				Proportional parts			
<i>n</i>	L Sin	<i>d</i>	L Tan	<i>c d</i>	L Ctn	L Cos	<i>d</i>	<i>n</i>	Proportional parts		
0	9.41 300	47	9.42 805	51	10.57 195	9.98 494	3	60			
1	9.41 347	47	9.42 856	50	10.57 144	9.98 491	3	59			
2	9.41 394	47	9.42 906	51	10.57 094	9.98 488	4	58			
3	9.41 441	47	9.42 957	50	10.57 043	9.98 484	3	57			
4	9.41 488	47	9.43 007	50	10.56 993	9.98 481	4	56			
5	9.41 535	47	9.43 057	51	10.56 943	9.98 477	3	55			
6	9.41 582	46	9.43 108	50	10.56 892	9.98 474	3	54			
7	9.41 628	47	9.43 158	50	10.56 842	9.98 471	4	53			
8	9.41 675	47	9.43 208	50	10.56 792	9.98 467	3	52			
9	9.41 722	46	9.43 258	50	10.56 742	9.98 464	4	51			
10	9.41 768	47	9.43 308	50	10.56 692	9.98 460	3	50			
11	9.41 815	46	9.43 358	50	10.56 642	9.98 457	4	49			
12	9.41 861	47	9.43 408	50	10.56 592	9.98 453	3	48			
13	9.41 908	46	9.43 458	50	10.56 542	9.98 450	3	47			
14	9.41 954	47	9.43 508	50	10.56 492	9.98 447	4	46			
15	9.42 001	46	9.43 558	49	10.56 442	9.98 443	3	45			
16	9.42 047	46	9.43 607	49	10.56 392	9.98 440	4	44			
17	9.42 093	47	9.43 657	50	10.56 343	9.98 436	3	43			
18	9.42 140	46	9.43 707	49	10.56 293	9.98 433	4	42			
19	9.42 186	46	9.43 756	50	10.56 244	9.98 429	3	41			
20	9.42 232	46	9.43 806	49	10.56 194	9.98 425	4	40			
21	9.42 278	46	9.43 855	50	10.56 145	9.98 422	3	39			
22	9.42 324	46	9.43 905	49	10.56 095	9.98 419	4	38			
23	9.42 370	46	9.43 954	50	10.56 046	9.98 415	3	37			
24	9.42 416	45	9.44 004	49	10.55 996	9.98 412	4	36			
25	9.42 461	46	9.44 053	49	10.55 947	9.98 409	3	35			
26	9.42 507	46	9.44 102	49	10.55 898	9.98 405	4	34			
27	9.42 553	46	9.44 151	50	10.55 849	9.98 402	3	33			
28	9.42 599	45	9.44 201	49	10.55 799	9.98 398	4	32			
29	9.42 644	46	9.44 250	49	10.55 750	9.98 395	3	31			
30	9.42 690	45	9.44 299	49	10.55 701	9.98 391	4	30			
31	9.42 735	46	9.44 348	49	10.55 652	9.98 388	3	29			
32	9.42 781	45	9.44 397	49	10.55 603	9.98 384	4	28			
33	9.42 826	46	9.44 446	49	10.55 554	9.98 381	3	27			
34	9.42 872	45	9.44 495	49	10.55 505	9.98 377	4	26			
35	9.42 917	45	9.44 544	48	10.55 456	9.98 373	3	25			
36	9.42 962	46	9.44 592	49	10.55 408	9.98 370	4	24			
37	9.43 008	45	9.44 641	49	10.55 359	9.98 366	3	23			
38	9.43 053	45	9.44 690	48	10.55 310	9.98 363	4	22			
39	9.43 098	45	9.44 738	49	10.55 262	9.98 359	3	21			
40	9.43 143	45	9.44 787	49	10.55 213	9.98 356	4	20			
41	9.43 188	45	9.44 836	48	10.55 164	9.98 352	3	19			
42	9.43 233	45	9.44 884	49	10.55 116	9.98 349	4	18			
43	9.43 278	45	9.44 933	48	10.55 067	9.98 345	3	17			
44	9.43 323	44	9.44 981	48	10.55 019	9.98 342	4	16			
45	9.43 367	45	9.45 029	49	10.54 971	9.98 338	3	15			
46	9.43 412	45	9.45 078	48	10.54 922	9.98 334	4	14			
47	9.43 457	45	9.45 126	48	10.54 874	9.98 331	3	13			
48	9.43 502	44	9.45 174	48	10.54 826	9.98 327	4	12			
49	9.43 546	45	9.45 222	49	10.54 778	9.98 324	3	11			
50	9.43 591	44	9.45 271	48	10.54 729	9.98 320	4	10			
51	9.43 635	45	9.45 319	48	10.54 681	9.98 317	3	9			
52	9.43 680	44	9.45 367	48	10.54 633	9.98 313	4	8			
53	9.43 724	45	9.45 415	48	10.54 585	9.98 309	3	7			
54	9.43 769	44	9.45 463	48	10.54 537	9.98 306	4	6			
55	9.43 813	44	9.45 511	48	10.54 489	9.98 302	3	5			
56	9.43 857	44	9.45 559	47	10.54 441	9.98 299	4	4			
57	9.43 901	45	9.45 606	48	10.54 394	9.98 295	3	3			
58	9.43 946	44	9.45 654	48	10.54 346	9.98 291	4	2			
59	9.43 990	44	9.45 702	48	10.54 298	9.98 288	3	1			
60	9.44 034		9.45 750		10.54 250	9.98 284		0			
<i>n</i>	L Cos	<i>d</i>	L Ctn	<i>c d</i>	L Tan	L Sin	<i>d</i>	<i>n</i>	Proportional parts		

105° (285°)

(254°) 74°

Table 6 COMMON LOGARITHMS OF TRIGONOMETRIC FUNCTIONS (continued)

The — 10 portion of the characteristic of the logarithm is not printed but must be written down whenever such a logarithm is used.

16° (196°)

(343°) 163°

°	L Sin		L Tan		L Ctn		L Cos		Proportional parts
	d	d	d	d	d	d	d		
0	9.44 034	44	9.45 750	47	10.54 250	9.98 284	3	60	
1	9.44 079	44	9.45 797	46	10.54 203	9.98 261	4	59	
2	9.44 122	44	9.45 848	47	10.54 155	9.98 277	4	58	
3	9.44 165	44	9.45 892	46	10.54 108	9.98 273	3	57	
4	9.44 210	43	9.45 940	47	10.54 060	9.98 270	4	56	
5	9.44 253	44	9.45 987	48	10.54 013	9.98 266	4	55	
6	9.44 297	44	9.45 035	47	10.53 965	9.98 262	3	54	48 47 46
7	9.44 341	44	9.45 082	46	10.53 918	9.98 259	4	53	1 0.8 0.8 0.8
8	9.44 385	43	9.45 130	47	10.53 870	9.98 255	4	52	2 1.6 1.6 1.5
9	9.44 428	43	9.45 177	47	10.53 823	9.98 251	3	51	3 2.4 2.4 2.3
10	9.44 472	44	9.45 224	47	10.53 776	9.98 248	4	50	4 3.2 3.1 3.1
11	9.44 515	43	9.45 271	46	10.53 729	9.98 244	1	49	
12	9.44 559	43	9.45 319	47	10.53 681	9.98 240	3	48	5 4.0 3.9 3.8
13	9.44 602	44	9.45 366	47	10.53 634	9.98 237	4	47	6 4.8 4.7 4.6
14	9.44 646	43	9.45 413	47	10.53 587	9.98 233	4	46	7 5.6 5.5 5.4
15	9.44 689	44	9.45 460	47	10.53 540	9.98 229	3	45	8 6.4 6.3 6.1
16	9.44 733	43	9.45 507	47	10.53 493	9.98 226	4	44	9 7.2 7.0 6.9
17	9.44 775	43	9.45 554	47	10.53 446	9.98 222	4	43	10 8.0 7.8 7.7
18	9.44 819	43	9.45 601	47	10.53 399	9.98 218	3	42	20 16.0 15.7 15.3
19	9.44 862	43	9.45 648	46	10.53 352	9.98 215	4	41	30 24.0 23.5 23.0
20	9.44 905	43	9.45 694	47	10.53 306	9.98 211	4	40	40 32.0 31.3 30.7
21	9.44 948	44	9.45 741	47	10.53 259	9.98 207	3	39	50 40.0 39.2 38.3
22	9.44 992	43	9.45 788	47	10.53 212	9.98 204	4	38	
23	9.45 035	42	9.45 835	46	10.53 165	9.98 200	4	37	45 44 43
24	9.45 077	43	9.45 881	47	10.53 118	9.98 196	4	36	1 0.8 0.7 0.7
25	9.45 120	43	9.45 928	47	10.53 072	9.98 192	3	35	2 1.6 1.5 1.4
26	9.45 163	43	9.45 975	46	10.53 025	9.98 189	4	34	3 2.2 2.2 2.2
27	9.45 206	43	9.47 021	47	10.52 979	9.98 185	4	33	4 3.0 2.9 2.9
28	9.45 249	43	9.47 068	46	10.52 932	9.98 181	4	32	
29	9.45 292	42	9.47 114	48	10.52 886	9.98 177	3	31	5 3.8 3.7 3.6
30	9.45 334	43	9.47 160	47	10.52 840	9.98 174	4	30	6 4.5 4.4 4.3
31	9.45 377	42	9.47 207	46	10.52 793	9.98 170	4	29	7 5.2 5.1 5.0
32	9.45 419	43	9.47 253	46	10.52 747	9.98 166	4	28	8 6.0 5.9 5.7
33	9.45 462	42	9.47 299	47	10.52 701	9.98 162	3	27	9 6.8 6.6 6.4
34	9.45 504	43	9.47 346	46	10.52 654	9.98 159	4	26	10 7.5 7.3 7.2
35	9.45 547	42	9.47 392	46	10.52 608	9.98 155	4	25	20 15.0 14.7 14.3
36	9.45 589	43	9.47 438	46	10.52 562	9.98 151	4	24	30 22.5 22.0 21.5
37	9.45 632	42	9.47 484	46	10.52 516	9.98 147	3	23	40 30.0 29.3 28.7
38	9.45 674	42	9.47 530	46	10.52 470	9.98 144	4	22	50 37.5 36.7 35.8
39	9.45 716	43	9.47 576	46	10.52 424	9.98 140	4	21	
40	9.45 758	43	9.47 622	46	10.52 378	9.98 136	4	20	42 41 4 3
41	9.45 801	42	9.47 668	46	10.52 332	9.98 132	3	19	1 0.7 0.7 0.7
42	9.45 843	42	9.47 714	46	10.52 286	9.98 129	4	18	2 1.4 1.4 0.1 0.1
43	9.45 885	42	9.47 760	46	10.52 240	9.98 125	4	17	3 2.1 2.0 0.2 0.2
44	9.45 927	42	9.47 806	46	10.52 194	9.98 121	4	16	4 2.8 2.7 0.1 0.2
45	9.45 969	42	9.47 852	45	10.52 148	9.98 117	4	15	5 3.5 3.4 0.3 0.2
46	9.46 011	42	9.47 897	46	10.52 103	9.98 113	3	14	6 4.2 4.1 0.4 0.3
47	9.46 053	42	9.47 943	46	10.52 057	9.98 110	4	13	7 4.9 4.8 0.5 0.4
48	9.46 095	41	9.47 989	46	10.52 011	9.98 106	4	12	8 5.6 5.5 0.5 0.4
49	9.46 136	42	9.48 035	45	10.51 965	9.98 102	4	11	9 6.3 6.2 0.6 0.4
50	9.46 178	42	9.48 080	46	10.51 920	9.98 098	4	10	10 7.0 6.8 0.7 0.5
51	9.46 220	42	9.48 126	45	10.51 874	9.98 094	4	9	20 14.0 13.7 1.3 1.0
52	9.46 262	41	9.48 171	46	10.51 829	9.98 090	3	8	30 21.0 20.5 2.0 1.5
53	9.46 303	42	9.48 217	45	10.51 783	9.98 087	4	7	40 28.0 27.3 2.7 2.0
54	9.46 345	41	9.48 262	43	10.51 738	9.98 083	4	6	50 35.0 34.2 3.3 2.5
55	9.46 386	42	9.48 307	46	10.51 693	9.98 079	4	5	
56	9.46 428	41	9.48 353	45	10.51 647	9.98 075	4	4	
57	9.46 469	42	9.48 398	45	10.51 602	9.98 071	4	3	
58	9.46 511	41	9.48 443	46	10.51 557	9.98 067	4	2	
59	9.46 552	42	9.48 489	43	10.51 511	9.98 063	3	1	
60	0.45 594		9.48 534		10.51 466	9.98 060		0	

106° (286°)

(253°) 73°

Proportional parts

Table 6 COMMON LOGARITHMS OF TRIGONOMETRIC FUNCTIONS (continued) 234

The — 10 portion of the characteristic of the logarithm is not printed but must be written down whenever such a logarithm is used.

17° (197°)

(342°) 162°

°	L Sin		d	L Tan		c d	L Ctn		L Cos		d	°	Proportional parts			
0	9.46	594	41	9.48	534	45	10.51	456	9.98	960	4	80				
1	9.46	635	41	9.48	579	45	10.51	421	9.98	956	4	59				
2	9.46	676	41	9.48	624	45	10.51	376	9.98	952	4	58				
3	9.46	717	41	9.48	669	45	10.51	331	9.98	948	4	57				
4	9.46	758	42	9.48	714	45	10.51	286	9.98	944	4	56				
5	9.46	800	42	9.48	759	45	10.51	241	9.98	940	4	55				
6	9.46	841	42	9.48	804	45	10.51	196	9.98	936	4	54				
7	9.46	882	42	9.48	849	45	10.51	151	9.98	932	4	53				
8	9.46	923	42	9.48	894	45	10.51	106	9.98	928	4	52				
9	9.46	964	42	9.48	939	45	10.51	61	9.98	924	4	51				
10	9.47	005	43	9.48	984	45	10.51	016	9.98	921	4	50				
11	9.47	045	43	9.49	029	44	10.50	971	9.98	917	4	49				
12	9.47	086	43	9.49	072	44	10.50	927	9.98	913	4	48				
13	9.47	127	43	9.49	115	44	10.50	882	9.98	909	4	47				
14	9.47	168	43	9.49	158	44	10.50	837	9.98	905	4	46				
15	9.47	209	43	9.49	207	44	10.50	792	9.98	901	4	45				
16	9.47	249	43	9.49	252	44	10.50	748	9.97	897	4	44				
17	9.47	290	43	9.49	295	44	10.50	703	9.97	893	4	43				
18	9.47	330	43	9.49	341	44	10.50	659	9.97	889	4	42				
19	9.47	371	43	9.49	385	44	10.50	615	9.97	885	4	41				
20	9.47	411	43	9.49	430	44	10.50	570	9.97	882	4	40				
21	9.47	452	43	9.49	474	44	10.50	526	9.97	878	4	39				
22	9.47	492	43	9.49	518	44	10.50	481	9.97	874	4	38				
23	9.47	533	43	9.49	563	44	10.50	437	9.97	870	4	37				
24	9.47	573	43	9.49	607	44	10.50	393	9.97	866	4	36				
25	9.47	613	43	9.49	652	44	10.50	348	9.97	862	4	35				
26	9.47	654	43	9.49	696	44	10.50	304	9.97	858	4	34				
27	9.47	694	43	9.49	740	44	10.50	260	9.97	854	4	33				
28	9.47	734	43	9.49	784	44	10.50	216	9.97	850	4	32				
29	9.47	774	43	9.49	828	44	10.50	172	9.97	846	4	31				
30	9.47	814	43	9.49	872	44	10.50	128	9.97	842	4	30				
31	9.47	854	43	9.49	916	44	10.50	84	9.97	838	4	29				
32	9.47	894	43	9.49	960	44	10.50	40	9.97	834	4	28				
33	9.47	934	43	9.50	004	44	10.49	96	9.97	830	4	27				
34	9.47	974	43	9.50	048	44	10.49	52	9.97	826	4	26				
35	9.48	014	43	9.50	092	44	10.49	908	9.97	822	4	25				
36	9.48	054	43	9.50	136	44	10.49	854	9.97	818	4	24				
37	9.48	094	43	9.50	180	44	10.49	800	9.97	814	4	23				
38	9.48	133	43	9.50	223	44	10.49	747	9.97	810	4	22				
39	9.48	173	43	9.50	267	44	10.49	693	9.97	806	4	21				
40	9.48	213	43	9.50	311	44	10.49	639	9.97	802	4	20				
41	9.48	252	43	9.50	355	44	10.49	585	9.97	798	4	19				
42	9.48	292	43	9.50	398	44	10.49	531	9.97	794	4	18				
43	9.48	332	43	9.50	442	44	10.49	477	9.97	790	4	17				
44	9.48	371	43	9.50	485	44	10.49	423	9.97	786	4	16				
45	9.48	411	43	9.50	528	44	10.49	369	9.97	782	4	15				
46	9.48	450	43	9.50	572	44	10.49	315	9.97	778	4	14				
47	9.48	490	43	9.50	616	44	10.49	261	9.97	774	4	13				
48	9.48	529	43	9.50	659	44	10.49	207	9.97	770	4	12				
49	9.48	568	43	9.50	703	44	10.49	153	9.97	766	4	11				
50	9.48	607	43	9.50	746	44	10.49	99	9.97	762	4	10				
51	9.48	647	43	9.50	789	44	10.49	45	9.97	758	4	9				
52	9.48	686	43	9.50	833	44	10.49	91	9.97	754	4	8				
53	9.48	725	43	9.50	876	44	10.49	37	9.97	750	4	7				
54	9.48	764	43	9.50	919	44	10.49	83	9.97	746	4	6				
55	9.48	803	43	9.50	962	44	10.49	29	9.97	742	4	5				
56	9.48	842	43	9.51	005	44	10.48	905	9.97	738	4	4				
57	9.48	881	43	9.51	048	44	10.48	851	9.97	734	4	3				
58	9.48	920	43	9.51	092	44	10.48	797	9.97	730	4	2				
59	9.48	959	43	9.51	135	44	10.48	743	9.97	726	4	1				
60	9.49	998		9.51	178		10.48	689	9.97	722		0				
°	L Cos	d	L Ctn	c d	L Tan	L Sin	d	°	Proportional parts							

107° (267°)

(252°) 72°

Table 6 COMMON LOGARITHMS OF TRIGONOMETRIC FUNCTIONS (continued) 235

The - 10 portion of the characteristic of the logarithm is not printed but must be written down whenever such a logarithm is used.

10° (190°)				(341°) 161°						
'	L Sin	d	L Tan	c d	L Ctn	L Cos	d	Proportional parts		
0	9.48 998	39	9.51 178	43	10.48 822	9.97 821	4	60		
1	9.49 037	39	9.51 221	43	10.48 779	9.97 817	5	69		
2	9.49 076	39	9.51 264	42	10.48 736	9.97 812	4	58		
3	9.49 115	38	9.51 306	43	10.48 694	9.97 808	4	87		
4	9.49 153	39	9.51 349	43	10.48 651	9.97 804	4	56		
5	9.49 192	39	9.51 392	43	10.48 608	9.97 800	4	55		
6	9.49 231	38	9.51 435	43	10.48 565	9.97 796	4	54		
7	9.49 269	39	9.51 478	42	10.48 522	9.97 792	4	53		
8	9.49 308	39	9.51 520	43	10.48 480	9.97 788	4	52		
9	9.49 347	38	9.51 563	43	10.48 437	9.97 784	5	51		
10	9.49 385	39	9.51 606	42	10.48 394	9.97 779	4	50		
11	9.49 424	38	9.51 648	43	10.48 352	9.97 775	4	49		
12	9.49 462	38	9.51 691	43	10.48 309	9.97 771	4	48		
13	9.49 500	39	9.51 734	42	10.48 266	9.97 767	4	47		
14	9.49 539	38	9.51 776	43	10.48 224	9.97 763	4	46		
15	9.49 577	38	9.51 819	42	10.48 181	9.97 758	5	45		
16	9.49 615	39	9.51 861	42	10.48 139	9.97 754	4	44		
17	9.49 654	38	9.51 903	43	10.48 097	9.97 750	4	43		
18	9.49 692	38	9.51 946	42	10.48 054	9.97 746	4	42		
19	9.49 730	38	9.51 988	43	10.48 012	9.97 742	4	41		
20	9.49 768	38	9.52 031	42	10.47 969	9.97 738	4	40		
21	9.49 806	38	9.52 073	42	10.47 927	9.97 734	5	39		
22	9.49 844	38	9.52 115	42	10.47 885	9.97 729	4	38		
23	9.49 882	38	9.52 157	43	10.47 843	9.97 725	4	37		
24	9.49 920	38	9.52 200	42	10.47 800	9.97 721	4	36		
25	9.49 958	38	9.52 242	42	10.47 758	9.97 717	4	35		
26	9.49 996	38	9.52 284	42	10.47 716	9.97 713	5	34		
27	9.50 034	38	9.52 326	43	10.47 674	9.97 708	4	33		
28	9.50 072	38	9.52 368	42	10.47 632	9.97 704	4	32		
29	9.50 110	38	9.52 410	42	10.47 590	9.97 700	4	31		
30	9.50 148	37	9.52 452	42	10.47 548	9.97 696	5	30		
31	9.50 185	38	9.52 494	42	10.47 506	9.97 691	4	29		
32	9.50 223	38	9.52 536	42	10.47 464	9.97 687	4	28		
33	9.50 261	37	9.52 578	42	10.47 422	9.97 683	4	27		
34	9.50 298	38	9.52 620	41	10.47 380	9.97 679	5	26		
35	9.50 336	38	9.52 661	42	10.47 339	9.97 674	4	25		
36	9.50 374	37	9.52 703	42	10.47 297	9.97 670	4	24		
37	9.50 411	38	9.52 745	42	10.47 255	9.97 666	4	23		
38	9.50 449	37	9.52 787	42	10.47 213	9.97 662	5	22		
39	9.50 486	37	9.52 829	41	10.47 171	9.97 657	4	21		
40	9.50 522	38	9.52 870	42	10.47 130	9.97 653	4	20		
41	9.50 561	37	9.52 912	41	10.47 088	9.97 648	4	19		
42	9.50 599	37	9.52 953	42	10.47 047	9.97 645	5	18		
43	9.50 635	38	9.52 995	42	10.47 005	9.97 640	4	17		
44	9.50 673	37	9.53 037	41	10.46 963	9.97 636	4	16		
45	9.50 710	37	9.53 078	42	10.46 922	9.97 632	4	15		
46	9.50 747	37	9.53 120	41	10.46 880	9.97 628	5	14		
47	9.50 784	37	9.53 161	41	10.46 839	9.97 623	4	13		
48	9.50 821	37	9.53 202	42	10.46 798	9.97 619	4	12		
49	9.50 858	38	9.53 244	41	10.46 756	9.97 615	5	11		
50	9.50 895	37	9.53 285	42	10.46 715	9.97 610	4	10		
51	9.50 933	37	9.53 327	41	10.46 673	9.97 606	4	9		
52	9.50 970	37	9.53 368	41	10.46 632	9.97 602	5	8		
53	9.51 007	36	9.53 409	41	10.46 591	9.97 597	4	7		
54	9.51 043	37	9.53 450	42	10.46 550	9.97 593	4	6		
55	9.51 080	37	9.53 492	41	10.46 508	9.97 589	5	5		
56	9.51 117	37	9.53 533	41	10.46 467	9.97 584	4	4		
57	9.51 154	37	9.53 574	41	10.46 426	9.97 580	4	3		
58	9.51 191	36	9.53 615	41	10.46 385	9.97 576	5	2		
59	9.51 227	37	9.53 656	41	10.46 344	9.97 571	4	1		
60	9.51 264		9.53 697		10.46 303	9.97 567		0		
'	L Cos	d	L Ctn	c d	L Tan	L Sin	d	Proportional parts		

108° (288°)

(251°) 71°

Table 6 COMMON LOGARITHMS OF TRIGONOMETRIC FUNCTIONS (continued) 236

The -10 portion of the characteristic of the logarithm is not printed but must be written down whenever such a logarithm is used.

19° (199°)				(340°) 160°				Proportional parts
θ	L Sin	d	L Tan	c d	L Ctn	L Cos	d	
0	9.51 264	37	9.53 897	41	10.46 303	9.97 567	4	60
1	9.51 301	37	9.53 738	41	10.46 262	9.97 563	4	59
2	9.51 338	36	9.53 779	41	10.46 221	9.97 568	4	58
3	9.51 374	37	9.53 820	41	10.46 180	9.97 554	4	57
4	9.51 411	36	9.53 861	41	10.46 139	9.97 550	5	56
5	9.51 447	37	9.53 902	41	10.46 098	9.97 545	4	55
6	9.51 484	36	9.53 943	41	10.46 057	9.97 541	5	54
7	9.51 520	37	9.53 984	41	10.46 016	9.97 536	4	53
8	9.51 557	36	9.54 025	40	10.45 975	9.97 532	4	52
9	9.51 593	36	9.54 065	41	10.45 935	9.97 528	5	51
10	9.51 629	37	9.54 106	41	10.45 894	9.97 523	4	50
11	9.51 666	36	9.54 147	40	10.45 853	9.97 519	4	49
12	9.51 702	36	9.54 187	41	10.45 813	9.97 515	5	48
13	9.51 738	36	9.54 228	41	10.45 772	9.97 510	4	47
14	9.51 774	37	9.54 268	40	10.45 731	9.97 506	5	46
15	9.51 811	36	9.54 309	41	10.45 691	9.97 501	4	45
16	9.51 847	36	9.54 350	40	10.45 650	9.97 497	5	44
17	9.51 883	36	9.54 390	41	10.45 610	9.97 492	4	43
18	9.51 919	36	9.54 431	40	10.45 569	9.97 488	4	42
19	9.51 955	36	9.54 471	41	10.45 529	9.97 484	5	41
20	9.51 991	36	9.54 512	40	10.45 488	9.97 479	4	40
21	9.52 027	36	9.54 552	41	10.45 448	9.97 475	5	39
22	9.52 063	36	9.54 593	40	10.45 407	9.97 470	4	38
23	9.52 099	36	9.54 633	40	10.45 367	9.97 466	5	37
24	9.52 135	36	9.54 673	41	10.45 327	9.97 461	4	36
25	9.52 171	36	9.54 714	40	10.45 286	9.97 457	4	35
26	9.52 207	36	9.54 754	40	10.45 246	9.97 453	5	34
27	9.52 242	36	9.54 794	41	10.45 206	9.97 448	4	33
28	9.52 278	36	9.54 835	40	10.45 165	9.97 444	5	32
29	9.52 314	36	9.54 875	40	10.45 125	9.97 439	4	31
30	9.52 350	35	9.54 915	40	10.45 085	9.97 435	5	30
31	9.52 385	35	9.54 955	40	10.45 045	9.97 430	4	29
32	9.52 421	35	9.54 995	40	10.45 005	9.97 426	5	28
33	9.52 456	36	9.55 035	40	10.44 965	9.97 421	4	27
34	9.52 492	35	9.55 075	40	10.44 925	9.97 417	5	26
35	9.52 527	36	9.55 115	40	10.44 885	9.97 412	4	25
36	9.52 563	35	9.55 155	40	10.44 845	9.97 408	5	24
37	9.52 598	36	9.55 195	40	10.44 805	9.97 403	4	23
38	9.52 634	35	9.55 235	40	10.44 765	9.97 399	5	22
39	9.52 669	36	9.55 275	40	10.44 725	9.97 394	4	21
40	9.52 705	35	9.55 315	40	10.44 685	9.97 390	5	20
41	9.52 740	35	9.55 355	40	10.44 645	9.97 385	4	19
42	9.52 775	36	9.55 395	39	10.44 605	9.97 381	5	18
43	9.52 811	35	9.55 434	40	10.44 566	9.97 376	4	17
44	9.52 846	35	9.55 474	40	10.44 526	9.97 372	5	16
45	9.52 881	35	9.55 514	40	10.44 486	9.97 367	4	15
46	9.52 916	35	9.55 554	39	10.44 446	9.97 363	5	14
47	9.52 951	35	9.55 593	40	10.44 407	9.97 358	5	13
48	9.52 986	35	9.55 633	40	10.44 367	9.97 353	4	12
49	9.53 021	35	9.55 672	39	10.44 327	9.97 349	5	11
50	9.53 056	36	9.55 712	40	10.44 288	9.97 344	4	10
51	9.53 092	34	9.55 752	39	10.44 248	9.97 340	5	9
52	9.53 126	35	9.55 791	40	10.44 209	9.97 335	4	8
53	9.53 161	35	9.55 831	39	10.44 169	9.97 331	5	7
54	9.53 195	35	9.55 870	40	10.44 130	9.97 326	4	6
55	9.53 231	35	9.55 910	39	10.44 090	9.97 322	5	5
56	9.53 266	35	9.55 949	40	10.44 051	9.97 317	5	4
57	9.53 301	35	9.55 989	39	10.44 011	9.97 312	4	3
58	9.53 336	36	9.56 028	39	10.43 972	9.97 308	5	2
59	9.53 370	35	9.56 067	40	10.43 933	9.97 303	4	1
60	9.53 405		9.56 107		10.43 893	9.97 299		0

19° (199°)		(250°) 70°		Proportional parts
θ	L Sin	L Cos	d	
0	9.51 264	9.53 897	41	60
1	9.51 301	9.53 738	41	59
2	9.51 338	9.53 779	41	58
3	9.51 374	9.53 820	41	57
4	9.51 411	9.53 861	41	56
5	9.51 447	9.53 902	41	55
6	9.51 484	9.53 943	41	54
7	9.51 520	9.53 984	41	53
8	9.51 557	9.54 025	40	52
9	9.51 593	9.54 065	41	51
10	9.51 629	9.54 106	41	50
11	9.51 666	9.54 147	40	49
12	9.51 702	9.54 187	41	48
13	9.51 738	9.54 228	41	47
14	9.51 774	9.54 268	40	46
15	9.51 811	9.54 309	41	45
16	9.51 847	9.54 350	40	44
17	9.51 883	9.54 390	41	43
18	9.51 919	9.54 431	40	42
19	9.51 955	9.54 471	41	41
20	9.51 991	9.54 512	40	40
21	9.52 027	9.54 552	41	39
22	9.52 063	9.54 593	40	38
23	9.52 099	9.54 633	40	37
24	9.52 135	9.54 673	41	36
25	9.52 171	9.54 714	40	35
26	9.52 207	9.54 754	40	34
27	9.52 242	9.54 794	41	33
28	9.52 278	9.54 835	40	32
29	9.52 314	9.54 875	40	31
30	9.52 350	9.54 915	40	30
31	9.52 385	9.54 955	40	29
32	9.52 421	9.54 995	40	28
33	9.52 456	9.55 035	40	27
34	9.52 492	9.55 075	40	26
35	9.52 527	9.55 115	40	25
36	9.52 563	9.55 155	40	24
37	9.52 598	9.55 195	40	23
38	9.52 634	9.55 235	40	22
39	9.52 669	9.55 275	40	21
40	9.52 705	9.55 315	40	20
41	9.52 740	9.55 355	40	19
42	9.52 775	9.55 395	39	18
43	9.52 811	9.55 434	40	17
44	9.52 846	9.55 474	40	16
45	9.52 881	9.55 514	40	15
46	9.52 916	9.55 554	39	14
47	9.52 951	9.55 593	40	13
48	9.52 986	9.55 633	40	12
49	9.53 021	9.55 672	39	11
50	9.53 056	9.55 712	40	10
51	9.53 092	9.55 752	39	9
52	9.53 126	9.55 791	40	8
53	9.53 161	9.55 831	39	7
54	9.53 195	9.55 870	40	6
55	9.53 231	9.55 910	39	5
56	9.53 266	9.55 949	40	4
57	9.53 301	9.55 989	39	3
58	9.53 336	9.56 028	39	2
59	9.53 370	9.56 067	40	1
60	9.53 405	9.56 107		0

309° (289°)

(250°) 70°

Table 6 COMMON LOGARITHMS OF TRIGONOMETRIC FUNCTIONS (continued) 237

The —10 portion of the characteristic of the logarithm is not printed but must be written down whenever such a logarithm is used.

20° (200°)						(339°) 159°					
	L. Sin	d	L. Tan	c d	L. Cot	L. Cos	d	Proportional parts			
0	9.53 405	35	9.55 107	39	10.43 893	9.97 230	5	60			
1	9.53 440	35	9.55 146	39	10.43 854	9.97 234	5	59			
2	9.53 475	34	9.55 185	38	10.43 815	9.97 238	4	58			
3	9.53 509	34	9.55 224	40	10.43 776	9.97 242	5	57			
4	9.53 544	34	9.55 264	39	10.43 736	9.97 246	4	56			
5	9.53 578	35	9.55 303	39	10.43 697	9.97 276	5	55			
6	9.53 613	34	9.55 342	38	10.43 658	9.97 271	5	54			
7	9.53 647	35	9.55 381	39	10.43 619	9.97 266	4	53			
8	9.53 682	34	9.55 420	39	10.43 580	9.97 262	5	52			
9	9.53 716	35	9.55 459	39	10.43 541	9.97 257	5	51			
10	9.53 751	34	9.55 498	39	10.43 502	9.97 252	4	50			
11	9.53 785	34	9.55 537	39	10.43 463	9.97 248	5	49			
12	9.53 819	35	9.55 576	39	10.43 424	9.97 243	5	48			
13	9.53 854	34	9.55 615	38	10.43 385	9.97 238	4	47			
14	9.53 888	34	9.55 654	39	10.43 346	9.97 234	5	46			
15	9.53 922	35	9.55 693	39	10.43 307	9.97 229	5	45			
16	9.53 957	34	9.55 732	35	10.43 268	9.97 224	4	44			
17	9.53 991	34	9.55 771	39	10.43 229	9.97 220	5	43			
18	9.54 025	34	9.55 810	38	10.43 190	9.97 215	5	42			
19	9.54 059	34	9.55 849	38	10.43 151	9.97 210	4	41			
20	9.54 093	34	9.55 887	39	10.43 113	9.97 206	5	40			
21	9.54 127	34	9.55 926	39	10.43 074	9.97 201	5	39			
22	9.54 161	34	9.55 965	39	10.43 035	9.97 196	4	38			
23	9.54 195	34	9.55 004	38	10.42 996	9.97 192	5	37			
24	9.54 229	34	9.55 042	39	10.42 958	9.97 187	5	36			
25	9.54 263	34	9.55 081	39	10.42 919	9.97 182	4	35			
26	9.54 297	34	9.55 120	38	10.42 880	9.97 178	5	34			
27	9.54 331	34	9.55 158	39	10.42 842	9.97 173	5	33			
28	9.54 365	34	9.55 197	38	10.42 803	9.97 168	5	32			
29	9.54 399	34	9.55 235	39	10.42 765	9.97 163	4	31			
30	9.54 433	33	9.55 274	38	10.42 726	9.97 159	5	30			
31	9.54 466	34	9.55 312	39	10.42 688	9.97 154	5	29			
32	9.54 500	34	9.55 351	38	10.42 649	9.97 149	4	28			
33	9.54 534	33	9.55 389	38	10.42 611	9.97 145	5	27			
34	9.54 567	34	9.55 428	38	10.42 572	9.97 140	5	26			
35	9.54 601	34	9.55 466	38	10.42 534	9.97 135	6	25			
36	9.54 635	33	9.55 504	39	10.42 496	9.97 130	4	24			
37	9.54 668	34	9.55 543	38	10.42 457	9.97 126	5	23			
38	9.54 702	33	9.55 581	38	10.42 419	9.97 121	5	22			
39	9.54 735	34	9.55 619	39	10.42 381	9.97 116	5	21			
40	9.54 769	33	9.55 658	38	10.42 342	9.97 111	4	20			
41	9.54 802	34	9.55 696	38	10.42 304	9.97 107	5	19			
42	9.54 836	33	9.55 734	38	10.42 266	9.97 102	5	18			
43	9.54 869	34	9.55 772	38	10.42 228	9.97 97	5	17			
44	9.54 903	33	9.55 810	39	10.42 190	9.97 92	5	16			
45	9.54 936	33	9.55 849	38	10.42 151	9.97 87	4	15			
46	9.54 969	34	9.55 887	38	10.42 113	9.97 83	5	14			
47	9.55 003	33	9.55 925	38	10.42 075	9.97 78	5	13			
48	9.55 036	33	9.55 963	38	10.42 037	9.97 73	5	12			
49	9.55 069	33	9.55 001	38	10.41 999	9.97 68	5	11			
50	9.55 102	34	9.55 039	36	10.41 961	9.97 63	4	10			
51	9.55 136	33	9.55 077	29	10.41 923	9.97 59	5	9			
52	9.55 169	33	9.55 115	38	10.41 885	9.97 54	5	8			
53	9.55 202	33	9.55 153	38	10.41 847	9.97 49	5	7			
54	9.55 235	33	9.55 191	38	10.41 809	9.97 44	5	6			
55	9.55 268	33	9.55 229	38	10.41 771	9.97 39	4	5			
56	9.55 301	33	9.55 267	37	10.41 733	9.97 35	5	4			
57	9.55 334	33	9.55 304	33	10.41 695	9.97 30	5	3			
58	9.55 367	33	9.55 342	35	10.41 658	9.97 25	5	2			
59	9.55 400	33	9.55 380	35	10.41 620	9.97 20	5	1			
60	9.55 433		9.55 418		10.41 582	9.97 015		0			
	L. Cos	d	L. Cot	c d	L. Tan	L. Sin	d	Proportional parts			

116° (390°)

(349°) 69°

Table 6 COMMON LOGARITHMS OF TRIGONOMETRIC FUNCTIONS (continued)

The — 10 portion of the characteristic of the logarithm is not printed but must be written down whenever such a logarithm is used.

21° (201°)

(336°) 150°

21° (201°)			(336°) 150°			Proportional parts					
L Sin	d	L Tan	c d	L Ctn	L Cos	d					
0	9.55 433	33	9.58 418	37	10.41 582	9.97 015	5				
1	9.55 466	33	9.58 456	38	10.41 545	9.97 010	5				
2	9.55 499	33	9.58 493	38	10.41 507	9.97 005	5				
3	9.55 532	33	9.58 531	38	10.41 469	9.97 001	5				
4	9.55 564	33	9.58 569	37	10.41 431	9.96 996	5				
5	9.55 597	33	9.58 606	38	10.41 394	9.96 991	5				
6	9.55 630	33	9.58 644	37	10.41 356	9.96 986	5				
7	9.55 663	33	9.58 681	38	10.41 318	9.96 981	5				
8	9.55 695	33	9.58 719	38	10.41 281	9.96 976	5				
9	9.55 728	33	9.58 757	37	10.41 243	9.96 971	5				
10	9.55 761	33	9.58 794	38	10.41 206	9.96 966	5				
11	9.55 793	33	9.58 832	37	10.41 168	9.96 962	4				
12	9.55 826	33	9.58 869	38	10.41 131	9.96 957	5				
13	9.55 858	33	9.58 907	37	10.41 093	9.96 952	5				
14	9.55 891	33	9.58 944	37	10.41 056	9.96 947	5				
15	9.55 923	33	9.58 981	38	10.41 019	9.96 942	5				
16	9.55 956	33	9.59 019	37	10.40 981	9.96 937	5				
17	9.55 988	33	9.59 056	38	10.40 944	9.96 932	5				
18	9.56 021	32	9.59 094	37	10.40 906	9.96 927	5				
19	9.56 053	32	9.59 131	37	10.40 869	9.96 922	5				
20	9.56 085	31	9.59 168	37	10.40 832	9.96 917	5				
21	9.56 118	31	9.59 205	38	10.40 795	9.96 912	5				
22	9.56 150	32	9.59 243	37	10.40 757	9.96 907	4				
23	9.56 182	33	9.59 280	37	10.40 720	9.96 903	5				
24	9.56 215	32	9.59 317	37	10.40 683	9.96 898	5				
25	9.56 247	32	9.59 354	37	10.40 646	9.96 893	5				
26	9.56 279	32	9.59 391	38	10.40 609	9.96 888	5				
27	9.56 311	32	9.59 429	37	10.40 571	9.96 883	5				
28	9.56 343	32	9.59 466	37	10.40 534	9.96 878	5				
29	9.56 375	31	9.59 503	37	10.40 497	9.96 873	5				
30	9.56 408	32	9.59 540	37	10.40 460	9.96 868	5				
31	9.56 440	32	9.59 577	37	10.40 423	9.96 863	5				
32	9.56 472	32	9.59 614	37	10.40 386	9.96 858	5				
33	9.56 504	32	9.59 651	37	10.40 349	9.96 853	5				
34	9.56 536	32	9.59 688	37	10.40 312	9.96 848	5				
35	9.56 568	31	9.59 725	37	10.40 275	9.96 843	5				
36	9.56 599	32	9.59 762	37	10.40 238	9.96 838	5				
37	9.56 631	32	9.59 799	36	10.40 201	9.96 833	5				
38	9.56 663	32	9.59 835	37	10.40 165	9.96 828	5				
39	9.56 695	32	9.59 872	37	10.40 128	9.96 823	5				
40	9.56 727	32	9.59 909	37	10.40 091	9.96 818	5				
41	9.56 759	31	9.59 946	37	10.40 054	9.96 813	5				
42	9.56 790	32	9.59 983	36	10.40 017	9.96 808	5				
43	9.56 822	32	9.60 019	37	10.39 981	9.96 803	5				
44	9.56 854	32	9.60 056	37	10.39 944	9.96 798	5				
45	9.56 885	31	9.60 093	37	10.39 907	9.96 793	5				
46	9.56 917	32	9.60 130	36	10.39 870	9.96 788	5				
47	9.56 949	31	9.60 166	37	10.39 834	9.96 783	5				
48	9.56 980	32	9.60 203	37	10.39 797	9.96 778	5				
49	9.57 012	32	9.60 240	36	10.39 760	9.96 772	5				
50	9.57 044	31	9.60 276	37	10.39 724	9.96 767	5				
51	9.57 075	32	9.60 313	36	10.39 687	9.96 762	5				
52	9.57 107	31	9.60 349	37	10.39 651	9.96 757	5				
53	9.57 138	32	9.60 386	36	10.39 614	9.96 752	5				
54	9.57 169	31	9.60 422	37	10.39 578	9.96 747	5				
55	9.57 201	31	9.60 459	36	10.39 541	9.96 742	5				
56	9.57 232	32	9.60 495	37	10.39 505	9.96 737	5				
57	9.57 264	31	9.60 532	36	10.39 468	9.96 732	5				
58	9.57 295	31	9.60 568	37	10.39 432	9.96 727	5				
59	9.57 326	32	9.60 605	36	10.39 395	9.96 722	5				
60	9.57 358		9.60 641		10.39 359	9.96 717					
L Cos	d	L Ctn	c d	L Tan	L Sin	d	Proportional parts				

111° (291°)

(246°) 68°

Table 6 COMMON LOGARITHMS OF TRIGONOMETRIC FUNCTIONS (continued) 239

The -10 portion of the characteristic of the logarithm is not printed but must be written down whenever such a logarithm is used.

32° (302°)					337° 157°					
'	L Sin	d	L Tan	c d	L Ctn	L Cos	d	Proportional parts		
0	9.57 358	31	9.60 641	36	10.39 359	9.96 717	6	60		
1	9.57 389	31	9.60 677	37	10.39 323	9.96 711	6	59		
2	9.57 420	31	9.60 714	36	10.39 286	9.96 705	5	58		
3	9.57 451	31	9.60 750	36	10.39 250	9.95 701	5	57		
4	9.57 482	32	9.60 786	37	10.39 214	9.95 695	5	56		
5	9.57 514	31	9.60 823	36	10.39 177	9.96 691	5	55		
6	9.57 545	31	9.60 859	36	10.39 141	9.96 685	5	54		
7	9.57 576	31	9.60 895	36	10.39 105	9.96 681	5	53		
8	9.57 607	31	9.60 931	36	10.39 069	9.96 675	5	52		
9	9.57 638	31	9.60 967	37	10.39 033	9.96 670	5	51		
10	9.57 669	31	9.61 004	36	10.38 996	9.96 665	5	50		
11	9.57 700	31	9.61 040	36	10.38 960	9.96 660	5	49		
12	9.57 731	31	9.61 076	36	10.38 924	9.96 655	5	48		
13	9.57 762	31	9.61 112	36	10.38 888	9.96 650	5	47		
14	9.57 793	31	9.61 148	36	10.38 852	9.96 645	5	46		
15	9.57 824	31	9.61 184	36	10.38 816	9.96 640	5	45		
16	9.57 855	30	9.61 220	36	10.38 780	9.96 635	5	44		
17	9.57 885	31	9.61 256	36	10.38 744	9.96 629	5	43		
18	9.57 916	31	9.61 292	36	10.38 708	9.96 624	5	42		
19	9.57 947	31	9.61 328	36	10.38 672	9.96 619	5	41		
20	9.57 978	30	9.61 364	36	10.38 636	9.96 614	5	40		
21	9.58 008	31	9.61 400	36	10.38 600	9.96 608	5	39		
22	9.58 039	31	9.61 436	36	10.38 564	9.96 603	5	38		
23	9.58 070	31	9.61 472	36	10.38 528	9.96 598	5	37		
24	9.58 101	30	9.61 508	36	10.38 492	9.96 593	5	36		
25	9.58 131	31	9.61 544	35	10.38 456	9.96 588	4	35		
26	9.58 162	30	9.61 579	36	10.38 421	9.96 582	5	34		
27	9.58 192	31	9.61 615	36	10.38 385	9.96 577	5	33		
28	9.58 223	30	9.61 651	36	10.38 349	9.96 572	5	32		
29	9.58 253	31	9.61 687	35	10.38 313	9.96 567	5	31		
30	9.58 284	30	9.61 722	36	10.38 278	9.96 562	5	30		
31	9.58 314	31	9.61 758	36	10.38 242	9.96 555	5	29		
32	9.58 345	30	9.61 794	36	10.38 206	9.96 551	5	28		
33	9.58 375	31	9.61 830	35	10.38 170	9.96 545	5	27		
34	9.58 406	30	9.61 865	36	10.38 135	9.96 541	5	26		
35	9.58 436	31	9.61 901	35	10.38 099	9.96 535	5	25		
36	9.58 467	30	9.61 936	36	10.38 064	9.96 530	5	24		
37	9.58 497	30	9.61 972	36	10.38 028	9.96 525	5	23		
38	9.58 527	30	9.62 008	35	10.37 992	9.96 520	5	22		
39	9.58 557	31	9.62 043	36	10.37 957	9.96 514	5	21		
40	9.58 588	30	9.62 079	35	10.37 921	9.96 509	5	20		
41	9.58 618	30	9.62 114	36	10.37 886	9.96 504	5	19		
42	9.58 648	30	9.62 150	35	10.37 850	9.96 498	5	18		
43	9.58 678	31	9.62 185	36	10.37 815	9.96 493	5	17		
44	9.58 709	30	9.62 221	35	10.37 779	9.96 488	5	16		
45	9.58 739	30	9.62 256	36	10.37 744	9.96 483	5	15		
46	9.58 769	30	9.62 292	35	10.37 708	9.96 477	5	14		
47	9.58 799	30	9.62 327	35	10.37 673	9.96 472	5	13		
48	9.58 829	30	9.62 362	36	10.37 638	9.96 467	5	12		
49	9.58 859	30	9.62 398	35	10.37 602	9.96 461	5	11		
50	9.58 889	30	9.62 433	35	10.37 567	9.96 456	5	10		
51	9.58 919	32	9.62 468	36	10.37 532	9.96 451	5	9		
52	9.58 949	30	9.62 504	35	10.37 496	9.96 445	5	8		
53	9.58 979	30	9.62 539	35	10.37 461	9.96 440	5	7		
54	9.59 009	30	9.62 574	35	10.37 426	9.96 435	5	6		
55	9.59 039	32	9.62 609	36	10.37 391	9.96 429	5	5		
56	9.59 069	32	9.62 645	35	10.37 355	9.96 424	5	4		
57	9.59 098	30	9.62 680	36	10.37 320	9.96 419	5	3		
58	9.59 128	32	9.62 715	35	10.37 285	9.96 413	5	2		
59	9.59 158	32	9.62 750	35	10.37 250	9.96 408	5	1		
60	9.59 188		9.62 785		10.37 215	9.96 403		0		
'	L Cos	d	L Ctn	c d	L Tan	L Sin	d	Proportional parts		

112° (302°)

(247°) 67°

Table 6 COMMON LOGARITHMS OF TRIGONOMETRIC FUNCTIONS (continued) 240

The -10 portion of the characteristic of the logarithm is not printed but must be written down whenever such a logarithm is used.

23° (203°)										(386°) 156°									
°	L Sin	d	L Tan	o d	L Ctn	L Cos	d	°		Proportional parts									
0	9.59 188	30	9.62 785	35	10.37 215	9.96 403	6	60											
1	9.59 218	29	9.62 820	35	10.37 180	9.96 397	5	59											
2	9.59 247	30	9.62 855	35	10.37 145	9.96 392	5	58											
3	9.59 277	30	9.62 890	35	10.37 110	9.96 387	5	57											
4	9.59 307	29	9.62 925	35	10.37 074	9.96 381	5	56											
5	9.59 336	30	9.62 961	35	10.37 039	9.96 376	5	55											
6	9.59 366	30	9.62 996	35	10.37 004	9.96 370	5	54											
7	9.59 396	29	9.63 031	35	10.36 969	9.96 365	5	53											
8	9.59 425	30	9.63 066	35	10.36 934	9.96 360	5	52											
9	9.59 455	29	9.63 101	34	10.36 899	9.96 354	5	51											
10	9.59 484	30	9.63 135	35	10.36 865	9.96 349	5	50											
11	9.59 514	29	9.63 170	35	10.36 830	9.96 343	5	49											
12	9.59 543	30	9.63 205	35	10.36 795	9.96 338	5	48											
13	9.59 573	29	9.63 240	35	10.36 760	9.96 333	5	47											
14	9.59 602	30	9.63 275	35	10.36 725	9.96 327	5	46											
15	9.59 632	29	9.63 310	35	10.36 690	9.96 322	5	45											
16	9.59 661	29	9.63 345	34	10.36 655	9.96 316	5	44											
17	9.59 690	30	9.63 379	35	10.36 621	9.96 311	5	43											
18	9.59 720	29	9.63 414	35	10.36 586	9.96 305	5	42											
19	9.59 749	29	9.63 449	35	10.36 551	9.96 300	5	41											
20	9.59 778	30	9.63 484	35	10.36 516	9.96 294	5	40											
21	9.59 808	29	9.63 519	34	10.36 481	9.96 289	5	39											
22	9.59 837	29	9.63 553	35	10.36 447	9.96 284	5	38											
23	9.59 866	29	9.63 588	35	10.36 412	9.96 278	5	37											
24	9.59 895	29	9.63 623	34	10.36 377	9.96 273	5	36											
25	9.59 924	30	9.63 657	35	10.36 343	9.96 267	5	35											
26	9.59 954	29	9.63 692	34	10.36 308	9.96 262	5	34											
27	9.59 983	29	9.63 726	35	10.36 274	9.96 256	5	33											
28	9.60 012	29	9.63 761	35	10.36 239	9.96 251	5	32											
29	9.60 041	29	9.63 795	34	10.36 204	9.96 245	5	31											
30	9.60 070	29	9.63 830	35	10.36 170	9.96 240	5	30											
31	9.60 099	29	9.63 865	34	10.36 135	9.96 234	5	29											
32	9.60 128	29	9.63 899	35	10.36 101	9.96 229	5	28											
33	9.60 157	29	9.63 934	34	10.36 066	9.96 223	5	27											
34	9.60 186	29	9.63 968	35	10.36 032	9.96 218	5	26											
35	9.60 215	29	9.64 003	34	10.35 997	9.96 212	5	25											
36	9.60 244	29	9.64 037	35	10.35 963	9.96 207	5	24											
37	9.60 273	29	9.64 072	34	10.35 928	9.96 201	5	23											
38	9.60 302	29	9.64 106	34	10.35 894	9.96 195	5	22											
39	9.60 331	28	9.64 140	35	10.35 860	9.96 189	5	21											
40	9.60 359	29	9.64 175	34	10.35 825	9.96 183	5	20											
41	9.60 388	29	9.64 209	34	10.35 791	9.96 179	5	19											
42	9.60 417	29	9.64 243	35	10.35 757	9.96 174	5	18											
43	9.60 446	29	9.64 278	34	10.35 722	9.96 168	5	17											
44	9.60 474	29	9.64 312	34	10.35 688	9.96 162	5	16											
45	9.60 503	29	9.64 346	35	10.35 654	9.96 157	5	15											
46	9.60 532	29	9.64 381	34	10.35 619	9.96 151	5	14											
47	9.60 561	28	9.64 415	34	10.35 585	9.96 146	5	13											
48	9.60 589	29	9.64 449	34	10.35 551	9.96 140	5	12											
49	9.60 618	28	9.64 483	34	10.35 517	9.96 135	5	11											
50	9.60 646	29	9.64 517	35	10.35 483	9.96 129	5	10											
51	9.60 675	29	9.64 552	34	10.35 448	9.96 123	5	9											
52	9.60 704	28	9.64 586	34	10.35 414	9.96 118	5	8											
53	9.60 732	29	9.64 620	34	10.35 380	9.96 112	5	7											
54	9.60 761	28	9.64 654	34	10.35 346	9.96 107	5	6											
55	9.60 789	29	9.64 688	34	10.35 312	9.96 101	5	5											
56	9.60 818	28	9.64 722	34	10.35 278	9.96 095	5	4											
57	9.60 846	29	9.64 756	34	10.35 244	9.96 090	5	3											
58	9.60 875	28	9.64 790	34	10.35 210	9.96 084	5	2											
59	9.60 903	28	9.64 824	34	10.35 176	9.96 079	5	1											
60	9.60 931		9.64 858		10.35 142	9.96 073		0											
°	L Cos	d	L Ctn	e d	L Tan	L Sin	d	°		Proportional parts									

Table 6 COMMON LOGARITHMS OF TRIGONOMETRIC FUNCTIONS (continued) 241

The — 10 portion of the characteristic of the logarithm is not printed but must be written down whenever such a logarithm is used.

24° (204°)						(335°) 155°					
'	L Sin	d	L Tan	c d	L Ctn	L Cos	d	Proportional parts			
0	9.60 931	23	9.64 858	34	10.35 142	9.95 073	6	60			
1	9.60 960	28	9.64 892	34	10.35 168	9.95 067	5	59			
2	9.60 988	28	9.64 926	34	10.35 074	9.95 062	6	58			
3	9.61 016	29	9.64 960	34	10.35 040	9.95 055	6	57			
4	9.61 045	28	9.64 994	34	10.35 066	9.95 050	6	56			
5	9.61 073	28	9.65 028	34	10.34 972	9.95 045	6	55			
6	9.61 101	28	9.65 062	34	10.34 938	9.95 039	5	54			
7	9.61 129	29	9.65 096	34	10.34 904	9.95 034	6	53			
8	9.61 158	28	9.65 130	34	10.34 870	9.95 028	6	52			
9	9.61 186	28	9.65 164	34	10.34 835	9.95 022	5	51			
10	9.61 214	28	9.65 197	34	10.34 803	9.95 017	6	50			
11	9.61 242	28	9.65 231	34	10.34 769	9.95 011	6	49			
12	9.61 270	28	9.65 265	34	10.34 735	9.95 005	5	48			
13	9.61 298	28	9.65 299	34	10.34 701	9.95 000	6	47			
14	9.61 326	28	9.65 333	34	10.34 667	9.95 994	6	46			
15	9.61 354	28	9.65 366	34	10.34 634	9.95 988	6	45			
16	9.61 382	29	9.65 400	34	10.34 600	9.95 982	5	44			
17	9.61 411	27	9.65 434	34	10.34 566	9.95 977	6	43			
18	9.61 438	28	9.65 467	34	10.34 533	9.95 971	6	42			
19	9.61 466	28	9.65 501	34	10.34 499	9.95 965	5	41			
20	9.61 494	28	9.65 535	34	10.34 465	9.95 960	6	40			
21	9.61 522	28	9.65 568	34	10.34 432	9.95 954	6	39			
22	9.61 550	28	9.65 602	34	10.34 398	9.95 948	6	38			
23	9.61 578	28	9.65 636	34	10.34 364	9.95 942	5	37			
24	9.61 606	28	9.65 669	34	10.34 331	9.95 937	6	36			
25	9.61 634	28	9.65 703	34	10.34 297	9.95 931	6	35			
26	9.61 662	27	9.65 736	34	10.34 264	9.95 925	5	34			
27	9.61 689	28	9.65 770	34	10.34 230	9.95 920	6	33			
28	9.61 717	28	9.65 803	34	10.34 197	9.95 914	6	32			
29	9.61 745	28	9.65 837	34	10.34 163	9.95 908	6	31			
30	9.61 773	27	9.65 870	34	10.34 130	9.95 902	5	30			
31	9.61 800	28	9.65 904	34	10.34 096	9.95 897	6	29			
32	9.61 828	28	9.65 937	34	10.34 063	9.95 891	6	28			
33	9.61 856	28	9.65 971	34	10.34 029	9.95 885	6	27			
34	9.61 883	28	9.65 004	34	10.33 996	9.95 879	6	26			
35	9.61 911	28	9.66 038	34	10.33 962	9.95 873	5	25			
36	9.61 939	27	9.66 071	34	10.33 929	9.95 868	6	24			
37	9.61 966	28	9.66 104	34	10.33 896	9.95 862	6	23			
38	9.61 994	27	9.66 138	34	10.33 862	9.95 856	6	22			
39	9.62 021	28	9.66 171	34	10.33 829	9.95 850	6	21			
40	9.62 049	27	9.66 204	34	10.33 796	9.95 844	5	20			
41	9.62 076	28	9.66 238	34	10.33 762	9.95 839	6	19			
42	9.62 104	27	9.66 271	34	10.33 729	9.95 833	6	18			
43	9.62 131	28	9.66 304	34	10.33 696	9.95 827	6	17			
44	9.62 159	27	9.66 337	34	10.33 663	9.95 821	6	16			
45	9.62 186	28	9.66 371	34	10.33 629	9.95 815	5	15			
46	9.62 214	27	9.66 404	34	10.33 596	9.95 810	6	14			
47	9.62 241	27	9.66 437	34	10.33 563	9.95 804	6	13			
48	9.62 268	28	9.66 470	34	10.33 530	9.95 798	6	12			
49	9.62 296	27	9.66 503	34	10.33 497	9.95 792	6	11			
50	9.62 323	27	9.66 537	34	10.33 463	9.95 786	5	10			
51	9.62 350	27	9.66 570	34	10.33 430	9.95 780	5	9			
52	9.62 377	28	9.66 603	34	10.33 397	9.95 775	6	8			
53	9.62 405	27	9.66 636	34	10.33 364	9.95 769	6	7			
54	9.62 432	27	9.66 669	34	10.33 331	9.95 763	6	6			
55	9.62 459	27	9.66 702	34	10.33 298	9.95 757	6	5			
56	9.62 486	27	9.66 735	34	10.33 265	9.95 751	6	4			
57	9.62 513	28	9.66 768	34	10.33 232	9.95 745	6	3			
58	9.62 541	27	9.66 801	34	10.33 199	9.95 739	6	2			
59	9.62 568	27	9.66 834	34	10.33 166	9.95 733	5	1			
60	9.62 595		9.66 867		10.33 133	9.95 728		0			
'	L Cos	d	L Ctn	c d	L Tan	L Stu	d	Proportional parts			

134° (204°)

(335°) 65°

Table 6 COMMON LOGARITHMS OF TRIGONOMETRIC FUNCTIONS (continued) 242

The — 10 portion of the characteristic of the logarithm is not printed but must be written down whenever such a logarithm is used.

25° (265°)

(334°) 154°

°	L Sin		L Tan		L Ctn		L Cos		°	Proportional parts					
	d		c d		d		d								
0	9.62	595	27	9.66	887	32	10.33	133	9.95	728	5	60			
1	9.62	622	27	9.66	900	32	10.33	100	9.95	722	6	59			
2	9.62	649	27	9.66	933	32	10.33	067	9.95	716	6	58			
3	9.62	676	27	9.66	966	32	10.33	034	9.95	710	6	57			
4	9.62	703	27	9.66	999	32	10.33	001	9.95	704	6	56			
5	9.62	730	27	9.67	032	32	10.32	968	9.95	698	5	55			
6	9.62	757	27	9.67	065	32	10.32	935	9.95	692	6	54			
7	9.62	784	27	9.67	098	32	10.32	902	9.95	686	6	53			
8	9.62	811	27	9.67	131	32	10.32	869	9.95	680	6	52			
9	9.62	838	27	9.67	163	32	10.32	837	9.95	674	6	51			
10	9.62	865	27	9.67	196	32	10.32	804	9.95	668	5	50			
11	9.62	892	26	9.67	229	32	10.32	771	9.95	663	6	49			
12	9.62	918	27	9.67	262	32	10.32	738	9.95	657	6	48			
13	9.62	945	27	9.67	295	32	10.32	705	9.95	651	6	47			
14	9.62	972	27	9.67	327	32	10.32	673	9.95	645	6	46			
15	9.62	999	27	9.67	360	32	10.32	640	9.95	639	5	45			
16	9.63	026	26	9.67	393	32	10.32	607	9.95	633	6	44			
17	9.63	053	26	9.67	426	32	10.32	574	9.95	627	6	43			
18	9.63	079	27	9.67	458	32	10.32	542	9.95	621	6	42			
19	9.63	106	27	9.67	491	32	10.32	509	9.95	615	6	41			
20	9.63	133	26	9.67	524	32	10.32	476	9.95	609	5	40			
21	9.63	159	27	9.67	556	32	10.32	444	9.95	603	6	39			
22	9.63	186	27	9.67	589	32	10.32	411	9.95	597	6	38			
23	9.63	213	26	9.67	622	32	10.32	378	9.95	591	6	37			
24	9.63	239	27	9.67	654	32	10.32	346	9.95	585	6	36			
25	9.63	266	26	9.67	687	32	10.32	313	9.95	579	4	35			
26	9.63	292	27	9.67	719	32	10.32	281	9.95	573	6	34			
27	9.63	319	26	9.67	752	32	10.32	248	9.95	567	6	33			
28	9.63	345	27	9.67	785	32	10.32	215	9.95	561	6	32			
29	9.63	372	26	9.67	817	32	10.32	183	9.95	555	6	31			
30	9.63	398	27	9.67	850	32	10.32	150	9.95	549	6	30			
31	9.63	425	26	9.67	882	32	10.32	118	9.95	543	6	29			
32	9.63	451	27	9.67	915	32	10.32	085	9.95	537	6	28			
33	9.63	478	26	9.67	947	32	10.32	053	9.95	531	6	27			
34	9.63	504	27	9.67	980	32	10.32	020	9.95	525	6	26			
35	9.63	531	26	9.68	012	32	10.31	988	9.95	519	4	25			
36	9.63	557	26	9.68	044	32	10.31	956	9.95	513	6	24			
37	9.63	583	27	9.68	077	32	10.31	923	9.95	507	7	23			
38	9.63	610	26	9.68	109	32	10.31	891	9.95	500	6	22			
39	9.63	636	26	9.68	142	32	10.31	858	9.95	494	6	21			
40	9.63	662	27	9.68	174	32	10.31	826	9.95	488	6	20			
41	9.63	689	26	9.68	206	32	10.31	794	9.95	482	6	19			
42	9.63	715	26	9.68	239	32	10.31	761	9.95	476	6	18			
43	9.63	741	26	9.68	271	32	10.31	729	9.95	470	6	17			
44	9.63	767	27	9.68	303	32	10.31	697	9.95	464	6	16			
45	9.63	794	26	9.68	336	32	10.31	664	9.95	458	6	15			
46	9.63	820	26	9.68	368	32	10.31	632	9.95	452	6	14			
47	9.63	846	26	9.68	400	32	10.31	600	9.95	446	6	13			
48	9.63	872	26	9.68	432	32	10.31	568	9.95	440	6	12			
49	9.63	898	26	9.68	465	32	10.31	535	9.95	434	7	11			
50	9.63	924	25	9.68	497	32	10.31	503	9.95	427	4	10			
51	9.63	950	25	9.68	529	32	10.31	471	9.95	421	6	9			
52	9.63	976	26	9.68	561	32	10.31	439	9.95	415	6	8			
53	9.64	002	26	9.68	593	32	10.31	407	9.95	409	6	7			
54	9.64	028	26	9.68	626	32	10.31	374	9.95	403	6	6			
55	9.64	054	25	9.68	658	32	10.31	342	9.95	397	6	5			
56	9.64	080	25	9.68	690	32	10.31	310	9.95	391	7	4			
57	9.64	106	26	9.68	722	32	10.31	278	9.95	384	4	3			
58	9.64	132	26	9.68	754	32	10.31	246	9.95	378	6	2			
59	9.64	158	26	9.68	786	32	10.31	214	9.95	372	6	1			
60	9.64	184		9.68	818		10.31	182	9.95	366		0			
°	L Cos	d		L Ctn	c d		L Tan		L Sin	d		°	Proportional parts		

315° (295°)

(344°) 64°

Table 6 COMMON LOGARITHMS OF TRIGONOMETRIC FUNCTIONS (continued) 243

The -10 portion of the characteristic of the logarithm is not printed but must be written down whenever such a logarithm is used.

26° (206")				(333") 153"				Proportional parts		
°	L Sin	d	L Tan	c d	L Ctn	L Cos	d			
0	9.64 184	25	9.62 818	32	10.31 182	9.95 366	6	60		
1	9.64 210	25	9.62 850	32	10.31 150	9.95 380	6	59		
2	9.64 236	25	9.62 882	32	10.31 118	9.95 394	6	58		
3	9.64 262	25	9.62 914	32	10.31 086	9.95 408	6	57		
4	9.64 288	25	9.62 946	32	10.31 054	9.95 421	6	56		
5	9.64 313	25	9.62 978	32	10.31 022	9.95 435	6	55		
6	9.64 339	25	9.62 010	32	10.30 990	9.95 449	6	54		
7	9.64 365	25	9.62 042	32	10.30 958	9.95 463	6	53		
8	9.64 391	25	9.62 074	32	10.30 926	9.95 477	6	52		
9	9.64 417	25	9.62 106	32	10.30 894	9.95 491	6	51		
10	9.64 442	25	9.62 138	32	10.30 862	9.95 504	6	50		
11	9.64 468	25	9.62 170	32	10.30 830	9.95 518	6	49		
12	9.64 494	25	9.62 202	32	10.30 798	9.95 532	6	48		
13	9.64 519	25	9.62 234	32	10.30 766	9.95 546	6	47		
14	9.64 545	25	9.62 266	32	10.30 734	9.95 560	6	46		
15	9.64 571	25	9.62 298	31	10.30 702	9.95 573	6	45		
16	9.64 596	25	9.62 329	32	10.30 671	9.95 587	6	44		
17	9.64 622	25	9.62 361	32	10.30 639	9.95 601	6	43		
18	9.64 647	25	9.62 393	32	10.30 607	9.95 615	6	42		
19	9.64 673	25	9.62 425	32	10.30 575	9.95 629	6	41		
20	9.64 698	25	9.62 457	31	10.30 543	9.95 642	6	40		
21	9.64 724	25	9.62 488	32	10.30 512	9.95 656	6	39		
22	9.64 749	25	9.62 520	32	10.30 480	9.95 670	6	38		
23	9.64 775	25	9.62 552	32	10.30 448	9.95 684	6	37		
24	9.64 800	25	9.62 584	31	10.30 416	9.95 698	6	36		
25	9.64 826	25	9.62 615	32	10.30 385	9.95 711	6	35		
26	9.64 851	25	9.62 647	32	10.30 353	9.95 725	6	34		
27	9.64 877	25	9.62 679	31	10.30 321	9.95 739	6	33		
28	9.64 902	25	9.62 710	32	10.30 290	9.95 753	6	32		
29	9.64 927	25	9.62 742	32	10.30 258	9.95 767	6	31		
30	9.64 953	25	9.62 774	31	10.30 226	9.95 780	6	30		
31	9.64 978	25	9.62 805	32	10.30 195	9.95 794	6	29		
32	9.65 003	25	9.62 837	31	10.30 163	9.95 808	6	28		
33	9.65 029	25	9.62 868	32	10.30 132	9.95 822	6	27		
34	9.65 054	25	9.62 900	32	10.30 100	9.95 836	6	26		
35	9.65 079	25	9.62 932	31	10.30 068	9.95 850	6	25		
36	9.65 104	25	9.62 963	32	10.30 037	9.95 864	6	24		
37	9.65 130	25	9.62 995	31	10.30 005	9.95 878	6	23		
38	9.65 155	25	9.70 026	32	10.29 974	9.95 892	6	22		
39	9.65 180	25	9.70 058	31	10.29 942	9.95 906	6	21		
40	9.65 205	25	9.70 089	32	10.29 911	9.95 920	6	20		
41	9.65 230	25	9.70 121	31	10.29 879	9.95 934	6	19		
42	9.65 255	25	9.70 152	32	10.29 848	9.95 948	6	18		
43	9.65 281	25	9.70 184	31	10.29 816	9.95 962	6	17		
44	9.65 306	25	9.70 215	32	10.29 785	9.95 976	6	16		
45	9.65 331	25	9.70 247	31	10.29 753	9.95 990	6	15		
46	9.65 356	25	9.70 278	32	10.29 722	9.95 104	6	14		
47	9.65 381	25	9.70 309	31	10.29 691	9.95 118	6	13		
48	9.65 406	25	9.70 341	32	10.29 659	9.95 132	6	12		
49	9.65 431	25	9.70 372	31	10.29 628	9.95 146	6	11		
50	9.65 456	25	9.70 404	32	10.29 596	9.95 160	6	10		
51	9.65 481	25	9.70 435	31	10.29 565	9.95 174	6	9		
52	9.65 506	25	9.70 466	32	10.29 534	9.95 188	6	8		
53	9.65 531	25	9.70 498	31	10.29 502	9.95 202	6	7		
54	9.65 556	25	9.70 529	32	10.29 471	9.95 216	6	6		
55	9.65 580	25	9.70 560	31	10.29 440	9.95 230	6	5		
56	9.65 605	25	9.70 592	32	10.29 408	9.95 244	6	4		
57	9.65 630	25	9.70 623	31	10.29 377	9.95 258	6	3		
58	9.65 655	25	9.70 654	32	10.29 346	9.95 272	6	2		
59	9.65 680	25	9.70 685	31	10.29 315	9.95 286	6	1		
60	9.65 705		9.70 717		10.29 283	9.94 988		0		
°	L Cos	d	L Ctn	c d	L Tan	L Sin	d	Proportional parts		

Table 6 COMMON LOGARITHMS OF TRIGONOMETRIC FUNCTIONS (continued) 244

The -10 portion of the characteristic of the logarithm is not printed but must be written down whenever such a logarithm is used.

27° (207°)						(332°) 152°					
'	L Sin	d	L Tan	c d	L Ctn	L Cos	d	'	Proportional parts		
0	9.65 706	24	9.70 717	31	10.29 283	9.94 988	6	60			
1	9.65 729	25	9.70 748	31	10.29 252	9.94 982	7	59			
2	9.65 754	25	9.70 779	31	10.29 221	9.94 975	6	58			
3	9.65 779	25	9.70 810	31	10.29 190	9.94 969	7	57			
4	9.65 804	24	9.70 841	32	10.29 159	9.94 962	6	56			
5	9.65 828	25	9.70 873	31	10.29 127	9.94 956	7	55			
6	9.65 853	25	9.70 904	31	10.29 096	9.94 949	6	54			
7	9.65 878	24	9.70 936	31	10.29 065	9.94 943	7	53			
8	9.65 902	25	9.70 966	31	10.29 034	9.94 936	6	52			
9	9.65 927	25	9.70 997	31	10.29 003	9.94 930	7	51			
10	9.65 952	24	9.71 028	31	10.28 972	9.94 923	6	50			
11	9.65 976	25	9.71 059	31	10.28 941	9.94 917	7	49			
12	9.66 001	24	9.71 090	31	10.28 910	9.94 911	6	48			
13	9.66 025	25	9.71 121	32	10.28 879	9.94 904	7	47			
14	9.66 050	25	9.71 153	31	10.28 847	9.94 898	6	46			
15	9.66 075	24	9.71 184	31	10.28 816	9.94 891	7	45			
16	9.66 099	25	9.71 215	31	10.28 785	9.94 885	6	44			
17	9.66 124	24	9.71 246	31	10.28 754	9.94 878	7	43			
18	9.66 148	25	9.71 277	31	10.28 723	9.94 871	6	42			
19	9.66 173	24	9.71 308	32	10.28 692	9.94 865	7	41			
20	9.66 197	24	9.71 339	31	10.28 661	9.94 858	6	40			
21	9.66 221	25	9.71 370	31	10.28 630	9.94 852	7	39			
22	9.66 246	24	9.71 401	30	10.28 599	9.94 845	6	38			
23	9.66 270	25	9.71 431	31	10.28 568	9.94 839	7	37			
24	9.66 295	24	9.71 462	31	10.28 538	9.94 832	6	36			
25	9.66 319	24	9.71 493	31	10.28 507	9.94 826	7	35			
26	9.66 343	25	9.71 524	31	10.28 476	9.94 819	6	34			
27	9.66 368	24	9.71 555	31	10.28 445	9.94 813	7	33			
28	9.66 392	25	9.71 586	31	10.28 414	9.94 806	6	32			
29	9.66 416	25	9.71 617	31	10.28 383	9.94 799	7	31			
30	9.66 441	24	9.71 648	31	10.28 352	9.94 793	6	30			
31	9.66 465	24	9.71 679	30	10.28 321	9.94 786	7	29			
32	9.66 489	24	9.71 709	31	10.28 291	9.94 780	6	28			
33	9.66 513	24	9.71 740	31	10.28 260	9.94 773	7	27			
34	9.66 537	25	9.71 771	31	10.28 229	9.94 767	6	26			
35	9.66 562	24	9.71 802	31	10.28 198	9.94 760	7	25			
36	9.66 586	24	9.71 833	30	10.28 167	9.94 753	6	24			
37	9.66 610	24	9.71 863	31	10.28 137	9.94 747	7	23			
38	9.66 634	24	9.71 894	31	10.28 106	9.94 740	6	22			
39	9.66 658	24	9.71 925	30	10.28 075	9.94 734	7	21			
40	9.66 682	24	9.71 955	31	10.28 045	9.94 727	6	20			
41	9.66 706	25	9.71 986	31	10.28 014	9.94 720	7	19			
42	9.66 731	24	9.72 017	31	10.27 983	9.94 714	6	18			
43	9.66 755	24	9.72 048	30	10.27 952	9.94 707	7	17			
44	9.66 779	24	9.72 078	31	10.27 922	9.94 700	6	16			
45	9.66 803	24	9.72 109	30	10.27 891	9.94 694	7	15			
46	9.66 827	24	9.72 140	30	10.27 860	9.94 687	6	14			
47	9.66 851	24	9.72 170	30	10.27 830	9.94 680	7	13			
48	9.66 875	24	9.72 201	30	10.27 799	9.94 674	6	12			
49	9.66 899	23	9.72 231	31	10.27 769	9.94 667	7	11			
50	9.66 922	24	9.72 262	31	10.27 738	9.94 660	6	10			
51	9.66 946	24	9.72 293	30	10.27 707	9.94 654	7	9			
52	9.66 970	24	9.72 323	31	10.27 677	9.94 647	6	8			
53	9.66 994	24	9.72 354	30	10.27 646	9.94 640	7	7			
54	9.67 018	24	9.72 384	31	10.27 616	9.94 634	6	6			
55	9.67 042	24	9.72 415	30	10.27 585	9.94 627	7	5			
56	9.67 066	24	9.72 445	31	10.27 555	9.94 620	6	4			
57	9.67 090	23	9.72 476	30	10.27 524	9.94 614	7	3			
58	9.67 113	24	9.72 506	31	10.27 494	9.94 607	6	2			
59	9.67 137	24	9.72 537	30	10.27 463	9.94 600	7	1			
60	9.67 161		9.72 567		10.27 433	9.94 593		0			
'	L Cos	d	L Ctn	c d	L Tan	L Sin	d	'	Proportional parts		

117° (297°)

(342°) 62°

Table 6 COMMON LOGARITHMS OF TRIGONOMETRIC FUNCTIONS (continued) 245

The — 10 portion of the characteristic of the logarithm is not printed but must be written down whenever such a logarithm is used.

20° (208°)

(331°) 151°

°	L Sin			L Tan			L Cot			L Cos			Proportional parts		
	d			c d			d			d					
0	9.67	161	24	9.72	567	31	10.27	433	9.94	593	6	60			
1	9.67	185	23	9.72	598	30	10.27	402	9.94	587	7	59			
2	9.67	208	25	9.72	628	31	10.27	372	9.94	580	7	58			
3	9.67	232	26	9.72	659	30	10.27	341	9.94	573	6	57			
4	9.67	256	24	9.72	689	21	10.27	311	9.94	567	7	56			
5	9.67	280	23	9.72	720	30	10.27	280	9.94	560	7	55			
6	9.67	303	24	9.72	750	30	10.27	250	9.94	553	7	54			
7	9.67	327	23	9.72	780	31	10.27	220	9.94	546	6	53			
8	9.67	350	24	9.72	811	30	10.27	189	9.94	540	7	52			
9	9.67	374	24	9.72	841	31	10.27	159	9.94	533	7	51			
10	9.67	398	23	9.72	872	30	10.27	128	9.94	525	7	50			
11	9.67	421	24	9.72	902	30	10.27	098	9.94	519	6	49			
12	9.67	445	23	9.72	932	31	10.27	068	9.94	513	7	48			
13	9.67	468	24	9.72	963	30	10.27	037	9.94	506	7	47			
14	9.67	492	23	9.72	993	30	10.27	007	9.94	499	7	46			
15	9.67	515	24	9.73	023	31	10.26	977	9.94	492	7	45			
16	9.67	539	23	9.73	054	30	10.26	946	9.94	485	6	44			
17	9.67	562	24	9.73	084	30	10.26	916	9.94	479	7	43			
18	9.67	586	23	9.73	114	30	10.26	886	9.94	472	7	42			
19	9.67	609	24	9.73	144	31	10.26	856	9.94	465	7	41			
20	9.67	633	23	9.73	175	30	10.26	825	9.94	458	7	40			
21	9.67	656	24	9.73	205	30	10.26	795	9.94	451	6	39			
22	9.67	680	23	9.73	235	30	10.26	765	9.94	445	7	38			
23	9.67	703	24	9.73	265	30	10.26	735	9.94	438	7	37			
24	9.67	726	23	9.73	295	31	10.26	705	9.94	431	7	36			
25	9.67	750	23	9.73	326	30	10.26	674	9.94	424	7	35			
26	9.67	773	24	9.73	356	30	10.26	644	9.94	417	7	34			
27	9.67	796	24	9.73	386	30	10.26	614	9.94	410	6	33			
28	9.67	820	23	9.73	416	30	10.26	584	9.94	404	7	32			
29	9.67	843	23	9.73	446	30	10.26	554	9.94	397	7	31			
30	9.67	866	24	9.73	476	31	10.26	524	9.94	390	7	30			
31	9.67	890	23	9.73	507	30	10.26	493	9.94	383	7	29			
32	9.67	913	23	9.73	537	30	10.26	463	9.94	376	7	28			
33	9.67	936	23	9.73	567	30	10.26	433	9.94	369	7	27			
34	9.67	959	23	9.73	597	30	10.26	403	9.94	362	7	26			
35	9.67	982	24	9.73	627	30	10.26	373	9.94	355	6	25			
36	9.68	006	23	9.73	657	30	10.26	343	9.94	348	7	24			
37	9.68	029	23	9.73	687	30	10.26	313	9.94	342	7	23			
38	9.68	052	23	9.73	717	30	10.26	283	9.94	335	7	22			
39	9.68	075	23	9.73	747	30	10.26	253	9.94	328	7	21			
40	9.68	098	23	9.73	777	30	10.26	223	9.94	321	7	20			
41	9.68	121	23	9.73	807	30	10.26	193	9.94	314	7	19			
42	9.68	144	23	9.73	837	30	10.26	163	9.94	307	7	18			
43	9.68	167	23	9.73	867	30	10.26	133	9.94	300	7	17			
44	9.68	190	23	9.73	897	30	10.26	103	9.94	293	7	16			
45	9.68	213	24	9.73	927	30	10.26	073	9.94	286	7	15			
46	9.68	237	23	9.73	957	30	10.26	043	9.94	279	6	14			
47	9.68	260	23	9.73	987	30	10.26	013	9.94	273	7	13			
48	9.68	283	22	9.74	017	30	10.25	983	9.94	266	7	12			
49	9.68	306	23	9.74	047	30	10.25	953	9.94	259	7	11			
50	9.68	328	23	9.74	077	30	10.25	923	9.94	252	7	10			
51	9.68	351	23	9.74	107	30	10.25	893	9.94	245	7	9			
52	9.68	374	23	9.74	137	29	10.25	863	9.94	238	7	8			
53	9.68	397	23	9.74	166	30	10.25	834	9.94	231	7	7			
54	9.68	420	23	9.74	196	30	10.25	804	9.94	224	7	6			
55	9.68	443	23	9.74	226	30	10.25	774	9.94	217	7	5			
56	9.68	466	23	9.74	256	30	10.25	744	9.94	210	7	4			
57	9.68	489	23	9.74	286	30	10.25	714	9.94	203	7	3			
58	9.68	512	23	9.74	316	29	10.25	684	9.94	196	7	2			
59	9.68	534	23	9.74	345	30	10.25	655	9.94	189	7	1			
60	9.68	557		9.74	375		10.25	625	9.94	182		0			
	L Cos	d		L Ctn	c d	L Tan		L Sin	d		Proportional parts				

118° (298°)

(341°) 61°

Table 6 COMMON LOGARITHMS OF TRIGONOMETRIC FUNCTIONS (continued) 246

The -10 portion of the characteristic of the logarithm is not printed but must be written down whenever such a logarithm is used.

29° (209°)				(330°) 150°				Proportional parts				
'	L Sin	d	L Tan	c d	L Ctn	L Cos	d	'				
0	9.68 567	23	9.74 375	30	10.25 625	9.94 182	7	60				
1	9.68 580	23	9.74 405	30	10.25 595	9.94 175	7	59				
2	9.68 603	22	9.74 435	30	10.25 565	9.94 168	7	58				
3	9.68 626	22	9.74 465	29	10.25 535	9.94 161	7	57				
4	9.68 648	22	9.74 494	30	10.25 506	9.94 154	7	56				
5	9.68 671	23	9.74 524	30	10.25 476	9.94 147	7	55				
6	9.68 694	22	9.74 554	29	10.25 446	9.94 140	7	54				
7	9.68 716	22	9.74 583	30	10.25 417	9.94 133	7	53				
8	9.68 739	22	9.74 613	30	10.25 387	9.94 126	7	52				
9	9.68 762	22	9.74 643	30	10.25 357	9.94 119	7	51				
10	9.68 784	23	9.74 673	29	10.25 327	9.94 112	7	50				
11	9.68 807	22	9.74 702	30	10.25 298	9.94 105	7	49				
12	9.68 829	22	9.74 732	30	10.25 268	9.94 098	7	48				
13	9.68 852	22	9.74 762	29	10.25 238	9.94 090	7	47				
14	9.68 875	22	9.74 791	30	10.25 209	9.94 083	7	46				
15	9.68 897	22	9.74 821	30	10.25 179	9.94 076	7	45				
16	9.68 920	22	9.74 851	29	10.25 149	9.94 069	7	44	30	29	23	
17	9.68 942	22	9.74 880	30	10.25 120	9.94 062	7	43	1	0.5	0.5	0.4
18	9.68 965	22	9.74 910	30	10.25 090	9.94 055	7	42	2	1.0	1.0	0.8
19	9.68 987	23	9.74 939	29	10.25 061	9.94 048	7	41	3	1.5	1.4	1.2
20	9.69 010	22	9.74 969	29	10.25 031	9.94 041	7	40	4	2.0	1.9	1.5
21	9.69 032	22	9.74 998	30	10.25 002	9.94 034	7	39	5	2.5	2.4	1.9
22	9.69 055	22	9.75 028	30	10.24 972	9.94 027	7	38	6	3.0	2.9	2.3
23	9.69 077	22	9.75 058	29	10.24 942	9.94 020	8	37	7	3.5	3.4	2.7
24	9.69 100	22	9.75 087	30	10.24 913	9.94 012	7	36	8	4.0	3.9	3.1
25	9.69 122	22	9.75 117	29	10.24 883	9.94 005	7	35	9	4.5	4.4	3.4
26	9.69 144	22	9.75 146	30	10.24 854	9.93 998	7	34	10	5.0	4.8	3.8
27	9.69 167	22	9.75 176	29	10.24 824	9.93 991	7	33	20	10.0	9.7	7.7
28	9.69 189	22	9.75 205	30	10.24 795	9.93 984	7	32	30	15.0	14.5	11.5
29	9.69 212	22	9.75 235	29	10.24 765	9.93 977	7	31	40	20.0	19.3	15.3
30	9.69 234	22	9.75 264	30	10.24 736	9.93 970	7	30	50	25.0	24.2	19.2
31	9.69 256	22	9.75 294	29	10.24 706	9.93 963	7	29				
32	9.69 279	22	9.75 323	30	10.24 677	9.93 956	7	28				
33	9.69 301	22	9.75 353	29	10.24 647	9.93 948	7	27				
34	9.69 323	22	9.75 382	29	10.24 618	9.93 941	7	26				
35	9.69 345	22	9.75 411	30	10.24 589	9.93 934	7	25				
36	9.69 368	22	9.75 441	29	10.24 559	9.93 927	7	24				
37	9.69 390	22	9.75 470	30	10.24 530	9.93 920	7	23				
38	9.69 412	22	9.75 500	29	10.24 500	9.93 912	7	22				
39	9.69 434	22	9.75 529	29	10.24 471	9.93 905	7	21				
40	9.69 456	22	9.75 558	30	10.24 442	9.93 898	7	20				
41	9.69 479	22	9.75 588	29	10.24 412	9.93 891	7	19				
42	9.69 501	22	9.75 617	30	10.24 383	9.93 884	7	18				
43	9.69 523	22	9.75 647	29	10.24 353	9.93 876	7	17				
44	9.69 545	22	9.75 676	29	10.24 324	9.93 869	7	16				
45	9.69 567	22	9.75 705	30	10.24 295	9.93 862	7	15				
46	9.69 589	22	9.75 735	29	10.24 265	9.93 855	7	14				
47	9.69 611	22	9.75 764	29	10.24 236	9.93 847	7	13				
48	9.69 633	22	9.75 793	29	10.24 207	9.93 840	7	12				
49	9.69 655	22	9.75 822	30	10.24 178	9.93 833	7	11				
50	9.69 677	22	9.75 852	29	10.24 148	9.93 826	7	10				
51	9.69 699	22	9.75 881	29	10.24 119	9.93 819	7	9				
52	9.69 721	22	9.75 910	29	10.24 090	9.93 811	7	8				
53	9.69 743	22	9.75 939	30	10.24 061	9.93 804	7	7				
54	9.69 765	22	9.75 969	29	10.24 031	9.93 797	7	6				
55	9.69 787	22	9.75 998	29	10.24 002	9.93 789	7	5				
56	9.69 809	22	9.76 027	29	10.23 973	9.93 782	7	4				
57	9.69 831	22	9.76 056	30	10.23 944	9.93 775	7	3				
58	9.69 853	22	9.76 086	29	10.23 914	9.93 768	7	2				
59	9.69 875	22	9.76 115	29	10.23 885	9.93 760	7	1				
60	9.69 897	22	9.76 144	30	10.23 856	9.93 753	7	0				
'	L Cos	d	L Ctn	c d	L Tan	L Sin	d	'	Proportional parts			

119° (299°)

(240°) 60°

Table 6 COMMON LOGARITHMS OF TRIGONOMETRIC FUNCTIONS (continued) 247

The — 10 portion of the characteristic of the logarithm is not printed but must be written down whenever such a logarithm is used.

30° (210°)										(329°) 149°									
<i>n</i>	L Sin	<i>d</i>	L Tan	<i>c d</i>	L Ctn	L Cos	<i>d</i>	<i>n</i>	<i>n</i>	Proportional parts									
0	9.69 897	22	9.76 144	28	10.23 866	9.93 753	7	60											
1	9.69 919	22	9.76 173	29	10.23 827	9.93 746	8	59											
2	9.69 941	22	9.76 202	29	10.23 798	9.93 738	7	58											
3	9.69 953	21	9.76 231	30	10.23 769	9.93 731	7	57											
4	9.69 984	22	9.76 261	29	10.23 739	9.93 724	7	56											
5	9.70 006	22	9.76 290	29	10.23 710	9.93 717	8	55											
6	9.70 028	22	9.76 319	29	10.23 681	9.93 709	7	54											
7	9.70 050	22	9.76 348	29	10.23 652	9.93 702	7	53											
8	9.70 072	21	9.76 377	29	10.23 623	9.93 695	8	52											
9	9.70 093	22	9.76 406	29	10.23 594	9.93 687	7	51											
10	9.70 115	22	9.76 435	29	10.23 565	9.93 680	7	50											
11	9.70 137	22	9.76 464	29	10.23 536	9.93 673	8	49											
12	9.70 159	21	9.76 493	29	10.23 507	9.93 665	7	48											
13	9.70 180	22	9.76 522	29	10.23 478	9.93 658	8	47											
14	9.70 202	22	9.76 551	29	10.23 449	9.93 650	7	46											
15	9.70 224	21	9.76 580	29	10.23 420	9.93 643	7	45											
16	9.70 246	22	9.76 609	30	10.23 391	9.93 636	8	44											
17	9.70 267	21	9.76 639	29	10.23 361	9.93 628	7	43											
18	9.70 288	22	9.76 668	29	10.23 332	9.93 621	7	42											
19	9.70 310	22	9.76 697	28	10.23 303	9.93 614	8	41											
20	9.70 332	21	9.76 726	29	10.23 275	9.93 606	7	40											
21	9.70 353	22	9.76 754	29	10.23 246	9.93 599	8	39											
22	9.70 375	21	9.76 783	29	10.23 217	9.93 591	7	38											
23	9.70 396	22	9.76 812	29	10.23 188	9.93 584	7	37											
24	9.70 418	21	9.76 841	29	10.23 159	9.93 577	8	36											
25	9.70 439	22	9.76 870	29	10.23 130	9.93 569	7	35											
26	9.70 461	21	9.76 899	29	10.23 101	9.93 562	8	34											
27	9.70 482	22	9.76 928	29	10.23 072	9.93 554	7	33											
28	9.70 504	21	9.76 957	29	10.23 043	9.93 547	8	32											
29	9.70 525	22	9.76 986	29	10.23 014	9.93 539	7	31											
30	9.70 547	21	9.77 015	29	10.22 985	9.93 532	7	30											
31	9.70 568	22	9.77 044	29	10.22 956	9.93 525	8	29											
32	9.70 590	21	9.77 073	28	10.22 927	9.93 517	7	28											
33	9.70 611	22	9.77 101	29	10.22 898	9.93 510	8	27											
34	9.70 633	21	9.77 130	29	10.22 870	9.93 502	7	26											
35	9.70 654	22	9.77 159	29	10.22 841	9.93 495	8	25											
36	9.70 675	21	9.77 188	29	10.22 812	9.93 487	7	24											
37	9.70 697	22	9.77 217	29	10.22 783	9.93 480	8	23											
38	9.70 718	21	9.77 246	28	10.22 754	9.93 472	7	22											
39	9.70 739	22	9.77 274	29	10.22 726	9.93 465	8	21											
40	9.70 761	21	9.77 303	29	10.22 697	9.93 457	7	20											
41	9.70 782	22	9.77 332	29	10.22 668	9.93 450	8	19											
42	9.70 803	21	9.77 361	29	10.22 639	9.93 442	7	18											
43	9.70 824	22	9.77 390	28	10.22 610	9.93 435	8	17											
44	9.70 846	21	9.77 418	29	10.22 582	9.93 427	7	16											
45	9.70 867	22	9.77 447	29	10.22 553	9.93 420	8	15											
46	9.70 888	21	9.77 476	29	10.22 524	9.93 412	7	14											
47	9.70 909	22	9.77 505	28	10.22 495	9.93 405	8	13											
48	9.70 931	21	9.77 533	29	10.22 467	9.93 397	7	12											
49	9.70 952	22	9.77 562	29	10.22 438	9.93 390	8	11											
50	9.70 973	21	9.77 591	28	10.22 409	9.93 382	7	10											
51	9.70 994	22	9.77 619	29	10.22 381	9.93 375	8	9											
52	9.71 015	21	9.77 648	29	10.22 352	9.93 367	7	8											
53	9.71 036	22	9.77 677	29	10.22 323	9.93 360	8	7											
54	9.71 058	21	9.77 706	28	10.22 294	9.93 352	7	6											
55	9.71 079	22	9.77 734	29	10.22 266	9.93 344	8	5											
56	9.71 100	21	9.77 763	28	10.22 237	9.93 337	7	4											
57	9.71 121	22	9.77 791	29	10.22 209	9.93 329	8	3											
58	9.71 142	21	9.77 820	29	10.22 180	9.93 322	7	2											
59	9.71 163	22	9.77 849	28	10.22 151	9.93 314	8	1											
60	9.71 184		9.77 877		10.22 123	9.93 307		0											
<i>n</i>	L Cos	<i>d</i>	L Ctn	<i>c d</i>	L Tan	L Sin	<i>d</i>	<i>n</i>	<i>n</i>	Proportional parts									

120° (300°)

(329°) 59°

Table 6 COMMON LOGARITHMS OF TRIGONOMETRIC FUNCTIONS (continued) 248

The — 10 portion of the characteristic of the logarithm is not printed but must be written down whenever such a logarithm is used.

31° (211°)				(328°) 148°				Proportional parts	
'	L Sin	d	L Tan	c d	L Ctn	L Cos	d	'	
0	9.77 184	21	9.77 877	25	10.22 123	9.93 307	8	60	
1	9.77 205	41	9.77 906	25	10.22 094	9.93 299	8	58	
2	9.77 226	23	9.77 935	28	10.22 065	9.93 291	7	58	
3	9.77 247	21	9.77 963	29	10.22 037	9.93 284	8	57	
4	9.77 268	21	9.77 992	28	10.22 008	9.93 276	7	56	
5	9.77 289	21	9.78 020	28	10.21 980	9.93 269	8	55	
6	9.77 310	21	9.78 049	28	10.21 951	9.93 261	8	54	
7	9.77 331	21	9.78 077	29	10.21 923	9.93 253	7	53	
8	9.77 352	21	9.78 106	29	10.21 894	9.93 246	8	52	
9	9.77 373	20	9.78 135	28	10.21 865	9.93 238	8	51	
10	9.77 393	20	9.78 163	29	10.21 837	9.93 230	7	50	
11	9.77 414	21	9.78 192	28	10.21 808	9.93 223	8	49	
12	9.77 435	21	9.78 220	29	10.21 780	9.93 215	8	48	
13	9.77 456	21	9.78 249	28	10.21 751	9.93 207	7	47	
14	9.77 477	21	9.78 277	29	10.21 723	9.93 200	8	46	
15	9.77 498	21	9.78 306	28	10.21 694	9.93 192	7	45	
16	9.77 519	20	9.78 334	29	10.21 666	9.93 184	7	44	
17	9.77 539	21	9.78 363	28	10.21 637	9.93 177	8	43	
18	9.77 560	21	9.78 391	28	10.21 609	9.93 169	8	42	
19	9.77 581	21	9.78 419	29	10.21 581	9.93 161	7	41	
20	9.77 602	20	9.78 448	28	10.21 552	9.93 154	8	40	
21	9.77 622	21	9.78 476	29	10.21 524	9.93 146	8	39	
22	9.77 643	21	9.78 505	28	10.21 495	9.93 138	7	38	
23	9.77 664	21	9.78 533	29	10.21 467	9.93 131	8	37	
24	9.77 685	20	9.78 562	28	10.21 438	9.93 123	8	36	
25	9.77 705	21	9.78 590	28	10.21 410	9.93 115	7	35	
26	9.77 726	21	9.78 618	29	10.21 382	9.93 108	8	34	
27	9.77 747	20	9.78 647	28	10.21 353	9.93 100	8	33	
28	9.77 767	21	9.78 675	29	10.21 325	9.93 092	8	32	
29	9.77 788	21	9.78 704	28	10.21 296	9.93 084	7	31	
30	9.77 808	20	9.78 732	29	10.21 268	9.93 077	8	30	
31	9.77 829	21	9.78 760	29	10.21 240	9.93 069	8	29	
32	9.77 850	20	9.78 789	28	10.21 211	9.93 061	8	28	
33	9.77 870	21	9.78 817	29	10.21 183	9.93 053	7	27	
34	9.77 891	20	9.78 845	29	10.21 155	9.93 046	8	26	
35	9.77 911	21	9.78 874	28	10.21 126	9.93 038	8	25	
36	9.77 932	20	9.78 902	29	10.21 098	9.93 030	8	24	
37	9.77 952	21	9.78 930	29	10.21 070	9.93 022	8	23	
38	9.77 973	21	9.78 959	28	10.21 041	9.93 014	7	22	
39	9.77 994	20	9.78 987	29	10.21 013	9.93 007	8	21	
40	9.72 014	20	9.79 015	28	10.20 985	9.92 999	8	20	
41	9.72 034	21	9.79 043	29	10.20 957	9.92 991	8	19	
42	9.72 055	20	9.79 072	28	10.20 928	9.92 983	7	18	
43	9.72 075	21	9.79 100	29	10.20 900	9.92 976	8	17	
44	9.72 096	20	9.79 128	29	10.20 872	9.92 968	8	16	
45	9.72 116	21	9.79 156	29	10.20 844	9.92 960	8	15	
46	9.72 137	20	9.79 185	28	10.20 815	9.92 952	8	14	
47	9.72 157	20	9.79 213	29	10.20 787	9.92 944	8	13	
48	9.72 177	21	9.79 241	28	10.20 759	9.92 936	7	12	
49	9.72 198	20	9.79 269	28	10.20 731	9.92 929	8	11	
50	9.72 218	20	9.79 297	29	10.20 703	9.92 921	8	10	
51	9.72 238	21	9.79 326	28	10.20 674	9.92 913	8	9	
52	9.72 259	20	9.79 354	29	10.20 646	9.92 905	8	8	
53	9.72 279	20	9.79 382	28	10.20 618	9.92 897	8	7	
54	9.72 299	21	9.79 410	28	10.20 590	9.92 889	8	6	
55	9.72 320	20	9.79 438	29	10.20 562	9.92 881	7	5	
56	9.72 340	20	9.79 466	29	10.20 534	9.92 874	8	4	
57	9.72 360	21	9.79 495	28	10.20 505	9.92 866	8	3	
58	9.72 381	20	9.79 523	29	10.20 477	9.92 858	8	2	
59	9.72 401	20	9.79 551	28	10.20 449	9.92 850	8	1	
60	9.72 421	---	9.79 579	---	10.20 421	9.92 842	---	0	
'	L Cos	d	L Ctn	c d	L Tan	L Sin	d	'	Proportional parts

121° (301°)

(238°) 58°

Table 6 COMMON LOGARITHMS OF TRIGONOMETRIC FUNCTIONS (continued) 249

The — 10 portion of the characteristic of the logarithm is not printed but must be written down whenever such a logarithm is used.

32° (212°)						(327°) 147°					
°	L. Sin	d	L. Tan	c d	L. Ctn	L. Cos	d	Proportional parts			
0	9.72 421	20	9.79 578	20	10.20 421	9.92 842	8				
1	9.72 441	20	9.79 607	28	10.20 393	9.92 834	8				
2	9.72 461	21	9.79 635	33	10.20 365	9.92 826	8				
3	9.72 482	20	9.79 663	39	10.20 337	9.92 818	8				
4	9.72 502	20	9.79 691	48	10.20 308	9.92 810	7				
5	9.72 522	20	9.79 719	59	10.20 281	9.92 803	8				
6	9.72 542	20	9.79 747	73	10.20 253	9.92 795	8				
7	9.72 562	20	9.79 776	89	10.20 224	9.92 787	8				
8	9.72 582	20	9.79 804	108	10.20 196	9.92 779	8				
9	9.72 602	20	9.79 832	130	10.20 168	9.92 771	8				
10	9.72 622	21	9.79 860	155	10.20 140	9.92 763	8				
11	9.72 643	20	9.79 888	183	10.20 112	9.92 755	8				
12	9.72 663	20	9.79 916	213	10.20 084	9.92 747	8				
13	9.72 683	20	9.79 944	245	10.20 056	9.92 739	8				
14	9.72 703	20	9.79 972	280	10.20 028	9.92 731	8				
15	9.72 723	20	9.80 000	317	10.20 000	9.92 723	8				
16	9.72 743	20	9.80 028	357	10.19 972	9.92 715	8				
17	9.72 763	20	9.80 056	400	10.19 944	9.92 707	8				
18	9.72 783	20	9.80 084	445	10.19 916	9.92 699	8				
19	9.72 803	20	9.80 112	493	10.19 888	9.92 691	8				
20	9.72 823	20	9.80 140	543	10.19 860	9.92 683	8				
21	9.72 843	20	9.80 168	595	10.19 832	9.92 675	8				
22	9.72 863	20	9.80 195	650	10.19 805	9.92 667	8				
23	9.72 883	19	9.80 223	707	10.19 777	9.92 659	8				
24	9.72 902	20	9.80 251	767	10.19 749	9.92 651	8				
25	9.72 922	20	9.80 279	829	10.19 721	9.92 643	8				
26	9.72 942	20	9.80 307	893	10.19 693	9.92 635	8				
27	9.72 962	20	9.80 335	960	10.19 665	9.92 627	8				
28	9.72 982	20	9.80 363	1029	10.19 637	9.92 619	8				
29	9.73 002	20	9.80 391	1101	10.19 609	9.92 611	8				
30	9.73 022	19	9.80 419	1175	10.19 581	9.92 603	8				
31	9.73 041	20	9.80 447	1251	10.19 553	9.92 595	8				
32	9.73 061	20	9.80 474	1329	10.19 526	9.92 587	8				
33	9.73 081	20	9.80 502	1409	10.19 498	9.92 579	8				
34	9.73 101	20	9.80 530	1491	10.19 470	9.92 571	8				
35	9.73 121	19	9.80 558	1575	10.19 442	9.92 563	8				
36	9.73 140	20	9.80 586	1661	10.19 414	9.92 555	8				
37	9.73 160	20	9.80 614	1749	10.19 386	9.92 547	8				
38	9.73 180	20	9.80 642	1839	10.19 358	9.92 539	8				
39	9.73 200	19	9.80 669	1931	10.19 331	9.92 531	8				
40	9.73 219	20	9.80 697	2025	10.19 303	9.92 522	8				
41	9.73 239	20	9.80 725	2121	10.19 275	9.92 514	8				
42	9.73 258	19	9.80 753	2219	10.19 247	9.92 506	8				
43	9.73 278	20	9.80 781	2319	10.19 219	9.92 498	8				
44	9.73 298	20	9.80 808	2421	10.19 192	9.92 490	8				
45	9.73 318	19	9.80 836	2525	10.19 164	9.92 482	8				
46	9.73 337	20	9.80 864	2631	10.19 136	9.92 473	8				
47	9.73 357	20	9.80 892	2739	10.19 108	9.92 465	8				
48	9.73 377	19	9.80 919	2849	10.19 081	9.92 457	8				
49	9.73 396	20	9.80 947	2961	10.19 053	9.92 449	8				
50	9.73 416	19	9.80 975	3075	10.19 025	9.92 441	8				
51	9.73 435	20	9.81 003	3191	10.19 997	9.92 433	8				
52	9.73 455	19	9.81 030	3309	10.19 970	9.92 425	8				
53	9.73 474	20	9.81 058	3429	10.19 942	9.92 418	8				
54	9.73 494	19	9.81 086	3551	10.19 914	9.92 410	8				
55	9.73 513	20	9.81 113	3675	10.19 887	9.92 402	8				
56	9.73 533	19	9.81 141	3801	10.19 859	9.92 394	8				
57	9.73 552	20	9.81 169	3929	10.19 831	9.92 386	8				
58	9.73 572	19	9.81 196	4059	10.19 804	9.92 378	8				
59	9.73 591	20	9.81 224	4191	10.19 776	9.92 370	8				
60	9.73 611		9.81 252	4325	10.19 748	9.92 362	8				
°	L. Cos	d	L. Ctn	c d	L. Tan	L. Sh	d	Proportional parts			

122° (302°)

(237°) 57°

Table 6 COMMON LOGARITHMS OF TRIGONOMETRIC FUNCTIONS (continued) 250

The -10 portion of the characteristic of the logarithm is not printed but must be written down whenever such a logarithm is used.

33° (213°)				(326°) 146°							
<i>r</i>	L Sin	<i>d</i>	L Tan	<i>c d</i>	L Ctn	L Cos	<i>d</i>	<i>r</i>	Proportional parts		
0	9.73 611	19	9.81 252	27	10.18 748	9.92 359	8	60			
1	9.73 630	20	9.81 279	26	10.18 721	9.92 351	8	59			
2	9.73 650	19	9.81 307	26	10.18 693	9.92 343	8	58			
3	9.73 669	20	9.81 335	27	10.18 665	9.92 335	9	57			
4	9.73 689	19	9.81 362	26	10.18 638	9.92 326	8	56			
5	9.73 708	19	9.81 390	26	10.18 610	9.92 318	8	55			
6	9.73 727	20	9.81 418	27	10.18 582	9.92 310	8	54			
7	9.73 747	19	9.81 445	26	10.18 555	9.92 302	9	53			
8	9.73 766	19	9.81 473	27	10.18 527	9.92 293	8	52			
9	9.73 785	20	9.81 500	26	10.18 500	9.92 285	8	51			
10	9.73 805	19	9.81 528	26	10.18 472	9.92 277	8	50			
11	9.73 824	19	9.81 556	27	10.18 444	9.92 269	9	49			
12	9.73 843	20	9.81 583	26	10.18 417	9.92 260	8	48			
13	9.73 863	19	9.81 611	27	10.18 389	9.92 252	8	47			
14	9.73 882	19	9.81 638	26	10.18 362	9.92 244	9	46			
15	9.73 901	20	9.81 665	26	10.18 334	9.92 235	8	45			
16	9.73 921	19	9.81 693	26	10.18 307	9.92 227	8	44			
17	9.73 940	19	9.81 721	27	10.18 279	9.92 219	8	43			
18	9.73 959	19	9.81 748	26	10.18 252	9.92 211	9	42			
19	9.73 978	19	9.81 776	27	10.18 224	9.92 202	8	41			
20	9.73 997	20	9.81 803	26	10.18 197	9.92 194	8	40			
21	9.74 017	19	9.81 831	27	10.18 169	9.92 185	8	39			
22	9.74 036	19	9.81 858	26	10.18 142	9.92 177	8	38			
23	9.74 055	19	9.81 886	27	10.18 114	9.92 169	8	37			
24	9.74 074	19	9.81 913	26	10.18 87	9.92 161	9	36			
25	9.74 093	20	9.81 941	27	10.18 059	9.92 152	8	35			
26	9.74 113	19	9.81 968	26	10.18 032	9.92 144	8	34			
27	9.74 132	19	9.81 996	27	10.18 004	9.92 136	9	33			
28	9.74 151	19	9.82 023	26	10.17 977	9.92 127	8	32			
29	9.74 170	19	9.82 051	27	10.17 949	9.92 119	8	31			
30	9.74 189	19	9.82 078	26	10.17 922	9.92 111	9	30			
31	9.74 208	19	9.82 106	27	10.17 894	9.92 102	8	29			
32	9.74 227	19	9.82 133	26	10.17 867	9.92 094	8	28			
33	9.74 246	19	9.82 161	27	10.17 839	9.92 086	8	27			
34	9.74 265	19	9.82 188	27	10.17 812	9.92 077	8	26			
35	9.74 284	19	9.82 215	26	10.17 785	9.92 069	9	25			
36	9.74 303	19	9.82 243	27	10.17 757	9.92 060	8	24			
37	9.74 322	19	9.82 270	26	10.17 730	9.92 052	8	23			
38	9.74 341	19	9.82 298	27	10.17 702	9.92 044	9	22			
39	9.74 360	19	9.82 325	27	10.17 675	9.92 036	8	21			
40	9.74 379	19	9.82 352	26	10.17 648	9.92 027	9	20			
41	9.74 398	19	9.82 380	27	10.17 620	9.92 018	8	19			
42	9.74 417	19	9.82 407	26	10.17 593	9.92 010	8	18			
43	9.74 436	19	9.82 435	27	10.17 565	9.92 002	9	17			
44	9.74 455	19	9.82 462	27	10.17 538	9.91 993	8	16			
45	9.74 474	19	9.82 489	26	10.17 511	9.91 985	9	15			
46	9.74 493	19	9.82 517	27	10.17 483	9.91 976	8	14			
47	9.74 512	19	9.82 544	27	10.17 456	9.91 968	9	13			
48	9.74 531	19	9.82 571	26	10.17 429	9.91 959	8	12			
49	9.74 549	19	9.82 599	27	10.17 401	9.91 951	9	11			
50	9.74 568	19	9.82 626	27	10.17 374	9.91 942	8	10			
51	9.74 587	19	9.82 653	26	10.17 347	9.91 934	9	9			
52	9.74 606	19	9.82 681	27	10.17 319	9.91 925	8	8			
53	9.74 625	19	9.82 708	27	10.17 292	9.91 917	9	7			
54	9.74 644	19	9.82 735	27	10.17 265	9.91 908	8	6			
55	9.74 662	19	9.82 762	26	10.17 238	9.91 900	9	5			
56	9.74 681	19	9.82 790	27	10.17 210	9.91 891	8	4			
57	9.74 700	19	9.82 817	27	10.17 183	9.91 883	9	3			
58	9.74 719	19	9.82 844	27	10.17 156	9.91 874	8	2			
59	9.74 737	19	9.82 871	26	10.17 128	9.91 865	9	1			
60	9.74 755	19	9.82 899	26	10.17 101	9.91 857	8	0			
<i>r</i>	L Cos	<i>d</i>	L Ctn	<i>c d</i>	L Tan	L Sin	<i>d</i>	<i>r</i>	Proportional parts		

123° (303°)

(236°) 56°

Table 6 COMMON LOGARITHMS OF TRIGONOMETRIC FUNCTIONS (continued) 251

The — 10 portion of the characteristic of the logarithm is not printed but must be written down whenever such a logarithm is used.

34° (214°)				325° 145°				Proportional parts					
	L Sin	d		L Tan	c d	L Cot		L Cos	d				
0	9.74 756	19		9.82 899	27	10.17 101		9.91 857	3		60		
1	9.74 775	19		9.82 926	27	10.17 074		9.91 849	3		59		
2	9.74 794	19		9.82 953	27	10.17 047		9.91 840	3		58		
3	9.74 812	19		9.82 980	27	10.17 020		9.91 832	3		57		
4	9.74 831	19		9.83 008	27	10.16 992		9.91 823	3		56		
5	9.74 850	18		9.83 035	27	10.16 965		9.91 815	3		55		
6	9.74 868	18		9.83 062	27	10.16 938		9.91 806	3		54		
7	9.74 887	18		9.83 089	28	10.16 911		9.91 798	3		53		
8	9.74 906	18		9.83 117	27	10.16 883		9.91 789	3		52		
9	9.74 924	18		9.83 144	27	10.16 856		9.91 781	3		51		
10	9.74 943	18		9.83 171	27	10.16 829		9.91 772	3		50		
11	9.74 961	18		9.83 198	27	10.16 802		9.91 763	3		49		
12	9.74 980	18		9.83 225	27	10.16 775		9.91 755	3		48		
13	9.74 999	18		9.83 252	28	10.16 748		9.91 746	3		47		
14	9.75 017	18		9.83 280	27	10.16 720		9.91 738	3		46		
15	9.75 036	18		9.83 307	27	10.16 693		9.91 729	3		45		
16	9.75 054	18		9.83 334	27	10.16 666		9.91 720	3		44		
17	9.75 073	18		9.83 361	27	10.16 639		9.91 712	3		43		
18	9.75 091	18		9.83 388	27	10.16 612		9.91 703	3		42		
19	9.75 110	18		9.83 415	27	10.16 585		9.91 695	3		41		
20	9.75 128	18		9.83 442	28	10.16 558		9.91 686	3		40		
21	9.75 147	18		9.83 470	27	10.16 530		9.91 677	3		39		
22	9.75 165	18		9.83 497	27	10.16 503		9.91 669	3		38		
23	9.75 184	18		9.83 524	27	10.16 476		9.91 660	3		37		
24	9.75 202	18		9.83 551	27	10.16 449		9.91 651	3		36		
25	9.75 221	18		9.83 578	27	10.16 422		9.91 643	3		35		
26	9.75 239	18		9.83 605	27	10.16 395		9.91 634	3		34		
27	9.75 258	18		9.83 632	27	10.16 368		9.91 625	3		33		
28	9.75 276	18		9.83 659	27	10.16 341		9.91 617	3		32		
29	9.75 294	18		9.83 686	27	10.16 314		9.91 608	3		31		
30	9.75 313	18		9.83 713	27	10.16 287		9.91 599	3		30		
31	9.75 331	18		9.83 740	28	10.16 260		9.91 591	3		29		
32	9.75 350	18		9.83 768	27	10.16 232		9.91 582	3		28		
33	9.75 368	18		9.83 795	27	10.16 205		9.91 573	3		27		
34	9.75 386	18		9.83 822	27	10.16 178		9.91 565	3		26		
35	9.75 405	18		9.83 849	27	10.16 151		9.91 556	3		25		
36	9.75 423	18		9.83 876	27	10.16 124		9.91 547	3		24		
37	9.75 441	18		9.83 903	27	10.16 097		9.91 538	3		23		
38	9.75 459	18		9.83 930	27	10.16 070		9.91 530	3		22		
39	9.75 478	18		9.83 957	27	10.16 043		9.91 521	3		21		
40	9.75 496	18		9.83 984	27	10.16 016		9.91 512	3		20		
41	9.75 514	18		9.84 011	27	10.15 989		9.91 504	3		19		
42	9.75 533	18		9.84 038	27	10.15 962		9.91 495	3		18		
43	9.75 551	18		9.84 065	27	10.15 935		9.91 486	3		17		
44	9.75 569	18		9.84 092	27	10.15 908		9.91 477	3		16		
45	9.75 587	18		9.84 119	27	10.15 881		9.91 469	3		15		
46	9.75 605	18		9.84 146	27	10.15 854		9.91 460	3		14		
47	9.75 624	18		9.84 173	27	10.15 827		9.91 451	3		13		
48	9.75 642	18		9.84 200	27	10.15 800		9.91 442	3		12		
49	9.75 660	18		9.84 227	27	10.15 773		9.91 433	3		11		
50	9.75 678	18		9.84 254	26	10.15 746		9.91 425	3		10		
51	9.75 696	18		9.84 280	27	10.15 720		9.91 416	3		9		
52	9.75 714	18		9.84 307	27	10.15 693		9.91 407	3		8		
53	9.75 733	18		9.84 334	27	10.15 666		9.91 398	3		7		
54	9.75 751	18		9.84 361	27	10.15 639		9.91 389	3		6		
55	9.75 769	18		9.84 388	27	10.15 612		9.91 381	3		5		
56	9.75 787	18		9.84 415	27	10.15 585		9.91 372	3		4		
57	9.75 805	18		9.84 442	27	10.15 558		9.91 363	3		3		
58	9.75 823	18		9.84 469	27	10.15 531		9.91 354	3		2		
59	9.75 841	18		9.84 496	27	10.15 504		9.91 345	3		1		
60	9.75 859	18		9.84 523	27	10.15 477		9.91 336	3		0		
	L Cos	d		L Ctn	c d	L Tan		L Sin	d				

124° (304°)

(325°) 55°

Table 6 COMMON LOGARITHMS OF TRIGONOMETRIC FUNCTIONS (continued) 252

The — 10 portion of the characteristic of the logarithm is not printed but must be written down whenever such a logarithm is used.

85° (215°)						324° 144°						Proportional parts			
	L Sin	d	L Tan	c d	L Cot	L Cos	d								
0	9.75 859	18	9.84 523	27	10.15 477	9.91 355	8	60							
1	9.75 877	16	9.84 550	26	10.15 450	9.91 328	9	59							
2	9.75 896	18	9.84 576	27	10.15 424	9.91 319	9	58							
3	9.75 913	18	9.84 602	27	10.15 397	9.91 310	9	57							
4	9.75 931	18	9.84 630	27	10.15 370	9.91 301	9	56							
5	9.75 949	18	9.84 657	27	10.15 343	9.91 292	9	55							
6	9.75 967	18	9.84 684	27	10.15 316	9.91 283	9	54							
7	9.75 986	18	9.84 711	27	10.15 289	9.91 274	8	53							
8	9.76 003	18	9.84 738	26	10.15 262	9.91 265	9	52							
9	9.76 021	18	9.84 764	27	10.15 236	9.91 257	9	51							
10	9.76 039	18	9.84 791	27	10.15 209	9.91 248	9	50							
11	9.76 057	18	9.84 818	27	10.15 182	9.91 239	9	49							
12	9.76 075	18	9.84 845	27	10.15 155	9.91 230	9	48							
13	9.76 093	18	9.84 872	27	10.15 128	9.91 221	9	47							
14	9.76 111	18	9.84 899	26	10.15 101	9.91 212	9	46							
15	9.76 129	17	9.84 925	27	10.15 075	9.91 203	9	45							
16	9.76 146	18	9.84 952	27	10.15 048	9.91 194	9	44							
17	9.76 164	18	9.84 979	27	10.15 021	9.91 185	9	43							
18	9.76 182	18	9.85 006	27	10.14 994	9.91 176	9	42							
19	9.76 200	18	9.85 033	26	10.14 967	9.91 167	9	41							
20	9.76 218	18	9.85 059	27	10.14 941	9.91 159	9	40							
21	9.76 236	17	9.85 086	27	10.14 914	9.91 149	9	39							
22	9.76 253	18	9.85 113	27	10.14 887	9.91 141	9	38							
23	9.76 271	18	9.85 140	26	10.14 860	9.91 132	9	37							
24	9.76 289	18	9.85 166	27	10.14 834	9.91 123	9	36							
25	9.76 307	17	9.85 193	27	10.14 807	9.91 114	9	35							
26	9.76 324	18	9.85 220	27	10.14 780	9.91 105	9	34							
27	9.76 342	18	9.85 247	26	10.14 753	9.91 096	9	33							
28	9.76 360	18	9.85 273	27	10.14 727	9.91 087	9	32							
29	9.76 378	17	9.85 300	27	10.14 700	9.91 078	9	31							
30	9.76 395	18	9.85 327	27	10.14 673	9.91 069	9	30							
31	9.76 413	18	9.85 354	26	10.14 646	9.91 060	9	29							
32	9.76 431	17	9.85 380	27	10.14 620	9.91 051	9	28							
33	9.76 448	18	9.85 407	27	10.14 593	9.91 042	9	27							
34	9.76 466	18	9.85 434	26	10.14 566	9.91 033	10	26							
35	9.76 484	17	9.85 460	27	10.14 540	9.91 023	9	25							
36	9.76 501	18	9.85 487	27	10.14 513	9.91 014	9	24							
37	9.76 519	18	9.85 513	26	10.14 486	9.91 006	9	23							
38	9.76 537	17	9.85 540	27	10.14 460	9.90 996	9	22							
39	9.76 554	18	9.85 567	27	10.14 433	9.90 987	9	21							
40	9.76 572	18	9.85 594	26	10.14 406	9.90 978	9	20							
41	9.76 590	17	9.85 620	27	10.14 380	9.90 969	9	19							
42	9.76 607	18	9.85 647	27	10.14 353	9.90 960	9	18							
43	9.76 625	17	9.85 674	26	10.14 326	9.90 951	9	17							
44	9.76 642	18	9.85 700	27	10.14 300	9.90 942	9	16							
45	9.76 660	17	9.85 727	27	10.14 273	9.90 933	9	15							
46	9.76 677	18	9.85 754	26	10.14 246	9.90 924	9	14							
47	9.76 695	17	9.85 780	27	10.14 220	9.90 915	9	13							
48	9.76 712	18	9.85 807	27	10.14 193	9.90 906	9	12							
49	9.76 730	17	9.85 834	26	10.14 166	9.90 896	10	11							
50	9.76 747	18	9.85 860	27	10.14 140	9.90 887	9	10							
51	9.76 765	17	9.85 887	26	10.14 113	9.90 878	9	9							
52	9.76 782	18	9.85 913	27	10.14 087	9.90 869	9	8							
53	9.76 800	17	9.85 940	27	10.14 060	9.90 860	9	7							
54	9.76 817	18	9.85 967	26	10.14 033	9.90 851	9	6							
55	9.76 835	17	9.85 993	27	10.14 007	9.90 842	10	5							
56	9.76 852	18	9.86 020	26	10.13 980	9.90 832	9	4							
57	9.76 870	17	9.86 046	27	10.13 954	9.90 823	9	3							
58	9.76 887	18	9.86 073	27	10.13 927	9.90 814	9	2							
59	9.76 904	18	9.86 100	26	10.13 900	9.90 805	9	1							
60	9.76 922		9.86 126		10.13 874	9.90 796		0							
	L Cos	d	L Cot	c d	L Tan	L Sin	d								

125° (305°)

(234°) 54°

Proportional parts		
27	26	18
1	0.4	0.4
2	0.9	0.5
3	1.4	1.3
4	1.8	1.7
5	2.2	2.2
6	2.7	2.6
7	3.2	3.0
8	3.6	3.5
9	4.0	3.9
10	4.5	4.3
20	9.0	8.7
30	13.5	13.0
40	18.0	17.3
50	22.5	21.7
17	10	
1	0.3	0.2
2	0.6	0.3
3	0.8	0.5
4	1.1	0.7
5	1.4	0.8
6	1.7	1.0
7	2.0	1.2
8	2.3	1.3
9	2.6	1.5
10	2.8	1.7
20	5.7	3.3
30	8.5	5.0
40	11.3	6.7
50	14.2	8.3
9	8	
1	0.2	0.1
2	0.3	0.3
3	0.4	0.4
4	0.6	0.5
5	0.8	0.7
6	0.9	0.8
7	1.0	0.9
8	1.2	1.1
9	1.4	1.2
10	1.5	1.3
20	3.0	2.7
30	4.5	4.0
40	6.0	5.3
50	7.5	6.7
Proportional parts		

Table 6 COMMON LOGARITHMS OF TRIGONOMETRIC FUNCTIONS (continued) 253

The -10 portion of the characteristic of the logarithm is not printed but must be written down whenever such a logarithm is used.

36° (216°)				(323°) 143°			
L Sin	d	L Tan	c d	L Ctn	L Cos	d	Proportional parts
0	9.76 922	17	9.86 126	27	10.13 874	9.90 796	9 60
1	9.76 939	18	9.86 153	26	10.13 847	9.90 787	10 59
2	9.76 957	17	9.86 179	27	10.13 821	9.90 777	5 58
3	9.76 974	17	9.86 206	26	10.13 794	9.90 768	3 57
4	9.76 991	18	9.86 232	27	10.13 768	9.90 759	9 56
5	9.77 009	17	9.86 259	26	10.13 741	9.90 750	9 55
6	9.77 026	17	9.86 285	27	10.13 715	9.90 741	10 54
7	9.77 043	18	9.86 312	26	10.13 688	9.90 731	5 53
8	9.77 061	17	9.86 338	27	10.13 662	9.90 722	3 52
9	9.77 078	17	9.86 365	27	10.13 635	9.90 713	9 51
10	9.77 095	17	9.86 392	26	10.13 608	9.90 704	10 50
11	9.77 112	18	9.86 418	27	10.13 582	9.90 694	9 49
12	9.77 130	17	9.86 445	26	10.13 555	9.90 685	9 48
13	9.77 147	17	9.86 471	27	10.13 529	9.90 676	9 47
14	9.77 164	17	9.86 498	26	10.13 502	9.90 667	10 46
15	9.77 181	18	9.86 524	27	10.13 476	9.90 657	9 45
16	9.77 199	17	9.86 551	26	10.13 449	9.90 648	9 44
17	9.77 216	17	9.86 577	26	10.13 423	9.90 639	9 43
18	9.77 233	17	9.86 603	27	10.13 397	9.90 630	10 42
19	9.77 250	18	9.86 630	26	10.13 370	9.90 620	9 41
20	9.77 268	17	9.86 656	27	10.13 344	9.90 611	9 40
21	9.77 285	17	9.86 683	26	10.13 317	9.90 602	10 39
22	9.77 302	17	9.86 709	27	10.13 291	9.90 592	9 38
23	9.77 319	17	9.86 735	26	10.13 264	9.90 583	9 37
24	9.77 336	17	9.86 762	27	10.13 238	9.90 574	9 36
25	9.77 353	17	9.86 789	26	10.13 211	9.90 565	10 35
26	9.77 370	17	9.86 815	27	10.13 185	9.90 555	9 34
27	9.77 387	18	9.86 842	26	10.13 158	9.90 546	9 33
28	9.77 405	17	9.86 868	26	10.13 132	9.90 537	10 32
29	9.77 422	17	9.86 894	27	10.13 106	9.90 527	9 31
30	9.77 439	17	9.86 921	26	10.13 079	9.90 518	9 30
31	9.77 456	17	9.86 947	27	10.13 053	9.90 509	9 29
32	9.77 473	17	9.86 974	26	10.13 026	9.90 499	9 28
33	9.77 490	17	9.87 000	27	10.13 000	9.90 490	10 27
34	9.77 507	17	9.87 027	26	10.12 973	9.90 480	9 26
35	9.77 524	17	9.87 053	26	10.12 947	9.90 471	9 25
36	9.77 541	18	9.87 079	27	10.12 921	9.90 462	10 24
37	9.77 558	17	9.87 106	26	10.12 894	9.90 452	9 23
38	9.77 575	17	9.87 132	26	10.12 868	9.90 443	9 22
39	9.77 592	17	9.87 158	27	10.12 842	9.90 434	10 21
40	9.77 609	17	9.87 185	26	10.12 815	9.90 424	9 20
41	9.77 626	17	9.87 211	27	10.12 789	9.90 415	10 19
42	9.77 643	17	9.87 238	26	10.12 762	9.90 405	9 18
43	9.77 660	17	9.87 264	26	10.12 735	9.90 396	10 17
44	9.77 677	17	9.87 290	27	10.12 710	9.90 386	9 16
45	9.77 694	17	9.87 317	26	10.12 683	9.90 377	9 15
46	9.77 711	17	9.87 343	26	10.12 657	9.90 368	10 14
47	9.77 728	16	9.87 369	27	10.12 631	9.90 358	9 13
48	9.77 744	17	9.87 396	26	10.12 604	9.90 349	10 12
49	9.77 761	17	9.87 422	26	10.12 578	9.90 339	9 11
50	9.77 778	17	9.87 448	27	10.12 552	9.90 330	10 10
51	9.77 795	17	9.87 475	26	10.12 525	9.90 320	9 9
52	9.77 812	17	9.87 501	26	10.12 499	9.90 311	10 8
53	9.77 829	17	9.87 527	27	10.12 473	9.90 301	9 7
54	9.77 846	16	9.87 554	26	10.12 446	9.90 292	10 6
55	9.77 862	17	9.87 580	26	10.12 420	9.90 282	9 5
56	9.77 879	17	9.87 606	27	10.12 394	9.90 273	10 4
57	9.77 896	17	9.87 633	26	10.12 367	9.90 263	9 3
58	9.77 913	17	9.87 659	26	10.12 341	9.90 254	10 2
59	9.77 930	16	9.87 685	26	10.12 315	9.90 244	9 1
60	9.77 946		9.87 711		10.12 289	9.90 235	0
	L Cos	d	L Ctn	c d	L Tan	L Sin	d

126° (306°)

(333°) 53°

Proportional parts		
27	26	
1	0.4	0.4
2	0.9	0.9
3	1.4	1.3
4	1.8	1.7
5	2.2	2.2
6	2.7	2.6
7	3.2	3.0
8	3.6	3.5
9	4.0	3.9
10	4.5	4.3
20	9.0	8.7
30	13.5	13.0
40	18.0	17.3
50	22.5	21.7
16	17	16
1	0.2	0.2
2	0.5	0.5
3	0.9	0.8
4	1.2	1.1
5	1.5	1.4
6	1.8	1.7
7	2.1	2.0
8	2.4	2.3
9	2.7	2.6
10	3.0	2.8
20	6.0	5.7
30	9.0	8.6
40	12.0	11.2
50	15.0	14.2
10	9	
1	0.2	0.2
2	0.3	0.3
3	0.5	0.4
4	0.7	0.6
5	0.8	0.6
6	1.0	0.9
7	1.2	1.0
8	1.3	1.2
9	1.5	1.4
10	1.7	1.5
20	3.3	3.0
30	5.0	4.5
40	6.7	6.0
50	8.3	7.5

Proportional parts

Table 6 COMMON LOGARITHMS OF TRIGONOMETRIC FUNCTIONS (continued) 254

The — 10 portion of the characteristic of the logarithm is not printed but must be written down whenever such a logarithm is used.

37° (217°)				(322°) 142°					
<i>n</i>	L Sin	<i>d</i>	L Tan	<i>n d</i>	L Cot	L Cos	<i>d</i>	Proportional parts	
0	9.77 946	17	9.87 711	27	10.12 289	9.90 235	10	60	
1	9.77 953	17	9.87 738	26	10.12 252	9.90 228	9	59	
2	9.77 980	17	9.87 754	26	10.12 236	9.90 216	10	58	
3	9.77 997	16	9.87 790	27	10.12 210	9.90 206	9	57	
4	9.78 013	17	9.87 817	26	10.12 183	9.90 197	10	56	
5	9.78 030	17	9.87 843	26	10.12 157	9.90 187	9	55	
6	9.78 047	16	9.87 866	26	10.12 131	9.90 178	10	54	" 27 26
7	9.78 053	17	9.87 895	27	10.12 105	9.90 168	9	53	1 5.4 0.4
8	9.78 080	17	9.87 922	26	10.12 078	9.90 159	10	52	2 0.9 0.9
9	9.78 097	16	9.87 948	26	10.12 052	9.90 149	10	51	3 1.4 1.3
10	9.78 113	17	9.87 974	26	10.12 026	9.90 139	9	50	4 1.8 1.7
11	9.78 130	17	9.88 000	27	10.12 000	9.90 130	10	49	
12	9.78 147	16	9.88 027	26	10.11 973	9.90 120	9	48	5 2.2 2.2
13	9.78 163	17	9.88 053	26	10.11 947	9.90 111	10	47	6 2.7 2.6
14	9.78 180	17	9.88 079	26	10.11 921	9.90 101	10	46	7 3.2 3.0
15	9.78 197	16	9.88 105	26	10.11 895	9.90 091	9	45	8 3.6 3.6
16	9.78 213	17	9.88 131	27	10.11 869	9.90 082	10	44	9 4.0 3.9
17	9.78 230	16	9.88 158	26	10.11 842	9.90 072	9	43	10 4.5 4.3
18	9.78 246	17	9.88 184	26	10.11 816	9.90 063	10	42	20 9.0 8.7
19	9.78 263	17	9.88 210	26	10.11 790	9.90 053	10	41	30 13.5 13.0
20	9.78 280	16	9.88 236	26	10.11 764	9.90 043	9	40	40 18.0 17.3
21	9.78 298	17	9.88 262	27	10.11 738	9.90 034	10	39	50 22.5 21.7
22	9.78 313	16	9.88 289	26	10.11 711	9.90 024	10	38	
23	9.78 329	17	9.88 315	26	10.11 685	9.90 014	9	37	" 17 16
24	9.78 345	16	9.88 341	26	10.11 659	9.90 005	10	36	1 0.3 0.3
25	9.78 362	17	9.88 367	26	10.11 633	9.89 995	10	35	2 0.6 0.6
26	9.78 379	16	9.88 393	27	10.11 607	9.89 985	9	34	3 0.8 0.8
27	9.78 395	17	9.88 420	24	10.11 580	9.89 976	10	33	4 1.1 1.1
28	9.78 412	16	9.88 446	26	10.11 554	9.89 966	10	32	
29	9.78 428	17	9.88 472	26	10.11 528	9.89 956	9	31	5 1.4 1.3
30	9.78 445	16	9.88 498	26	10.11 502	9.89 947	10	30	6 1.7 1.6
31	9.78 461	17	9.88 524	26	10.11 476	9.89 937	10	29	7 2.0 1.9
32	9.78 478	16	9.88 550	27	10.11 450	9.89 927	9	28	8 2.3 2.1
33	9.78 494	16	9.88 577	26	10.11 423	9.89 918	10	27	9 2.6 2.4
34	9.78 510	17	9.88 603	26	10.11 397	9.89 908	10	26	
35	9.78 527	16	9.88 629	26	10.11 371	9.89 898	10	25	10 2.6 2.7
36	9.78 543	17	9.88 655	26	10.11 345	9.89 888	9	24	20 5.7 5.3
37	9.78 560	16	9.88 681	26	10.11 319	9.89 879	10	23	30 8.5 8.0
38	9.78 575	16	9.88 707	26	10.11 293	9.89 869	10	22	40 11.3 10.7
39	9.78 592	17	9.88 733	26	10.11 267	9.89 859	10	21	50 14.7 13.3
40	9.78 609	16	9.88 759	27	10.11 241	9.89 849	9	20	
41	9.78 625	17	9.88 785	26	10.11 214	9.89 840	10	19	" 10 9
42	9.78 642	16	9.88 812	26	10.11 188	9.89 830	10	18	1 0.2 0.2
43	9.78 658	16	9.88 838	26	10.11 162	9.89 820	10	17	2 0.3 0.3
44	9.78 674	17	9.88 864	26	10.11 136	9.89 810	9	16	3 0.5 0.5
45	9.78 691	16	9.88 890	26	10.11 110	9.89 801	10	15	4 0.7 0.6
46	9.78 707	16	9.88 916	26	10.11 084	9.89 791	10	14	5 0.8 0.8
47	9.78 723	16	9.88 942	26	10.11 058	9.89 781	10	13	6 1.0 0.9
48	9.78 739	17	9.88 968	26	10.11 032	9.89 771	10	12	7 1.2 1.0
49	9.78 755	16	9.88 994	26	10.11 006	9.89 761	9	11	8 1.3 1.2
50	9.78 772	16	9.89 020	26	10.10 980	9.89 750	10	10	9 1.5 1.4
51	9.78 788	17	9.89 046	27	10.10 954	9.89 742	10	9	10 1.7 1.5
52	9.78 805	16	9.89 073	26	10.10 927	9.89 732	10	8	20 3.3 3.0
53	9.78 821	16	9.89 099	26	10.10 901	9.89 722	10	7	30 5.0 4.5
54	9.78 837	16	9.89 125	26	10.10 875	9.89 712	10	6	40 6.7 6.0
55	9.78 853	16	9.89 151	26	10.10 849	9.89 702	9	5	50 8.3 7.5
56	9.78 869	17	9.89 177	26	10.10 823	9.89 692	10	4	
57	9.78 886	16	9.89 203	26	10.10 797	9.89 683	10	3	
58	9.78 902	16	9.89 229	26	10.10 771	9.89 673	10	2	
59	9.78 918	16	9.89 255	26	10.10 745	9.89 663	10	1	
60	9.78 934		9.89 281		10.10 719	9.89 653		0	
<i>n</i>	L Cos	<i>d</i>	L Cot	<i>n d</i>	L Tan	L Sin	<i>d</i>	Proportional parts	

127° (307°)

(232°) 52°

Table 6 COMMON LOGARITHMS OF TRIGONOMETRIC FUNCTIONS (continued) 255

The -10 portion of the characteristic of the logarithm is not printed but must be written down whenever such a logarithm is used.

36° (218°)				(321°) 141°				Proportional parts		
	L Sin	d	L Tan	c d	L Ctn	L Cos	d			
0	9.78 934	16	9.89 281	26	10.10 719	9.89 653	10			
1	9.78 960	17	9.89 307	24	10.10 693	9.89 643	10			
2	9.78 987	16	9.89 333	26	10.10 667	9.89 633	9			
3	9.78 983	16	9.89 359	26	10.10 641	9.89 624	10			
4	9.78 999	16	9.89 385	26	10.10 615	9.89 614	10			
5	9.79 015	16	9.89 411	26	10.10 589	9.89 604	10			
6	9.79 021	16	9.89 437	26	10.10 563	9.89 594	10			
7	9.79 047	16	9.89 463	26	10.10 537	9.89 584	10			
8	9.79 053	16	9.89 489	26	10.10 511	9.89 574	10			
9	9.79 079	16	9.89 515	26	10.10 485	9.89 564	10			
10	9.79 095	16	9.89 541	26	10.10 459	9.89 554	10			
11	9.79 111	17	9.89 567	26	10.10 433	9.89 544	10			
12	9.79 128	16	9.89 593	26	10.10 407	9.89 534	10			
13	9.79 144	16	9.89 619	26	10.10 381	9.89 524	10			
14	9.79 160	16	9.89 645	26	10.10 355	9.89 514	10			
15	9.79 176	16	9.89 671	26	10.10 329	9.89 504	9			
16	9.79 192	16	9.89 697	26	10.10 303	9.89 495	10			
17	9.79 208	16	9.89 723	26	10.10 277	9.89 485	10			
18	9.79 224	16	9.89 749	26	10.10 251	9.89 475	10			
19	9.79 240	16	9.89 775	26	10.10 225	9.89 465	10			
20	9.79 256	16	9.89 801	26	10.10 199	9.89 455	10			
21	9.79 272	16	9.89 827	26	10.10 173	9.89 445	10			
22	9.79 288	16	9.89 853	26	10.10 147	9.89 435	10			
23	9.79 304	15	9.89 879	26	10.10 121	9.89 425	10			
24	9.79 319	16	9.89 905	26	10.10 095	9.89 415	10			
25	9.79 335	16	9.89 931	26	10.10 069	9.89 405	10			
26	9.79 351	16	9.89 957	26	10.10 043	9.89 395	10			
27	9.79 367	16	9.89 983	26	10.10 017	9.89 385	10			
28	9.79 383	16	9.90 009	26	10.09 991	9.89 375	11			
29	9.79 399	16	9.90 035	26	10.09 965	9.89 364	10			
30	9.79 415	16	9.90 061	25	10.09 939	9.89 354	10			
31	9.79 431	16	9.90 086	26	10.09 914	9.89 344	10			
32	9.79 447	16	9.90 112	24	10.09 888	9.89 334	10			
33	9.79 463	15	9.90 138	26	10.09 862	9.89 324	10			
34	9.79 478	16	9.90 164	26	10.09 836	9.89 314	10			
35	9.79 494	16	9.90 190	26	10.09 810	9.89 304	10			
36	9.79 510	16	9.90 216	26	10.09 784	9.89 294	10			
37	9.79 526	16	9.90 242	26	10.09 758	9.89 284	10			
38	9.79 542	16	9.90 268	26	10.09 732	9.89 274	10			
39	9.79 558	15	9.90 294	26	10.09 706	9.89 264	10			
40	9.79 573	16	9.90 320	26	10.09 680	9.89 254	10			
41	9.79 589	16	9.90 346	25	10.09 654	9.89 244	11			
42	9.79 605	16	9.90 371	26	10.09 629	9.89 233	10			
43	9.79 621	15	9.90 397	26	10.09 603	9.89 223	10			
44	9.79 636	16	9.90 423	26	10.09 577	9.89 213	10			
45	9.79 652	16	9.90 449	26	10.09 551	9.89 203	10			
46	9.79 668	16	9.90 475	26	10.09 525	9.89 193	10			
47	9.79 684	16	9.90 501	26	10.09 499	9.89 183	10			
48	9.79 699	16	9.90 527	26	10.09 473	9.89 173	11			
49	9.79 715	16	9.90 553	25	10.09 447	9.89 162	10			
50	9.79 731	15	9.90 578	26	10.09 422	9.89 152	10			
51	9.79 746	16	9.90 604	26	10.09 396	9.89 142	10			
52	9.79 762	15	9.90 630	26	10.09 370	9.89 132	10			
53	9.79 778	15	9.90 656	26	10.09 344	9.89 122	10			
54	9.79 793	16	9.90 682	26	10.09 318	9.89 112	11			
55	9.79 809	15	9.90 708	26	10.09 292	9.89 101	10			
56	9.79 825	15	9.90 734	25	10.09 266	9.89 091	10			
57	9.79 840	15	9.90 759	26	10.09 241	9.89 081	10			
58	9.79 856	16	9.90 785	26	10.09 215	9.89 071	11			
59	9.79 872	15	9.90 811	26	10.09 189	9.89 060	10			
60	9.79 887		9.90 837		10.09 163	9.89 050				
	L Cos	d	L Ctn	c d	L Tan	L Sin	d	Proportional parts		

128° (308°)

(321°) 51°

Table 6 COMMON LOGARITHMS OF TRIGONOMETRIC FUNCTIONS (continued) 256

The — 10 portion of the characteristic of the logarithm is not printed but must be written down whenever such a logarithm is used.

39° (319°)					(320°) 140°					Proportional parts	
'	L Sin	d	L Tan	c d	L Ctn	L Cos	d	'			
0	9.79 887	16	9.90 837	26	10.09 163	9.89 050	10	60			
1	9.79 903	15	9.90 863	26	10.09 137	9.89 040	10	59			
2	9.79 918	15	9.90 889	26	10.09 111	9.89 030	10	58			
3	9.79 934	15	9.90 914	26	10.09 086	9.89 020	11	57			
4	9.79 950	15	9.90 940	26	10.09 060	9.89 009	10	56			
5	9.79 965	15	9.90 966	26	10.09 034	9.88 999	10	55			
6	9.79 981	15	9.90 992	26	10.09 008	9.88 989	11	54		26 25	
7	9.79 996	15	9.91 018	25	10.08 982	9.88 978	10	53			
8	9.80 012	15	9.91 043	26	10.08 957	9.88 968	10	52		0.4 0.4	
9	9.80 027	15	9.91 069	26	10.08 931	9.88 958	10	51		0.5 0.8	
10	9.80 043	15	9.91 095	26	10.08 905	9.88 948	11	50		1.3 1.2	
11	9.80 058	15	9.91 121	26	10.08 879	9.88 937	10	49		1.7 1.7	
12	9.80 074	15	9.91 147	25	10.08 853	9.88 927	10	48		2.2 2.1	
13	9.80 089	15	9.91 172	26	10.08 828	9.88 917	10	47		2.6 2.5	
14	9.80 105	15	9.91 198	25	10.08 802	9.88 906	10	46		3.0 2.9	
15	9.80 120	15	9.91 224	25	10.08 776	9.88 896	10	45		3.5 3.3	
16	9.80 136	15	9.91 250	26	10.08 750	9.88 886	11	44		3.9 3.8	
17	9.80 151	15	9.91 276	25	10.08 724	9.88 875	10	43			
18	9.80 166	15	9.91 301	26	10.08 699	9.88 865	10	42		4.3 4.2	
19	9.80 182	15	9.91 327	26	10.08 673	9.88 855	11	41		4.7 4.3	
20	9.80 197	15	9.91 353	26	10.08 647	9.88 844	10	40		13.0 12.5	
21	9.80 213	15	9.91 379	25	10.08 621	9.88 834	10	39		17.3 16.7	
22	9.80 228	15	9.91 404	26	10.08 596	9.88 824	11	38		21.7 20.8	
23	9.80 244	15	9.91 430	26	10.08 570	9.88 813	10	37			
24	9.80 259	15	9.91 456	26	10.08 544	9.88 803	10	36			
25	9.80 274	15	9.91 482	25	10.08 518	9.88 793	11	35			
26	9.80 290	15	9.91 507	26	10.08 493	9.88 782	10	34			
27	9.80 305	15	9.91 533	26	10.08 467	9.88 772	11	33			
28	9.80 320	15	9.91 559	26	10.08 441	9.88 761	10	32			
29	9.80 336	15	9.91 585	25	10.08 415	9.88 751	10	31			
30	9.80 351	15	9.91 610	26	10.08 390	9.88 741	11	30			
31	9.80 366	15	9.91 636	26	10.08 364	9.88 730	10	29			
32	9.80 382	15	9.91 662	26	10.08 338	9.88 720	11	28			
33	9.80 397	15	9.91 688	25	10.08 312	9.88 709	10	27			
34	9.80 412	15	9.91 713	26	10.08 287	9.88 699	11	26			
35	9.80 428	15	9.91 739	26	10.08 261	9.88 688	10	25			
36	9.80 443	15	9.91 765	26	10.08 235	9.88 678	10	24			
37	9.80 458	15	9.91 791	25	10.08 209	9.88 668	11	23			
38	9.80 473	15	9.91 816	26	10.08 184	9.88 657	10	22			
39	9.80 489	15	9.91 842	26	10.08 158	9.88 647	11	21			
40	9.80 504	15	9.91 868	25	10.08 132	9.88 636	10	20			
41	9.80 519	15	9.91 893	26	10.08 107	9.88 625	11	19			
42	9.80 534	15	9.91 919	26	10.08 081	9.88 615	10	18			
43	9.80 550	15	9.91 945	26	10.08 055	9.88 605	11	17			
44	9.80 565	15	9.91 971	25	10.08 029	9.88 594	10	16			
45	9.80 580	15	9.91 996	26	10.08 004	9.88 584	11	15			
46	9.80 595	15	9.92 022	26	10.07 978	9.88 573	10	14			
47	9.80 610	15	9.92 048	25	10.07 952	9.88 563	11	13			
48	9.80 625	15	9.92 073	26	10.07 927	9.88 552	10	12			
49	9.80 641	15	9.92 099	26	10.07 901	9.88 542	11	11			
50	9.80 656	15	9.92 125	25	10.07 875	9.88 531	10	10			
51	9.80 671	15	9.92 150	26	10.07 850	9.88 521	11	9			
52	9.80 686	15	9.92 176	26	10.07 824	9.88 510	10	8			
53	9.80 701	15	9.92 202	25	10.07 798	9.88 499	10	7			
54	9.80 716	15	9.92 227	26	10.07 773	9.88 489	11	6			
55	9.80 731	15	9.92 253	26	10.07 747	9.88 478	10	5			
56	9.80 746	15	9.92 279	25	10.07 721	9.88 468	11	4			
57	9.80 762	15	9.92 304	26	10.07 696	9.88 457	10	3			
58	9.80 777	15	9.92 330	26	10.07 670	9.88 447	11	2			
59	9.80 792	15	9.92 356	25	10.07 644	9.88 436	10	1			
60	9.80 807		9.92 381		10.07 619	9.88 425		0			
'	L Cos	d	L Ctn	c d	L Tan	L Sin	d	'	Proportional parts		

129° (399°)

(230°) 50°

Table 6 COMMON LOGARITHMS OF TRIGONOMETRIC FUNCTIONS (continued) 257

The — 10 portion of the characteristic of the logarithm is not printed but must be written down whenever such a logarithm is used.

40° (220°)					(319°) 139°					Proportional parts		
<i>n</i>	L Sin	<i>d</i>	L Tan	<i>c d</i>	L Ctn	L Cos	<i>d</i>	<i>f</i>				
0	9.80 807	15	9.92 381	20	10.07 619	9.88 425	10	80				
1	9.80 822	15	9.92 407	26	10.07 593	9.88 415	11	89				
2	9.80 837	15	9.92 433	28	10.07 567	9.88 404	10	83				
3	9.80 852	15	9.92 458	35	10.07 542	9.88 394	11	87				
4	9.80 867	15	9.92 484	25	10.07 516	9.88 383	11	86				
5	9.80 882	15	9.92 510	25	10.07 490	9.88 372	10	55				
6	9.80 897	15	9.92 535	26	10.07 465	9.88 362	11	54				
7	9.80 912	15	9.92 561	28	10.07 439	9.88 351	11	53				
8	9.80 927	15	9.92 587	25	10.07 413	9.88 340	10	52				
9	9.80 942	15	9.92 612	26	10.07 388	9.88 330	11	51				
10	9.80 957	15	9.92 638	25	10.07 362	9.88 319	11	50				
11	9.80 972	15	9.92 663	26	10.07 337	9.88 308	10	49				
12	9.80 987	15	9.92 689	26	10.07 311	9.88 296	11	48				
13	9.81 002	15	9.92 715	23	10.07 285	9.88 287	11	47				
14	9.81 017	15	9.92 740	26	10.07 260	9.88 276	10	46				
15	9.81 032	15	9.92 766	26	10.07 234	9.88 266	11	45				
16	9.81 047	15	9.92 792	25	10.07 208	9.88 255	11	44				
17	9.81 061	15	9.92 817	26	10.07 183	9.88 244	10	43				
18	9.81 076	15	9.92 843	23	10.07 157	9.88 234	11	42				
19	9.81 091	15	9.92 868	26	10.07 132	9.88 223	11	41				
20	9.81 106	15	9.92 894	26	10.07 106	9.88 212	11	40				
21	9.81 121	15	9.92 920	25	10.07 080	9.88 201	10	39				
22	9.81 136	15	9.92 945	26	10.07 055	9.88 191	11	38				
23	9.81 151	15	9.92 971	25	10.07 029	9.88 180	11	37				
24	9.81 166	15	9.92 996	24	10.07 004	9.88 169	11	36				
25	9.81 180	15	9.93 022	26	10.06 978	9.88 158	10	35				
26	9.81 195	15	9.93 048	25	10.06 952	9.88 148	11	34				
27	9.81 210	15	9.93 073	26	10.06 927	9.88 137	11	33				
28	9.81 225	15	9.93 099	25	10.06 901	9.88 126	11	32				
29	9.81 240	15	9.93 124	26	10.06 876	9.88 115	10	31				
30	9.81 254	15	9.93 150	25	10.06 850	9.88 105	11	30				
31	9.81 269	15	9.93 175	26	10.06 825	9.88 094	11	29				
32	9.81 284	15	9.93 201	26	10.06 799	9.88 083	11	28				
33	9.81 299	15	9.93 227	25	10.06 773	9.88 072	11	27				
34	9.81 314	15	9.93 252	26	10.06 748	9.88 061	10	26				
35	9.81 328	15	9.93 278	25	10.06 722	9.88 051	11	25				
36	9.81 343	15	9.93 303	26	10.06 697	9.88 040	11	24				
37	9.81 358	15	9.93 328	25	10.06 671	9.88 029	11	23				
38	9.81 372	15	9.93 354	26	10.06 646	9.88 018	11	22				
39	9.81 387	15	9.93 380	26	10.06 620	9.88 007	11	21				
40	9.81 402	15	9.93 406	25	10.06 594	9.87 996	11	20				
41	9.81 417	15	9.93 431	26	10.06 568	9.87 985	10	19				
42	9.81 431	15	9.93 457	25	10.06 543	9.87 975	11	18				
43	9.81 446	15	9.93 482	26	10.06 518	9.87 964	11	17				
44	9.81 461	15	9.93 508	25	10.06 492	9.87 953	11	16				
45	9.81 475	15	9.93 533	26	10.06 467	9.87 942	11	15				
46	9.81 490	15	9.93 559	25	10.06 441	9.87 931	11	14				
47	9.81 505	15	9.93 584	26	10.06 416	9.87 920	11	13				
48	9.81 519	15	9.93 610	26	10.06 390	9.87 909	11	12				
49	9.81 534	15	9.93 636	25	10.06 364	9.87 898	11	11				
50	9.81 549	15	9.93 661	26	10.06 339	9.87 887	10	10				
51	9.81 563	15	9.93 687	25	10.06 313	9.87 877	11	9				
52	9.81 578	15	9.93 712	26	10.06 288	9.87 866	11	8				
53	9.81 592	15	9.93 738	25	10.06 262	9.87 855	11	7				
54	9.81 607	15	9.93 763	26	10.06 237	9.87 844	11	6				
55	9.81 622	15	9.93 789	25	10.06 211	9.87 833	11	5				
56	9.81 636	15	9.93 814	26	10.06 186	9.87 822	11	4				
57	9.81 651	15	9.93 840	25	10.06 160	9.87 811	11	3				
58	9.81 665	15	9.93 865	26	10.06 135	9.87 800	11	2				
59	9.81 680	15	9.93 891	25	10.06 109	9.87 789	11	1				
60	9.81 694		9.93 916		10.06 084	9.87 778		0				
<i>n</i>	L Cos	<i>d</i>	L Ctn	<i>c d</i>	L Tan	L Sin	<i>d</i>	<i>f</i>	Proportional parts			

130° (310°)

(229°) 49°

Table 6 COMMON LOGARITHMS OF TRIGONOMETRIC FUNCTIONS (continued) 253

The — 10 portion of the characteristic of the logarithm is not printed but must be written down whenever such a logarithm is used.

41° (221°)						(318°) 138°					
	L Sin	d	L Tan	c d	L Ctn	L Cos	d	Proportional parts			
0	9.81 694	15	9.93 916	26	10.06 084	9.87 778	11				
1	9.81 709	14	9.93 942	25	10.06 059	9.87 767	11				
2	9.81 723	13	9.93 967	24	10.06 032	9.87 758	11				
3	9.81 738	12	9.93 993	23	10.06 007	9.87 748	11				
4	9.81 752	11	9.94 018	22	10.05 982	9.87 734	11				
5	9.81 767	10	9.94 044	21	10.05 955	9.87 723	11				
6	9.81 781	9	9.94 069	20	10.05 931	9.87 712	11				
7	9.81 796	8	9.94 095	19	10.05 905	9.87 701	11				
8	9.81 810	7	9.94 120	18	10.05 880	9.87 690	11				
9	9.81 825	6	9.94 145	17	10.05 854	9.87 679	11				
10	9.81 839	5	9.94 171	16	10.05 829	9.87 668	11				
11	9.81 854	4	9.94 197	15	10.05 803	9.87 657	11				
12	9.81 868	3	9.94 222	14	10.05 778	9.87 646	11				
13	9.81 882	2	9.94 248	13	10.05 752	9.87 635	11				
14	9.81 897	1	9.94 273	12	10.05 727	9.87 624	11				
15	9.81 911	0	9.94 299	11	10.05 701	9.87 613	11				
16	9.81 926	15	9.94 324	10	10.05 676	9.87 601	11				
17	9.81 940	14	9.94 350	9	10.05 650	9.87 590	11				
18	9.81 955	13	9.94 375	8	10.05 625	9.87 579	11				
19	9.81 969	12	9.94 401	7	10.05 599	9.87 568	11				
20	9.81 983	11	9.94 426	6	10.05 574	9.87 557	11				
21	9.81 998	10	9.94 452	5	10.05 548	9.87 546	11				
22	9.82 012	9	9.94 477	4	10.05 523	9.87 535	11				
23	9.82 026	8	9.94 503	3	10.05 497	9.87 524	11				
24	9.82 041	7	9.94 528	2	10.05 472	9.87 513	11				
25	9.82 055	6	9.94 554	1	10.05 446	9.87 501	11				
26	9.82 069	15	9.94 579	25	10.05 421	9.87 490	11				
27	9.82 084	14	9.94 604	24	10.05 396	9.87 479	11				
28	9.82 098	13	9.94 630	23	10.05 370	9.87 468	11				
29	9.82 112	12	9.94 655	22	10.05 345	9.87 457	11				
30	9.82 126	11	9.94 681	21	10.05 319	9.87 446	11				
31	9.82 141	10	9.94 706	20	10.05 294	9.87 434	11				
32	9.82 155	9	9.94 732	19	10.05 268	9.87 423	11				
33	9.82 169	8	9.94 757	18	10.05 243	9.87 412	11				
34	9.82 184	7	9.94 783	17	10.05 217	9.87 401	11				
35	9.82 198	6	9.94 808	16	10.05 192	9.87 390	11				
36	9.82 212	5	9.94 834	15	10.05 166	9.87 379	11				
37	9.82 226	4	9.94 859	14	10.05 141	9.87 367	11				
38	9.82 240	3	9.94 884	13	10.05 116	9.87 356	11				
39	9.82 255	2	9.94 910	12	10.05 090	9.87 345	11				
40	9.82 269	1	9.94 935	11	10.05 065	9.87 334	11				
41	9.82 283	0	9.94 961	10	10.05 039	9.87 322	11				
42	9.82 297	15	9.94 986	25	10.05 014	9.87 311	11				
43	9.82 311	14	9.95 012	24	10.04 988	9.87 300	11				
44	9.82 326	13	9.95 037	23	10.04 963	9.87 288	11				
45	9.82 340	12	9.95 062	22	10.04 938	9.87 277	11				
46	9.82 354	11	9.95 088	21	10.04 912	9.87 266	11				
47	9.82 368	10	9.95 113	20	10.04 887	9.87 255	11				
48	9.82 382	9	9.95 139	19	10.04 861	9.87 243	11				
49	9.82 395	8	9.95 164	18	10.04 836	9.87 232	11				
50	9.82 410	7	9.95 190	17	10.04 810	9.87 221	11				
51	9.82 424	6	9.95 215	16	10.04 785	9.87 209	11				
52	9.82 439	5	9.95 240	15	10.04 760	9.87 198	11				
53	9.82 453	4	9.95 266	14	10.04 734	9.87 187	11				
54	9.82 467	3	9.95 291	13	10.04 709	9.87 175	11				
55	9.82 481	2	9.95 317	12	10.04 683	9.87 164	11				
56	9.82 495	1	9.95 342	11	10.04 658	9.87 153	11				
57	9.82 509	0	9.95 368	10	10.04 632	9.87 141	11				
58	9.82 523	15	9.95 393	25	10.04 607	9.87 130	11				
59	9.82 537	14	9.95 418	24	10.04 582	9.87 119	11				
60	9.82 551	13	9.95 444	23	10.04 556	9.87 107	11				
	L Cos	d	L Ctn	c d	L Tan	L Sin	d	Proportional parts			

Table 6 COMMON LOGARITHMS OF TRIGONOMETRIC FUNCTIONS (continued) 259

The -10 portion of the characteristic of the logarithm is not printed but must be written down whenever such a logarithm is used.

42° (222°)				(317°) 137°					
<i>n</i>	L Sin	<i>d</i>	L Tan	<i>c d</i>	L Ctn	L Cos	<i>d</i>	Proportional parts	
0	9.82 551	14	9.95 444	25	10.04 556	9.87 107	11	60	
1	9.82 565	14	9.95 469	26	10.04 531	9.87 096	11	59	
2	9.82 579	14	9.95 495	25	10.04 605	9.87 085	12	58	
3	9.82 593	14	9.95 520	25	10.04 480	9.87 073	11	57	
4	9.82 607	14	9.95 545	25	10.04 455	9.87 062	12	56	
5	9.82 621	14	9.95 571	25	10.04 429	9.87 050	11	55	
6	9.82 635	14	9.95 596	26	10.04 404	9.87 039	11	54	
7	9.82 649	14	9.95 622	25	10.04 378	9.87 028	12	53	
8	9.82 663	14	9.95 647	25	10.04 353	9.87 016	11	52	
9	9.82 677	14	9.95 672	26	10.04 328	9.87 005	12	51	
10	9.82 691	14	9.95 698	25	10.04 302	9.86 993	11	50	
11	9.82 705	14	9.95 723	25	10.04 277	9.86 982	12	49	
12	9.82 719	14	9.95 748	26	10.04 252	9.86 970	11	48	
13	9.82 733	14	9.95 774	25	10.04 226	9.86 958	12	47	
14	9.82 747	14	9.95 799	25	10.04 201	9.86 947	11	46	
15	9.82 761	14	9.95 825	25	10.04 175	9.86 936	12	45	
16	9.82 775	13	9.95 850	25	10.04 150	9.86 924	11	44	
17	9.82 788	14	9.95 875	26	10.04 125	9.86 913	11	43	
18	9.82 802	14	9.95 901	25	10.04 099	9.86 902	12	42	
19	9.82 816	14	9.95 926	26	10.04 074	9.86 890	11	41	
20	9.82 830	14	9.95 952	25	10.04 048	9.86 879	12	40	
21	9.82 844	14	9.95 977	25	10.04 023	9.86 867	12	39	
22	9.82 858	14	9.95 002	25	10.03 998	9.86 855	11	38	
23	9.82 872	13	9.95 028	25	10.03 972	9.86 844	12	37	
24	9.82 886	14	9.95 053	25	10.03 947	9.86 832	11	36	
25	9.82 899	14	9.95 078	26	10.03 922	9.86 821	12	35	
26	9.82 913	14	9.95 104	25	10.03 896	9.86 809	11	34	
27	9.82 927	14	9.95 129	26	10.03 871	9.86 798	12	33	
28	9.82 941	14	9.95 155	25	10.03 845	9.86 786	11	32	
29	9.82 955	13	9.95 180	25	10.03 820	9.86 775	12	31	
30	9.82 968	14	9.95 205	26	10.03 795	9.86 763	11	30	
31	9.82 982	14	9.95 231	25	10.03 769	9.86 752	12	29	
32	9.82 996	14	9.95 256	25	10.03 744	9.86 740	12	28	
33	9.83 010	13	9.95 281	25	10.03 719	9.86 728	11	27	
34	9.83 023	14	9.95 307	25	10.03 693	9.86 717	12	26	
35	9.83 037	14	9.95 332	25	10.03 668	9.86 705	11	25	
36	9.83 051	14	9.95 357	25	10.03 643	9.86 694	12	24	
37	9.83 065	13	9.95 383	25	10.03 617	9.86 682	12	23	
38	9.83 079	14	9.95 408	25	10.03 592	9.86 670	11	22	
39	9.83 092	14	9.95 433	26	10.03 567	9.86 659	12	21	
40	9.83 106	14	9.95 459	25	10.03 541	9.86 647	12	20	
41	9.83 120	13	9.95 484	26	10.03 516	9.86 635	11	19	
42	9.83 133	14	9.95 510	25	10.03 490	9.86 624	12	18	
43	9.83 147	14	9.95 535	25	10.03 465	9.86 612	12	17	
44	9.83 161	13	9.95 560	26	10.03 440	9.86 600	11	16	
45	9.83 174	14	9.95 586	25	10.03 414	9.86 589	12	15	
46	9.83 188	14	9.95 611	25	10.03 389	9.86 577	12	14	
47	9.83 202	13	9.95 636	26	10.03 364	9.86 565	11	13	
48	9.83 215	14	9.95 662	25	10.03 338	9.86 554	12	12	
49	9.83 229	13	9.95 687	25	10.03 313	9.86 542	12	11	
50	9.83 242	14	9.95 712	26	10.03 288	9.86 530	12	10	
51	9.83 256	14	9.95 738	25	10.03 262	9.86 518	11	9	
52	9.83 270	13	9.95 763	25	10.03 237	9.86 507	12	8	
53	9.83 283	14	9.95 788	26	10.03 212	9.86 495	12	7	
54	9.83 297	13	9.95 814	25	10.03 186	9.86 483	11	6	
55	9.83 310	14	9.95 839	25	10.03 161	9.86 472	12	5	
56	9.83 324	14	9.95 864	26	10.03 136	9.86 460	12	4	
57	9.83 338	13	9.95 890	25	10.03 110	9.86 448	12	3	
58	9.83 351	14	9.95 915	26	10.03 085	9.86 436	11	2	
59	9.83 365	13	9.95 940	26	10.03 060	9.86 425	12	1	
60	9.83 378		9.95 965		10.03 034	9.86 413		0	
<i>n</i>	L Cos	<i>d</i>	L Ctn	<i>c d</i>	L Tan	L Sin	<i>d</i>	Proportional parts	

132° (312°)

(227°) 47°

Table 6 COMMON LOGARITHMS OF TRIGONOMETRIC FUNCTIONS (continued) 260

The -10 portion of the characteristic of the logarithm is not printed but must be written down whenever such a logarithm is used.

43° (223°)				(316°) 136°				Proportional parts				
	L Sin	d		L Tan	d	L Ctn	L Cos	d				
0	9.83 378	14		9.96 966	25	10.03 034	9.86 413	12	60			
1	9.83 392	13		9.96 991	25	10.03 009	9.86 401	12	59			
2	9.83 405	14		9.97 016	26	10.02 984	9.86 389	12	58			
3	9.83 419	13		9.97 042	25	10.02 958	9.86 377	11	57			
4	9.83 432	14		9.97 067	25	10.02 933	9.86 366	12	56			
5	9.83 446	13		9.97 092	26	10.02 908	9.86 354	12	55			
6	9.83 459	14		9.97 118	25	10.02 882	9.86 342	12	54			
7	9.83 473	13		9.97 143	25	10.02 857	9.86 330	12	53			
8	9.83 486	14		9.97 168	25	10.02 832	9.86 318	12	52			
9	9.83 500	13		9.97 193	26	10.02 807	9.86 306	11	51			
10	9.83 513	14		9.97 219	25	10.02 781	9.86 295	12	50			
11	9.83 527	13		9.97 244	25	10.02 756	9.86 283	12	49			
12	9.83 540	14		9.97 269	26	10.02 731	9.86 271	12	48			
13	9.83 554	13		9.97 295	25	10.02 705	9.86 259	12	47			
14	9.83 567	14		9.97 320	25	10.02 680	9.86 247	12	46			
15	9.83 581	13		9.97 345	26	10.02 655	9.86 235	12	45			
16	9.83 594	14		9.97 371	25	10.02 629	9.86 223	12	44			
17	9.83 608	13		9.97 396	25	10.02 604	9.86 211	11	43			
18	9.83 621	14		9.97 421	26	10.02 579	9.86 200	12	42			
19	9.83 634	13		9.97 447	25	10.02 553	9.86 188	12	41			
20	9.83 648	14		9.97 472	25	10.02 528	9.86 176	12	40			
21	9.83 661	13		9.97 497	26	10.02 503	9.86 164	12	39			
22	9.83 674	14		9.97 523	25	10.02 477	9.86 152	12	38			
23	9.83 688	13		9.97 548	25	10.02 452	9.86 140	12	37			
24	9.83 701	14		9.97 573	25	10.02 427	9.86 128	12	36			
25	9.83 715	13		9.97 598	26	10.02 402	9.86 116	12	35			
26	9.83 728	14		9.97 624	25	10.02 376	9.86 104	12	34			
27	9.83 741	13		9.97 649	25	10.02 351	9.86 092	12	33			
28	9.83 755	14		9.97 674	26	10.02 326	9.86 080	12	32			
29	9.83 768	13		9.97 700	25	10.02 300	9.86 068	12	31			
30	9.83 781	14		9.97 725	25	10.02 275	9.86 056	12	30			
31	9.83 795	13		9.97 750	26	10.02 250	9.86 044	12	29			
32	9.83 808	14		9.97 776	25	10.02 224	9.86 032	12	28			
33	9.83 821	13		9.97 801	25	10.02 199	9.86 020	12	27			
34	9.83 834	14		9.97 826	25	10.02 174	9.86 008	12	26			
35	9.83 848	13		9.97 851	26	10.02 149	9.85 996	12	25			
36	9.83 861	14		9.97 877	25	10.02 123	9.85 984	12	24			
37	9.83 874	13		9.97 902	25	10.02 098	9.85 972	12	23			
38	9.83 887	14		9.97 927	26	10.02 073	9.85 960	12	22			
39	9.83 901	13		9.97 953	25	10.02 047	9.85 948	12	21			
40	9.83 914	14		9.97 978	26	10.02 022	9.85 936	12	20			
41	9.83 927	13		9.98 003	25	10.01 997	9.85 924	12	19			
42	9.83 940	14		9.98 029	25	10.01 971	9.85 912	12	18			
43	9.83 954	13		9.98 054	25	10.01 946	9.85 900	12	17			
44	9.83 967	14		9.98 079	25	10.01 921	9.85 888	12	16			
45	9.83 980	13		9.98 104	26	10.01 896	9.85 876	12	15			
46	9.83 993	14		9.98 130	25	10.01 870	9.85 864	12	14			
47	9.84 006	13		9.98 155	25	10.01 845	9.85 851	12	13			
48	9.84 020	14		9.98 180	26	10.01 820	9.85 839	12	12			
49	9.84 033	13		9.98 206	25	10.01 794	9.85 827	12	11			
50	9.84 046	14		9.98 231	25	10.01 769	9.85 815	12	10			
51	9.84 059	13		9.98 256	26	10.01 744	9.85 803	12	9			
52	9.84 072	14		9.98 281	25	10.01 719	9.85 791	12	8			
53	9.84 085	13		9.98 307	25	10.01 693	9.85 779	12	7			
54	9.84 098	14		9.98 332	25	10.01 668	9.85 766	12	6			
55	9.84 112	13		9.98 357	26	10.01 643	9.85 754	12	5			
56	9.84 125	14		9.98 383	25	10.01 617	9.85 742	12	4			
57	9.84 138	13		9.98 408	25	10.01 592	9.85 730	12	3			
58	9.84 151	14		9.98 433	25	10.01 567	9.85 718	12	2			
59	9.84 164	13		9.98 458	26	10.01 542	9.85 706	12	1			
60	9.84 177			9.98 484		10.01 516	9.85 693		0			
	L Cos	d		L Ctn	d	L Tan	L Sin	d		Proportional parts		

133° (313°)

(226°) 46°

Table 6 COMMON LOGARITHMS OF TRIGONOMETRIC FUNCTIONS (continued) 261

The -10 portion of the characteristic of the logarithm is not printed but must be written down whenever such a logarithm is used.

44° (224°)

(315°) 135°

°	L. Sin		L. Tan		L. Cot		L. Cos		Proportional parts			
	d		c d		d		d					
0	9.84 177	13	9.98 484	25	10.01 516		9.85 693	12	60			
1	9.84 190	13	9.98 509	25	10.01 491		9.85 681	12	59			
2	9.84 203	13	9.98 534	26	10.01 466		9.85 669	12	58			
3	9.84 216	13	9.98 560	25	10.01 440		9.85 657	12	57			
4	9.84 229	13	9.98 585	25	10.01 415		9.85 645	13	56			
5	9.84 242	13	9.98 610	25	10.01 390		9.85 632	12	55			
6	9.84 255	14	9.98 635	26	10.01 365		9.85 620	12	54			
7	9.84 268	13	9.98 661	25	10.01 339		9.85 608	12	53			
8	9.84 282	13	9.98 686	25	10.01 314		9.85 596	13	52			
9	9.84 295	13	9.98 711	26	10.01 289		9.85 583	12	51			
10	9.84 308	13	9.98 737	25	10.01 263		9.85 571	12	50			
11	9.84 321	13	9.98 762	25	10.01 238		9.85 559	12	49			
12	9.84 334	13	9.98 787	25	10.01 213		9.85 547	13	48			
13	9.84 347	13	9.98 812	26	10.01 188		9.85 534	12	47			
14	9.84 360	13	9.98 838	25	10.01 162		9.85 522	12	46			
15	9.84 373	12	9.98 863	25	10.01 137		9.85 510	13	45		26	25
16	9.84 385	13	9.98 888	25	10.01 112		9.85 497	12	44	1	0.4	0.4
17	9.84 398	13	9.98 913	25	10.01 087		9.85 485	12	43	2	0.9	0.8
18	9.84 411	13	9.98 939	25	10.01 061		9.85 473	13	42	3	1.3	1.2
19	9.84 424	13	9.98 964	25	10.01 036		9.85 460	12	41	4	1.7	1.7
20	9.84 437	13	9.98 989	26	10.01 011		9.85 448	12	40	5	2.2	2.1
21	9.84 450	13	9.99 015	25	10.00 985		9.85 436	13	39	6	2.6	2.5
22	9.84 463	13	9.99 040	25	10.00 960		9.85 423	12	38	7	3.0	2.9
23	9.84 476	13	9.99 065	25	10.00 935		9.85 411	12	37	8	3.5	3.2
24	9.84 488	13	9.99 090	26	10.00 910		9.85 399	13	36	9	3.9	3.6
25	9.84 502	13	9.99 116	26	10.00 884		9.85 386	12	35	10	4.3	4.2
26	9.84 515	13	9.99 141	25	10.00 859		9.85 374	13	34	20	4.7	4.3
27	9.84 528	12	9.99 166	25	10.00 834		9.85 361	12	33	30	5.0	4.5
28	9.84 540	13	9.99 191	26	10.00 809		9.85 349	12	32	40	5.3	4.7
29	9.84 553	13	9.99 217	25	10.00 783		9.85 337	13	31	50	5.7	5.0
30	9.84 566	13	9.99 242	25	10.00 758		9.85 324	12	30			
31	9.84 579	13	9.99 267	26	10.00 733		9.85 312	12	29			
32	9.84 592	13	9.99 293	25	10.00 707		9.85 299	12	28			
33	9.84 605	13	9.99 318	25	10.00 682		9.85 287	13	27			
34	9.84 618	12	9.99 343	25	10.00 657		9.85 274	12	26			
35	9.84 630	13	9.99 368	26	10.00 632		9.85 262	12	25			
36	9.84 643	13	9.99 394	25	10.00 606		9.85 250	13	24			
37	9.84 656	13	9.99 419	25	10.00 581		9.85 237	12	23			
38	9.84 669	13	9.99 444	25	10.00 556		9.85 225	13	22			
39	9.84 682	12	9.99 469	26	10.00 531		9.85 212	12	21			
40	9.84 694	13	9.99 495	25	10.00 505		9.85 200	13	20			
41	9.84 707	13	9.99 520	25	10.00 480		9.85 187	12	19			
42	9.84 720	13	9.99 545	25	10.00 455		9.85 175	13	18			
43	9.84 733	12	9.99 570	26	10.00 430		9.85 162	12	17			
44	9.84 745	13	9.99 596	25	10.00 404		9.85 150	13	16			
45	9.84 758	13	9.99 621	25	10.00 379		9.85 137	12	15			
46	9.84 771	13	9.99 646	26	10.00 354		9.85 125	13	14			
47	9.84 784	12	9.99 672	25	10.00 328		9.85 112	12	13			
48	9.84 796	13	9.99 697	25	10.00 303		9.85 100	13	12			
49	9.84 809	13	9.99 722	25	10.00 278		9.85 087	13	11			
50	9.84 822	13	9.99 747	26	10.00 253		9.85 074	12	10			
51	9.84 835	12	9.99 773	25	10.00 227		9.85 062	13	9			
52	9.84 847	13	9.99 798	25	10.00 202		9.85 049	12	8			
53	9.84 860	13	9.99 823	25	10.00 177		9.85 037	13	7			
54	9.84 873	12	9.99 848	26	10.00 152		9.85 024	12	6			
55	9.84 885	13	9.99 874	25	10.00 126		9.85 012	13	5			
56	9.84 898	13	9.99 899	25	10.00 101		9.84 999	13	4			
57	9.84 911	12	9.99 925	25	10.00 076		9.84 986	12	3			
58	9.84 923	13	9.99 949	26	10.00 051		9.84 974	13	2			
59	9.84 936	13	9.99 973	25	10.00 025		9.84 961	12	1			
60	9.84 949		10.00 000		10.00 000		9.84 949		0			

134° (314°)

(225°) 45°

Table 7 NATURAL TRIGONOMETRIC FUNCTIONS 262

0° (180°)		(359°) 179°		1° (181°)		(358°) 170°				
	Sin	Tan	Ctn	Cos		Sin	Tan	Ctn	Cos	
0	.00000	.00000	-----	1.0000	60	.01745	.01746	57.290	.99985	60
1	.00029	.00029	3437.7	1.0000	59	.01774	.01775	58.361	.99984	59
2	.00058	.00058	1718.9	1.0000	58	.01803	.01804	59.442	.99984	58
3	.00087	.00087	1145.9	1.0000	57	.01832	.01833	60.561	.99983	57
4	.00116	.00116	859.44	1.0000	56	.01862	.01862	61.709	.99983	56
5	.00145	.00145	687.55	1.0000	55	.01891	.01891	62.882	.99982	55
6	.00175	.00175	572.96	1.0000	54	.01920	.01920	64.081	.99982	54
7	.00204	.00204	491.11	1.0000	53	.01949	.01949	65.303	.99981	53
8	.00233	.00233	429.72	1.0000	52	.01978	.01978	66.549	.99980	52
9	.00262	.00262	381.97	1.0000	51	.02007	.02007	67.816	.99980	51
10	.00291	.00291	348.77	1.0000	50	.02036	.02036	69.104	.99979	50
11	.00320	.00320	312.52	.99999	49	.02065	.02066	70.412	.99979	49
12	.00349	.00349	286.48	.99999	48	.02094	.02095	71.740	.99978	48
13	.00378	.00378	264.44	.99999	47	.02123	.02124	73.085	.99977	47
14	.00407	.00407	245.55	.99999	46	.02152	.02153	74.449	.99977	46
15	.00436	.00436	229.18	.99998	45	.02181	.02182	75.829	.99976	45
16	.00465	.00465	214.86	.99998	44	.02211	.02211	77.226	.99976	44
17	.00495	.00495	202.22	.99998	43	.02240	.02240	78.639	.99975	43
18	.00524	.00524	190.98	.99998	42	.02269	.02269	80.068	.99974	42
19	.00553	.00553	180.99	.99998	41	.02298	.02298	81.513	.99974	41
20	.00582	.00582	171.89	.99998	40	.02327	.02328	82.964	.99973	40
21	.00611	.00611	163.70	.99998	39	.02356	.02357	84.433	.99972	39
22	.00640	.00640	156.26	.99998	38	.02385	.02386	85.916	.99972	38
23	.00669	.00669	149.47	.99998	37	.02414	.02415	87.411	.99971	37
24	.00698	.00698	143.24	.99998	36	.02443	.02444	88.917	.99970	36
25	.00727	.00727	137.51	.99997	35	.02472	.02472	90.436	.99969	35
26	.00756	.00756	132.22	.99997	34	.02501	.02502	91.965	.99969	34
27	.00785	.00785	127.32	.99997	33	.02530	.02531	93.506	.99968	33
28	.00814	.00814	122.77	.99997	32	.02559	.02560	95.057	.99967	32
29	.00844	.00844	118.54	.99996	31	.02588	.02589	96.618	.99966	31
30	.00873	.00873	114.59	.99996	30	.02618	.02619	98.188	.99966	30
31	.00902	.00902	110.89	.99996	29	.02647	.02648	99.769	.99965	29
32	.00931	.00931	107.43	.99996	28	.02676	.02677	101.358	.99964	28
33	.00960	.00960	104.17	.99995	27	.02705	.02706	102.955	.99963	27
34	.00989	.00989	101.11	.99995	26	.02734	.02735	104.563	.99963	26
35	.01018	.01018	98.218	.99995	25	.02763	.02764	106.178	.99962	25
36	.01047	.01047	95.489	.99995	24	.02792	.02793	107.801	.99961	24
37	.01076	.01076	92.908	.99994	23	.02821	.02822	109.431	.99960	23
38	.01105	.01105	90.463	.99994	22	.02850	.02851	111.070	.99959	22
39	.01134	.01134	88.144	.99994	21	.02879	.02881	112.715	.99959	21
40	.01164	.01164	85.940	.99993	20	.02908	.02910	114.368	.99958	20
41	.01193	.01193	83.844	.99993	19	.02938	.02939	116.027	.99957	19
42	.01222	.01222	81.847	.99993	18	.02967	.02968	117.694	.99956	18
43	.01251	.01251	79.943	.99992	17	.02996	.02997	119.366	.99955	17
44	.01280	.01280	78.126	.99992	16	.03025	.03026	121.045	.99954	16
45	.01309	.01309	76.390	.99991	15	.03054	.03055	122.730	.99953	15
46	.01338	.01338	74.729	.99991	14	.03083	.03084	124.421	.99952	14
47	.01367	.01367	73.139	.99991	13	.03112	.03114	126.118	.99951	13
48	.01396	.01396	71.615	.99990	12	.03141	.03143	127.821	.99950	12
49	.01425	.01425	70.153	.99990	11	.03170	.03172	129.528	.99950	11
50	.01454	.01455	68.750	.99989	10	.03199	.03201	131.242	.99949	10
51	.01483	.01484	67.402	.99989	9	.03228	.03230	132.960	.99948	9
52	.01513	.01513	66.105	.99989	8	.03257	.03259	134.683	.99947	8
53	.01542	.01542	64.858	.99988	7	.03286	.03288	136.412	.99946	7
54	.01571	.01571	63.657	.99988	6	.03316	.03317	138.145	.99945	6
55	.01600	.01600	62.499	.99987	5	.03345	.03346	139.882	.99944	5
56	.01629	.01629	61.383	.99987	4	.03374	.03376	141.624	.99943	4
57	.01658	.01658	60.306	.99986	3	.03403	.03405	143.371	.99942	3
58	.01687	.01687	59.266	.99985	2	.03432	.03434	145.122	.99941	2
59	.01716	.01716	58.261	.99985	1	.03461	.03463	146.877	.99940	1
60	.01745	.01745	57.290	.99985	0	.03490	.03492	148.636	.99939	0
	Cos	Ctn	Tan	Sin		Cos	Ctn	Tan	Sin	
90° (270°)					(269°) 89°	91° (271°)			(268°) 88°	

For degrees indicated at top (bottom) of page use column headings at top (bottom). With degrees at left (right) of each block (top or bottom), use minute column at left (right). The correct sign (plus or minus) must be prefixed in accordance with Sec. 54.

Table 7 NATURAL TRIGONOMETRIC FUNCTIONS (continued)

2° (182°)					(357°) 177°					3° (183°)					(356°) 176°				
°	Sin	Tan	Ctn	Cos	°	Sin	Tan	Ctn	Cos	°	Sin	Tan	Ctn	Cos	°	Sin	Tan	Ctn	Cos
0	.03490	.03492	28.688	.99939	80	.05234	.05241	19.081	.99863	60	.05234	.05241	19.081	.99863	60	.05234	.05241	19.081	.99863
1	.03519	.03521	28.399	.99938	79	.05263	.05270	18.976	.99861	59	.05263	.05270	18.976	.99861	59	.05263	.05270	18.976	.99861
2	.03548	.03550	28.166	.99937	58	.05292	.05299	18.871	.99860	58	.05292	.05299	18.871	.99860	58	.05292	.05299	18.871	.99860
3	.03577	.03579	27.937	.99936	57	.05321	.05328	18.768	.99858	57	.05321	.05328	18.768	.99858	57	.05321	.05328	18.768	.99858
4	.03606	.03609	27.712	.99935	56	.05350	.05357	18.666	.99857	56	.05350	.05357	18.666	.99857	56	.05350	.05357	18.666	.99857
5	.03635	.03638	27.490	.99934	55	.05379	.05387	18.564	.99855	55	.05379	.05387	18.564	.99855	55	.05379	.05387	18.564	.99855
6	.03664	.03667	27.271	.99933	54	.05408	.05416	18.464	.99854	54	.05408	.05416	18.464	.99854	54	.05408	.05416	18.464	.99854
7	.03693	.03696	27.057	.99932	53	.05437	.05446	18.366	.99852	53	.05437	.05446	18.366	.99852	53	.05437	.05446	18.366	.99852
8	.03723	.03725	26.845	.99931	52	.05466	.05474	18.268	.99851	52	.05466	.05474	18.268	.99851	52	.05466	.05474	18.268	.99851
9	.03752	.03754	26.637	.99930	51	.05495	.05503	18.171	.99849	51	.05495	.05503	18.171	.99849	51	.05495	.05503	18.171	.99849
10	.03781	.03783	26.432	.99929	50	.05524	.05533	18.075	.99847	50	.05524	.05533	18.075	.99847	50	.05524	.05533	18.075	.99847
11	.03810	.03812	26.230	.99927	49	.05553	.05562	17.980	.99846	49	.05553	.05562	17.980	.99846	49	.05553	.05562	17.980	.99846
12	.03839	.03842	26.031	.99926	48	.05582	.05591	17.886	.99844	48	.05582	.05591	17.886	.99844	48	.05582	.05591	17.886	.99844
13	.03868	.03871	25.835	.99925	47	.05611	.05620	17.793	.99842	47	.05611	.05620	17.793	.99842	47	.05611	.05620	17.793	.99842
14	.03897	.03900	25.642	.99924	46	.05640	.05649	17.702	.99841	46	.05640	.05649	17.702	.99841	46	.05640	.05649	17.702	.99841
15	.03926	.03929	25.452	.99923	45	.05669	.05678	17.611	.99839	45	.05669	.05678	17.611	.99839	45	.05669	.05678	17.611	.99839
16	.03955	.03958	25.264	.99922	44	.05698	.05708	17.521	.99838	44	.05698	.05708	17.521	.99838	44	.05698	.05708	17.521	.99838
17	.03984	.03987	25.080	.99921	43	.05727	.05737	17.431	.99836	43	.05727	.05737	17.431	.99836	43	.05727	.05737	17.431	.99836
18	.04013	.04016	24.898	.99919	42	.05756	.05766	17.343	.99834	42	.05756	.05766	17.343	.99834	42	.05756	.05766	17.343	.99834
19	.04042	.04046	24.719	.99918	41	.05785	.05795	17.256	.99833	41	.05785	.05795	17.256	.99833	41	.05785	.05795	17.256	.99833
20	.04071	.04075	24.542	.99917	40	.05814	.05824	17.169	.99831	40	.05814	.05824	17.169	.99831	40	.05814	.05824	17.169	.99831
21	.04100	.04104	24.368	.99915	39	.05843	.05854	17.084	.99829	39	.05843	.05854	17.084	.99829	39	.05843	.05854	17.084	.99829
22	.04129	.04133	24.196	.99915	38	.05872	.05883	16.999	.99827	38	.05872	.05883	16.999	.99827	38	.05872	.05883	16.999	.99827
23	.04158	.04162	24.026	.99913	37	.05902	.05912	16.915	.99826	37	.05902	.05912	16.915	.99826	37	.05902	.05912	16.915	.99826
24	.04188	.04191	23.859	.99912	36	.05931	.05941	16.832	.99824	36	.05931	.05941	16.832	.99824	36	.05931	.05941	16.832	.99824
25	.04217	.04220	23.695	.99911	35	.05960	.05970	16.750	.99822	35	.05960	.05970	16.750	.99822	35	.05960	.05970	16.750	.99822
26	.04246	.04250	23.532	.99910	34	.05989	.05999	16.668	.99821	34	.05989	.05999	16.668	.99821	34	.05989	.05999	16.668	.99821
27	.04276	.04279	23.372	.99909	33	.06018	.06029	16.587	.99819	33	.06018	.06029	16.587	.99819	33	.06018	.06029	16.587	.99819
28	.04304	.04308	23.214	.99907	32	.06047	.06058	16.507	.99817	32	.06047	.06058	16.507	.99817	32	.06047	.06058	16.507	.99817
29	.04333	.04337	23.058	.99906	31	.06076	.06087	16.428	.99815	31	.06076	.06087	16.428	.99815	31	.06076	.06087	16.428	.99815
30	.04362	.04366	22.904	.99905	30	.06105	.06116	16.350	.99813	30	.06105	.06116	16.350	.99813	30	.06105	.06116	16.350	.99813
31	.04391	.04395	22.752	.99904	29	.06134	.06145	16.272	.99812	29	.06134	.06145	16.272	.99812	29	.06134	.06145	16.272	.99812
32	.04420	.04424	22.602	.99902	28	.06163	.06175	16.195	.99810	28	.06163	.06175	16.195	.99810	28	.06163	.06175	16.195	.99810
33	.04449	.04454	22.454	.99901	27	.06192	.06204	16.119	.99808	27	.06192	.06204	16.119	.99808	27	.06192	.06204	16.119	.99808
34	.04478	.04483	22.308	.99900	26	.06221	.06233	16.043	.99806	26	.06221	.06233	16.043	.99806	26	.06221	.06233	16.043	.99806
35	.04507	.04512	22.164	.99898	25	.06250	.06262	15.969	.99804	25	.06250	.06262	15.969	.99804	25	.06250	.06262	15.969	.99804
36	.04536	.04541	22.022	.99897	24	.06279	.06291	15.895	.99803	24	.06279	.06291	15.895	.99803	24	.06279	.06291	15.895	.99803
37	.04565	.04570	21.881	.99895	23	.06308	.06321	15.821	.99801	23	.06308	.06321	15.821	.99801	23	.06308	.06321	15.821	.99801
38	.04594	.04599	21.743	.99894	22	.06337	.06350	15.749	.99799	22	.06337	.06350	15.749	.99799	22	.06337	.06350	15.749	.99799
39	.04623	.04628	21.606	.99893	21	.06366	.06379	15.676	.99797	21	.06366	.06379	15.676	.99797	21	.06366	.06379	15.676	.99797
40	.04653	.04658	21.470	.99892	20	.06395	.06408	15.605	.99795	20	.06395	.06408	15.605	.99795	20	.06395	.06408	15.605	.99795
41	.04682	.04687	21.337	.99890	19	.06424	.06438	15.534	.99793	19	.06424	.06438	15.534	.99793	19	.06424	.06438	15.534	.99793
42	.04711	.04716	21.206	.99889	18	.06453	.06467	15.464	.99792	18	.06453	.06467	15.464	.99792	18	.06453	.06467	15.464	.99792
43	.04740	.04746	21.076	.99888	17	.06482	.06496	15.394	.99790	17	.06482	.06496	15.394	.99790	17	.06482	.06496	15.394	.99790
44	.04769	.04774	20.946	.99888	16	.06511	.06525	15.325	.99788	16	.06511	.06525	15.325	.99788	16	.06511	.06525	15.325	.99788
45	.04798	.04803	20.819	.99885	15	.06540	.06554	15.257	.99786	15	.06540	.06554	15.257	.99786	15	.06540	.06554	15.257	.99786
46	.04827	.04833	20.693	.99883	14	.06569	.06584	15.189	.99784	14	.06569	.06584	15.189	.99784	14	.06569	.06584	15.189	.99784
47	.04856	.04862	20.569	.99882	13	.06598	.06613	15.122	.99782	13	.06598	.06613	15.122	.99782	13	.06598	.06613	15.122	.99782
48	.04885	.04891	20.446	.99881	12	.06627	.06642	15.056	.99780	12	.06627	.06642	15.056	.99780	12	.06627	.06642	15.056	.99780
49	.04914	.04920	20.326	.99879	11	.06656	.06671	14.990	.99778	11	.06656	.06671	14.990	.99778	11	.06656	.06671	14.990	.99778
50	.04943	.04949	20.206	.99878	10	.06685	.06700	14.924	.99776	10	.06685	.06700	14.924	.99776	10	.06685	.06700	14.924	.99776
51	.04972	.04978	20.087	.99876	9	.06714	.06730	14.860	.99774	9	.06714	.06730	14.860	.99774	9	.06714	.06730	14.860	.99774
52	.05001	.05007	19.970	.99875	8	.06743	.06759	14.795	.99772	8	.06743	.06759	14.795	.99772	8	.06743	.06759	14.795	.99772
53	.05030	.05037	19.855	.99873	7	.06773	.06788	14.732	.99770	7	.06773	.06788	14.732	.99770	7	.06773	.06788	14.732	.99770
54	.05059	.05066	19.740	.99872	6	.06802	.06817	14.669	.99768	6	.06802	.06817	14.669	.99768	6	.06802	.06817	14.669	.99768
55	.05088	.05095	19.627	.99870	5	.06831	.06847	14.606	.99766	5	.06831	.06847	14.606	.99766	5	.06831	.06847	14.606	.99766
56	.05117	.05124	19.516	.99869	4	.06860	.06876	14.544	.99764	4	.06860	.06876	14.544	.99764	4	.06860	.06876	14.544	.99764
57	.05146	.05153	19.405	.99867	3	.06889	.06905	14.482	.99762	3	.06889	.06905	14.482	.99762	3	.06889	.06905	14.482	.99762
58	.05175	.05182	19.296	.99866	2	.06918	.06934	14.421	.99760	2	.06918	.06934	14.421	.99760	2	.06918	.06934	14.421	.99760
59	.05205	.05212	19.188	.99864	1	.06947	.06963	14.361	.99758	1	.06947	.06963	14.361	.99758	1	.06947	.06963	14.361	.99758
60	.05234	.05241	19.081	.99863	0	.06976	.06993	14.301	.99756	0	.06976	.06993	14.301	.99756	0	.06976	.06993	14.301	.99756
°	Cos	Ctn	Tan	Sin	°														

Table 7 NATURAL TRIGONOMETRIC FUNCTIONS (continued)

4° (184°)					$(355^\circ) 175^\circ$					5° (185°)					$(354^\circ) 174^\circ$				
	Sin	Tan	Ctn	Cos							Sin	Tan	Ctn	Cos		Sin	Tan	Ctn	Cos
0	.06976	.06983	14.301	.99756	60	.08716	.08749	11.430	.99619	60	.09716	.09749	11.430	.99619		.09716	.09749	11.430	.99619
1	.07005	.07022	14.241	.99764	59	.08745	.08778	11.392	.99617	59	.09745	.09778	11.392	.99617		.09745	.09778	11.392	.99617
2	.07034	.07051	14.182	.99762	58	.08774	.08807	11.354	.99614	58	.09774	.09807	11.354	.99614		.09774	.09807	11.354	.99614
3	.07063	.07080	14.124	.99750	57	.08802	.08837	11.316	.99612	57	.09802	.09837	11.316	.99612		.09802	.09837	11.316	.99612
4	.07092	.07110	14.065	.99748	56	.08831	.08866	11.279	.99609	56	.09831	.09866	11.279	.99609		.09831	.09866	11.279	.99609
5	.07121	.07139	14.006	.99746	55	.08860	.08895	11.242	.99607	55	.09860	.09895	11.242	.99607		.09860	.09895	11.242	.99607
6	.07150	.07168	13.951	.99744	54	.08889	.08925	11.205	.99604	54	.09889	.08925	11.205	.99604		.09889	.08925	11.205	.99604
7	.07179	.07197	13.894	.99742	53	.08918	.08954	11.168	.99602	53	.08918	.08954	11.168	.99602		.08918	.08954	11.168	.99602
8	.07208	.07227	13.838	.99740	52	.08947	.08983	11.132	.99599	52	.08947	.08983	11.132	.99599		.08947	.08983	11.132	.99599
9	.07237	.07256	13.782	.99738	51	.08976	.09013	11.095	.99596	51	.08976	.09013	11.095	.99596		.08976	.09013	11.095	.99596
10	.07266	.07285	13.727	.99736	50	.09005	.09042	11.059	.99594	50	.09005	.09042	11.059	.99594		.09005	.09042	11.059	.99594
11	.07295	.07314	13.672	.99734	49	.09034	.09071	11.024	.99591	49	.09034	.09071	11.024	.99591		.09034	.09071	11.024	.99591
12	.07324	.07344	13.617	.99731	48	.09063	.09101	10.988	.99588	48	.09063	.09101	10.988	.99588		.09063	.09101	10.988	.99588
13	.07353	.07373	13.563	.99729	47	.09092	.09130	10.953	.99585	47	.09092	.09130	10.953	.99585		.09092	.09130	10.953	.99585
14	.07382	.07402	13.510	.99727	46	.09121	.09159	10.918	.99582	46	.09121	.09159	10.918	.99582		.09121	.09159	10.918	.99582
15	.07411	.07431	13.457	.99725	45	.09150	.09189	10.883	.99580	45	.09150	.09189	10.883	.99580		.09150	.09189	10.883	.99580
16	.07440	.07461	13.404	.99723	44	.09179	.09218	10.848	.99578	44	.09179	.09218	10.848	.99578		.09179	.09218	10.848	.99578
17	.07469	.07490	13.352	.99721	43	.09208	.09247	10.813	.99576	43	.09208	.09247	10.813	.99576		.09208	.09247	10.813	.99576
18	.07498	.07519	13.300	.99719	42	.09237	.09277	10.778	.99574	42	.09237	.09277	10.778	.99574		.09237	.09277	10.778	.99574
19	.07527	.07548	13.248	.99718	41	.09266	.09306	10.743	.99570	41	.09266	.09306	10.743	.99570		.09266	.09306	10.743	.99570
20	.07556	.07578	13.197	.99714	40	.09295	.09335	10.712	.99567	40	.09295	.09335	10.712	.99567		.09295	.09335	10.712	.99567
21	.07585	.07607	13.146	.99712	39	.09324	.09365	10.678	.99564	39	.09324	.09365	10.678	.99564		.09324	.09365	10.678	.99564
22	.07614	.07636	13.096	.99710	38	.09353	.09394	10.645	.99562	38	.09353	.09394	10.645	.99562		.09353	.09394	10.645	.99562
23	.07643	.07665	13.046	.99708	37	.09382	.09423	10.612	.99559	37	.09382	.09423	10.612	.99559		.09382	.09423	10.612	.99559
24	.07672	.07695	12.996	.99706	36	.09411	.09453	10.579	.99556	36	.09411	.09453	10.579	.99556		.09411	.09453	10.579	.99556
25	.07701	.07724	12.947	.99703	35	.09440	.09482	10.546	.99553	35	.09440	.09482	10.546	.99553		.09440	.09482	10.546	.99553
26	.07730	.07753	12.898	.99701	34	.09469	.09511	10.514	.99551	34	.09469	.09511	10.514	.99551		.09469	.09511	10.514	.99551
27	.07759	.07782	12.850	.99699	33	.09498	.09541	10.481	.99548	33	.09498	.09541	10.481	.99548		.09498	.09541	10.481	.99548
28	.07788	.07812	12.801	.99696	32	.09527	.09570	10.449	.99545	32	.09527	.09570	10.449	.99545		.09527	.09570	10.449	.99545
29	.07817	.07841	12.754	.99694	31	.09556	.09600	10.417	.99542	31	.09556	.09600	10.417	.99542		.09556	.09600	10.417	.99542
30	.07846	.07870	12.706	.99692	30	.09585	.09629	10.385	.99540	30	.09585	.09629	10.385	.99540		.09585	.09629	10.385	.99540
31	.07875	.07899	12.659	.99689	29	.09614	.09658	10.354	.99537	29	.09614	.09658	10.354	.99537		.09614	.09658	10.354	.99537
32	.07904	.07929	12.612	.99687	28	.09642	.09688	10.322	.99534	28	.09642	.09688	10.322	.99534		.09642	.09688	10.322	.99534
33	.07933	.07958	12.566	.99685	27	.09671	.09717	10.291	.99531	27	.09671	.09717	10.291	.99531		.09671	.09717	10.291	.99531
34	.07962	.07987	12.520	.99683	26	.09700	.09746	10.260	.99528	26	.09700	.09746	10.260	.99528		.09700	.09746	10.260	.99528
35	.07991	.08017	12.474	.99680	25	.09729	.09776	10.229	.99526	25	.09729	.09776	10.229	.99526		.09729	.09776	10.229	.99526
36	.08020	.08046	12.429	.99678	24	.09758	.09805	10.199	.99523	24	.09758	.09805	10.199	.99523		.09758	.09805	10.199	.99523
37	.08049	.08075	12.384	.99676	23	.09787	.09834	10.168	.99520	23	.09787	.09834	10.168	.99520		.09787	.09834	10.168	.99520
38	.08078	.08104	12.339	.99673	22	.09816	.09863	10.138	.99517	22	.09816	.09863	10.138	.99517		.09816	.09863	10.138	.99517
39	.08107	.08134	12.295	.99671	21	.09845	.09893	10.108	.99514	21	.09845	.09893	10.108	.99514		.09845	.09893	10.108	.99514
40	.08136	.08163	12.251	.99668	20	.09874	.09923	10.078	.99511	20	.09874	.09923	10.078	.99511		.09874	.09923	10.078	.99511
41	.08165	.08192	12.207	.99666	19	.09903	.09952	10.048	.99508	19	.09903	.09952	10.048	.99508		.09903	.09952	10.048	.99508
42	.08194	.08221	12.163	.99664	18	.09932	.09981	10.019	.99506	18	.09932	.09981	10.019	.99506		.09932	.09981	10.019	.99506
43	.08223	.08251	12.120	.99661	17	.09961	.10011	9.990	.99503	17	.09961	.10011	9.990	.99503		.09961	.10011	9.990	.99503
44	.08252	.08280	12.077	.99659	16	.09990	.10040	9.960	.99500	16	.09990	.10040	9.960	.99500		.09990	.10040	9.960	.99500
45	.08281	.08309	12.035	.99657	15	.10019	.10069	9.930	.99497	15	.10019	.10069	9.930	.99497		.10019	.10069	9.930	.99497
46	.08310	.08339	11.992	.99655	14	.10048	.10099	9.901	.99494	14	.10048	.10099	9.901	.99494		.10048	.10099	9.901	.99494
47	.08339	.08368	11.950	.99652	13	.10077	.10128	9.873	.99491	13	.10077	.10128	9.873	.99491		.10077	.10128	9.873	.99491
48	.08368	.08397	11.909	.99649	12	.10106	.10158	9.844	.99488	12	.10106	.10158	9.844	.99488		.10106	.10158	9.844	.99488
49	.08397	.08427	11.867	.99647	11	.10135	.10187	9.816	.99485	11	.10135	.10187	9.816	.99485		.10135	.10187	9.816	.99485
50	.08426	.08456	11.826	.99644	10	.10164	.10216	9.788	.99482	10	.10164	.10216	9.788	.99482		.10164	.10216	9.788	.99482
51	.08455	.08485	11.785	.99642	9	.10192	.10246	9.760	.99479	9	.10192	.10246	9.760	.99479		.10192	.10246	9.760	.99479
52	.08484	.08514	11.745	.99639	8	.10221	.10275	9.732	.99476	8	.10221	.10275	9.732	.99476		.10221	.10275	9.732	.99476
53	.08513	.08544	11.705	.99637	7	.10250	.10305	9.704	.99473	7	.10250	.10305	9.704	.99473		.10250	.10305	9.704	.99473
54	.08542	.08573	11.664	.99635	6	.10279	.10334	9.676	.99470	6	.10279	.10334	9.676	.99470		.10279	.10334	9.676	.99470
55	.08571	.08602	11.625	.99632	5	.10308	.10363	9.649	.99467	5	.10308	.10363	9.649	.99467		.10308	.10363	9.649	.99467
56	.08600	.08632	11.585	.99630	4	.10337	.10393	9.620	.99464	4	.10337	.10393	9.620	.99464		.10337	.10393	9.620	.99464
57	.08629	.08661	11.546	.99627	3	.10366	.10422	9.594	.99461	3	.10366	.10422	9.594	.99461		.10366	.10422	9.594	.99461
58	.08658	.08690	11.507	.99625	2	.10395	.10452	9.567	.99458	2	.10395	.10452	9.567	.99458		.10395	.10452	9.567	.99458
59	.08687	.08720	11.468	.99622	1	.10424	.10481	9.541	.99455	1	.10424	.10481	9.541	.99455		.10424	.10481	9.541	.99455
60	.08716	.08749	11.430	.99619	0	.10453	.10510	9.514	.99452	0	.10453	.10510	9.514	.99452		.10453	.10510	9.514	.99452

94° (274°)

(265°) 85°

95° (275°)

(264°) 84°

Table 7 NATURAL TRIGONOMETRIC FUNCTIONS (continued)

6° (186°)				(353°) 173°				7° (187°)				(352°) 172°			
'	Sin	Tan	Ctn	Cos	'	Sin	Tan	Ctn	Cos	'	Sin	Tan	Ctn	Cos	'
0	.10453	.10510	9.5144	.99452	60	.12187	.12278	8.1443	.99255	60	.12187	.12278	8.1443	.99255	60
1	.10492	.10540	9.4878	.99449	59	.12216	.12308	8.1248	.99251	59	.12216	.12308	8.1248	.99251	59
2	.10511	.10563	9.4614	.99448	58	.12245	.12338	8.1054	.99248	58	.12245	.12338	8.1054	.99248	58
3	.10540	.10599	9.4352	.99443	57	.12274	.12367	8.0860	.99244	57	.12274	.12367	8.0860	.99244	57
4	.10569	.10628	9.4090	.99440	56	.12302	.12397	8.0667	.99240	56	.12302	.12397	8.0667	.99240	56
5	.10597	.10657	9.3831	.99437	55	.12331	.12426	8.0476	.99237	55	.12331	.12426	8.0476	.99237	55
6	.10626	.10687	9.3572	.99434	54	.12360	.12456	8.0285	.99233	54	.12360	.12456	8.0285	.99233	54
7	.10655	.10716	9.3315	.99431	53	.12389	.12485	8.0095	.99230	53	.12389	.12485	8.0095	.99230	53
8	.10684	.10746	9.3050	.99423	52	.12418	.12515	7.9905	.99226	52	.12418	.12515	7.9905	.99226	52
9	.10713	.10775	9.2806	.99424	51	.12447	.12544	7.9718	.99222	51	.12447	.12544	7.9718	.99222	51
10	.10742	.10805	9.2553	.99421	50	.12476	.12574	7.9530	.99219	50	.12476	.12574	7.9530	.99219	50
11	.10771	.10834	9.2302	.99418	49	.12504	.12603	7.9344	.99215	49	.12504	.12603	7.9344	.99215	49
12	.10800	.10863	9.2052	.99415	48	.12533	.12633	7.9168	.99211	48	.12533	.12633	7.9168	.99211	48
13	.10829	.10893	9.1803	.99412	47	.12562	.12662	7.8993	.99208	47	.12562	.12662	7.8993	.99208	47
14	.10858	.10922	9.1555	.99409	46	.12591	.12692	7.8821	.99204	46	.12591	.12692	7.8821	.99204	46
15	.10887	.10952	9.1308	.99406	45	.12620	.12722	7.8650	.99200	45	.12620	.12722	7.8650	.99200	45
16	.10916	.10981	9.1065	.99402	44	.12649	.12751	7.8484	.99197	44	.12649	.12751	7.8484	.99197	44
17	.10945	.11011	9.0821	.99399	43	.12678	.12781	7.8323	.99193	43	.12678	.12781	7.8323	.99193	43
18	.10973	.11040	9.0579	.99396	42	.12706	.12810	7.8166	.99189	42	.12706	.12810	7.8166	.99189	42
19	.11002	.11070	9.0338	.99393	41	.12735	.12840	7.7998	.99185	41	.12735	.12840	7.7998	.99185	41
20	.11031	.11099	9.0098	.99390	40	.12764	.12869	7.7704	.99182	40	.12764	.12869	7.7704	.99182	40
21	.11060	.11128	8.9860	.99386	39	.12793	.12899	7.7528	.99178	39	.12793	.12899	7.7528	.99178	39
22	.11089	.11158	8.9623	.99383	38	.12822	.12929	7.7346	.99175	38	.12822	.12929	7.7346	.99175	38
23	.11118	.11187	8.9387	.99380	37	.12851	.12958	7.7171	.99171	37	.12851	.12958	7.7171	.99171	37
24	.11147	.11217	8.9152	.99377	36	.12880	.12988	7.6996	.99167	36	.12880	.12988	7.6996	.99167	36
25	.11176	.11246	8.8919	.99374	35	.12908	.13017	7.6821	.99163	35	.12908	.13017	7.6821	.99163	35
26	.11205	.11276	8.8686	.99370	34	.12937	.13047	7.6647	.99160	34	.12937	.13047	7.6647	.99160	34
27	.11234	.11305	8.8455	.99367	33	.12966	.13076	7.6473	.99156	33	.12966	.13076	7.6473	.99156	33
28	.11263	.11335	8.8225	.99364	32	.12995	.13106	7.6301	.99152	32	.12995	.13106	7.6301	.99152	32
29	.11291	.11364	8.7996	.99360	31	.13024	.13136	7.6129	.99148	31	.13024	.13136	7.6129	.99148	31
30	.11320	.11394	8.7769	.99357	30	.13053	.13165	7.5958	.99144	30	.13053	.13165	7.5958	.99144	30
31	.11349	.11423	8.7542	.99354	29	.13081	.13195	7.5787	.99141	29	.13081	.13195	7.5787	.99141	29
32	.11378	.11452	8.7317	.99351	28	.13110	.13224	7.5618	.99137	28	.13110	.13224	7.5618	.99137	28
33	.11407	.11482	8.7093	.99347	27	.13139	.13254	7.5449	.99133	27	.13139	.13254	7.5449	.99133	27
34	.11436	.11511	8.6870	.99344	26	.13168	.13284	7.5281	.99129	26	.13168	.13284	7.5281	.99129	26
35	.11465	.11541	8.6648	.99341	25	.13197	.13313	7.5113	.99125	25	.13197	.13313	7.5113	.99125	25
36	.11494	.11570	8.6427	.99337	24	.13226	.13343	7.4947	.99122	24	.13226	.13343	7.4947	.99122	24
37	.11523	.11600	8.6208	.99334	23	.13254	.13372	7.4781	.99118	23	.13254	.13372	7.4781	.99118	23
38	.11552	.11629	8.5989	.99331	22	.13283	.13402	7.4615	.99114	22	.13283	.13402	7.4615	.99114	22
39	.11580	.11659	8.5772	.99327	21	.13312	.13432	7.4451	.99110	21	.13312	.13432	7.4451	.99110	21
40	.11609	.11688	8.5555	.99324	20	.13341	.13461	7.4287	.99106	20	.13341	.13461	7.4287	.99106	20
41	.11638	.11718	8.5340	.99320	19	.13370	.13491	7.4124	.99102	19	.13370	.13491	7.4124	.99102	19
42	.11667	.11747	8.5126	.99317	18	.13399	.13521	7.3962	.99098	18	.13399	.13521	7.3962	.99098	18
43	.11696	.11777	8.4913	.99314	17	.13427	.13550	7.3800	.99094	17	.13427	.13550	7.3800	.99094	17
44	.11725	.11806	8.4701	.99310	16	.13456	.13580	7.3639	.99091	16	.13456	.13580	7.3639	.99091	16
45	.11754	.11836	8.4490	.99307	15	.13485	.13609	7.3479	.99087	15	.13485	.13609	7.3479	.99087	15
46	.11783	.11865	8.4280	.99303	14	.13514	.13639	7.3319	.99083	14	.13514	.13639	7.3319	.99083	14
47	.11812	.11895	8.4071	.99300	13	.13543	.13669	7.3160	.99079	13	.13543	.13669	7.3160	.99079	13
48	.11840	.11924	8.3863	.99297	12	.13572	.13698	7.3002	.99075	12	.13572	.13698	7.3002	.99075	12
49	.11869	.11954	8.3656	.99293	11	.13600	.13728	7.2844	.99071	11	.13600	.13728	7.2844	.99071	11
50	.11898	.11983	8.3450	.99289	10	.13629	.13758	7.2687	.99067	10	.13629	.13758	7.2687	.99067	10
51	.11927	.12013	8.3245	.99288	9	.13658	.13787	7.2531	.99063	9	.13658	.13787	7.2531	.99063	9
52	.11956	.12042	8.3041	.99283	8	.13687	.13817	7.2375	.99059	8	.13687	.13817	7.2375	.99059	8
53	.11985	.12072	8.2838	.99279	7	.13716	.13846	7.2220	.99055	7	.13716	.13846	7.2220	.99055	7
54	.12014	.12101	8.2635	.99276	6	.13744	.13876	7.2066	.99051	6	.13744	.13876	7.2066	.99051	6
55	.12043	.12131	8.2434	.99272	5	.13773	.13906	7.1912	.99047	5	.13773	.13906	7.1912	.99047	5
56	.12071	.12160	8.2234	.99269	4	.13802	.13935	7.1759	.99043	4	.13802	.13935	7.1759	.99043	4
57	.12100	.12190	8.2035	.99265	3	.13831	.13965	7.1607	.99039	3	.13831	.13965	7.1607	.99039	3
58	.12129	.12219	8.1837	.99262	2	.13860	.13995	7.1455	.99035	2	.13860	.13995	7.1455	.99035	2
59	.12158	.12249	8.1640	.99258	1	.13889	.14024	7.1304	.99031	1	.13889	.14024	7.1304	.99031	1
60	.12187	.12278	8.1443	.99255	0	.13917	.14054	7.1154	.99027	0	.13917	.14054	7.1154	.99027	0
	Cos	Ctn	Tan	Sin		Cos	Ctn	Tan	Sin		Cos	Ctn	Tan	Sin	

96° (276°)

(263°) 83°

97° (277°)

(262°) 82°

Table 7 NATURAL TRIGONOMETRIC FUNCTIONS (continued)

8° (188°)					(351°) 171°					9° (189°)					(350°) 170°				
	Sin	Tan	Ctn	Cos		Sin	Tan	Ctn	Cos		Sin	Tan	Ctn	Cos		Sin	Tan	Ctn	Cos
0	.13917	.14054	7.1184	.99027	60	.15643	.15838	6.3138	.98769	60	.15643	.15838	6.3138	.98769	60	.15643	.15838	6.3138	.98769
1	.13946	.14084	7.1004	.99023	59	.15672	.15868	6.3019	.98764	59	.15672	.15868	6.3019	.98764	59	.15672	.15868	6.3019	.98764
2	.13975	.14113	7.0855	.99019	58	.15701	.15898	6.2901	.98760	58	.15701	.15898	6.2901	.98760	58	.15701	.15898	6.2901	.98760
3	.14004	.14143	7.0706	.99015	57	.15730	.15928	6.2783	.98756	57	.15730	.15928	6.2783	.98756	57	.15730	.15928	6.2783	.98756
4	.14033	.14173	7.0558	.99011	56	.15758	.15958	6.2666	.98751	56	.15758	.15958	6.2666	.98751	56	.15758	.15958	6.2666	.98751
5	.14061	.14202	7.0410	.99006	55	.15787	.15988	6.2549	.98746	55	.15787	.15988	6.2549	.98746	55	.15787	.15988	6.2549	.98746
6	.14090	.14232	7.0264	.99002	54	.15816	.16017	6.2432	.98741	54	.15816	.16017	6.2432	.98741	54	.15816	.16017	6.2432	.98741
7	.14119	.14262	7.0117	.98998	53	.15845	.16047	6.2316	.98737	53	.15845	.16047	6.2316	.98737	53	.15845	.16047	6.2316	.98737
8	.14148	.14291	6.9972	.98994	52	.15873	.16077	6.2200	.98732	52	.15873	.16077	6.2200	.98732	52	.15873	.16077	6.2200	.98732
9	.14177	.14321	6.9827	.98990	51	.15902	.16107	6.2085	.98728	51	.15902	.16107	6.2085	.98728	51	.15902	.16107	6.2085	.98728
10	.14205	.14351	6.9682	.98986	50	.15931	.16137	6.1970	.98723	50	.15931	.16137	6.1970	.98723	50	.15931	.16137	6.1970	.98723
11	.14234	.14381	6.9538	.98982	49	.15959	.16167	6.1856	.98718	49	.15959	.16167	6.1856	.98718	49	.15959	.16167	6.1856	.98718
12	.14263	.14410	6.9395	.98978	48	.15988	.16196	6.1742	.98714	48	.15988	.16196	6.1742	.98714	48	.15988	.16196	6.1742	.98714
13	.14292	.14440	6.9252	.98973	47	.16017	.16226	6.1628	.98709	47	.16017	.16226	6.1628	.98709	47	.16017	.16226	6.1628	.98709
14	.14320	.14470	6.9110	.98969	46	.16046	.16256	6.1515	.98704	46	.16046	.16256	6.1515	.98704	46	.16046	.16256	6.1515	.98704
15	.14349	.14499	6.8969	.98965	45	.16074	.16286	6.1402	.98700	45	.16074	.16286	6.1402	.98700	45	.16074	.16286	6.1402	.98700
16	.14378	.14529	6.8828	.98961	44	.16103	.16316	6.1290	.98695	44	.16103	.16316	6.1290	.98695	44	.16103	.16316	6.1290	.98695
17	.14407	.14559	6.8687	.98957	43	.16132	.16346	6.1178	.98690	43	.16132	.16346	6.1178	.98690	43	.16132	.16346	6.1178	.98690
18	.14436	.14588	6.8546	.98953	42	.16160	.16376	6.1066	.98686	42	.16160	.16376	6.1066	.98686	42	.16160	.16376	6.1066	.98686
19	.14464	.14618	6.8408	.98948	41	.16189	.16405	6.0955	.98681	41	.16189	.16405	6.0955	.98681	41	.16189	.16405	6.0955	.98681
20	.14493	.14648	6.8269	.98944	40	.16218	.16435	6.0844	.98676	40	.16218	.16435	6.0844	.98676	40	.16218	.16435	6.0844	.98676
21	.14522	.14678	6.8131	.98940	39	.16246	.16465	6.0734	.98671	39	.16246	.16465	6.0734	.98671	39	.16246	.16465	6.0734	.98671
22	.14551	.14707	6.7994	.98936	38	.16275	.16495	6.0624	.98667	38	.16275	.16495	6.0624	.98667	38	.16275	.16495	6.0624	.98667
23	.14580	.14737	6.7856	.98931	37	.16304	.16525	6.0514	.98662	37	.16304	.16525	6.0514	.98662	37	.16304	.16525	6.0514	.98662
24	.14608	.14767	6.7720	.98927	36	.16333	.16555	6.0405	.98657	36	.16333	.16555	6.0405	.98657	36	.16333	.16555	6.0405	.98657
25	.14637	.14796	6.7584	.98923	35	.16361	.16585	6.0296	.98652	35	.16361	.16585	6.0296	.98652	35	.16361	.16585	6.0296	.98652
26	.14666	.14826	6.7448	.98918	34	.16390	.16615	6.0188	.98648	34	.16390	.16615	6.0188	.98648	34	.16390	.16615	6.0188	.98648
27	.14695	.14856	6.7313	.98914	33	.16419	.16645	6.0080	.98643	33	.16419	.16645	6.0080	.98643	33	.16419	.16645	6.0080	.98643
28	.14723	.14886	6.7179	.98910	32	.16447	.16674	5.9972	.98638	32	.16447	.16674	5.9972	.98638	32	.16447	.16674	5.9972	.98638
29	.14752	.14915	6.7045	.98906	31	.16476	.16704	5.9865	.98633	31	.16476	.16704	5.9865	.98633	31	.16476	.16704	5.9865	.98633
30	.14781	.14945	6.6912	.98902	30	.16505	.16734	5.9758	.98629	30	.16505	.16734	5.9758	.98629	30	.16505	.16734	5.9758	.98629
31	.14810	.14975	6.6779	.98897	29	.16533	.16764	5.9651	.98624	29	.16533	.16764	5.9651	.98624	29	.16533	.16764	5.9651	.98624
32	.14838	.15005	6.6646	.98893	28	.16562	.16794	5.9545	.98619	28	.16562	.16794	5.9545	.98619	28	.16562	.16794	5.9545	.98619
33	.14867	.15034	6.6514	.98888	27	.16591	.16824	5.9439	.98614	27	.16591	.16824	5.9439	.98614	27	.16591	.16824	5.9439	.98614
34	.14896	.15064	6.6383	.98884	26	.16620	.16854	5.9333	.98609	26	.16620	.16854	5.9333	.98609	26	.16620	.16854	5.9333	.98609
35	.14925	.15094	6.6252	.98880	25	.16648	.16884	5.9228	.98604	25	.16648	.16884	5.9228	.98604	25	.16648	.16884	5.9228	.98604
36	.14954	.15124	6.6122	.98876	24	.16677	.16914	5.9124	.98600	24	.16677	.16914	5.9124	.98600	24	.16677	.16914	5.9124	.98600
37	.14982	.15153	6.5992	.98871	23	.16706	.16944	5.9019	.98596	23	.16706	.16944	5.9019	.98596	23	.16706	.16944	5.9019	.98596
38	.15011	.15183	6.5863	.98867	22	.16734	.16974	5.8915	.98591	22	.16734	.16974	5.8915	.98591	22	.16734	.16974	5.8915	.98591
39	.15040	.15213	6.5734	.98863	21	.16763	.17004	5.8811	.98586	21	.16763	.17004	5.8811	.98586	21	.16763	.17004	5.8811	.98586
40	.15069	.15243	6.5606	.98858	20	.16792	.17033	5.8708	.98580	20	.16792	.17033	5.8708	.98580	20	.16792	.17033	5.8708	.98580
41	.15097	.15272	6.5478	.98854	19	.16820	.17063	5.8605	.98575	19	.16820	.17063	5.8605	.98575	19	.16820	.17063	5.8605	.98575
42	.15126	.15302	6.5350	.98849	18	.16849	.17093	5.8502	.98570	18	.16849	.17093	5.8502	.98570	18	.16849	.17093	5.8502	.98570
43	.15155	.15332	6.5223	.98845	17	.16878	.17123	5.8400	.98566	17	.16878	.17123	5.8400	.98566	17	.16878	.17123	5.8400	.98566
44	.15184	.15362	6.5097	.98841	16	.16906	.17153	5.8298	.98561	16	.16906	.17153	5.8298	.98561	16	.16906	.17153	5.8298	.98561
45	.15212	.15391	6.4971	.98836	15	.16935	.17183	5.8197	.98556	15	.16935	.17183	5.8197	.98556	15	.16935	.17183	5.8197	.98556
46	.15241	.15421	6.4846	.98832	14	.16964	.17213	5.8095	.98551	14	.16964	.17213	5.8095	.98551	14	.16964	.17213	5.8095	.98551
47	.15270	.15451	6.4721	.98827	13	.16992	.17243	5.7994	.98546	13	.16992	.17243	5.7994	.98546	13	.16992	.17243	5.7994	.98546
48	.15299	.15481	6.4596	.98823	12	.17021	.17273	5.7894	.98541	12	.17021	.17273	5.7894	.98541	12	.17021	.17273	5.7894	.98541
49	.15327	.15511	6.4472	.98818	11	.17050	.17303	5.7794	.98536	11	.17050	.17303	5.7794	.98536	11	.17050	.17303	5.7794	.98536
50	.15356	.15540	6.4348	.98814	10	.17078	.17333	5.7694	.98531	10	.17078	.17333	5.7694	.98531	10	.17078	.17333	5.7694	.98531
51	.15385	.15570	6.4225	.98809	9	.17107	.17363	5.7594	.98526	9	.17107	.17363	5.7594	.98526	9	.17107	.17363	5.7594	.98526
52	.15414	.15600	6.4103	.98805	8	.17136	.17393	5.7495	.98521	8	.17136	.17393	5.7495	.98521	8	.17136	.17393	5.7495	.98521
53	.15442	.15630	6.3980	.98800	7	.17164	.17423	5.7396	.98516	7	.17164	.17423	5.7396	.98516	7	.17164	.17423	5.7396	.98516
54	.15471	.15660	6.3859	.98796	6	.17193	.17453	5.7297	.98511	6	.17193	.17453	5.7297	.98511	6	.17193	.17453	5.7297	.98511
55	.15500	.15689	6.3737	.98791	5	.17222	.17483	5.7199	.98506	5	.17222	.17483	5.7199	.98506	5	.17222	.17483	5.7199	.98506
56	.15529	.15719	6.3617	.98787	4	.17250	.17513	5.7101	.98501	4	.17250	.17513	5.7101	.98501	4	.17250	.17513	5.7101	.98501
57	.15557	.15749	6.3496	.98782	3	.17279	.17543	5.7004	.98496	3	.17279	.17543	5.7004	.98496	3	.17279	.17543	5.7004	.98496
58	.15586	.15779	6.3376	.98778	2	.17308	.17573	5.6906	.98491	2	.17308	.17573	5.6906	.98491	2	.17308	.17573	5.6906	.98491
59	.15615	.15809	6.3257	.98773	1	.17336	.17603	5.6809	.98486	1	.17336	.17603	5.6809	.98486	1	.17336	.17603	5.6809	.98486
60	.15643	.15838	6.3138	.98769	0	.17365	.17633	5.6713	.98481	0	.17365	.17633	5.6713	.98481	0	.17365	.17633	5.6713	.98481
	Cos	Ctn	Tan	Sin		Cos	Ctn</												

10° (190°)				(349°) 169°				11° (191°)				(348°) 168°			
°	Sin	Tan	Ctn.	Cos	°	Sin	Tan	Ctn.	Cos	°	Sin	Tan	Ctn.	Cos	°
0	.17365	.17633	5.6713	.98481	60	.19081	.19438	5.1446	.98163	60	.19081	.19438	5.1446	.98163	60
1	.17393	.17663	5.6617	.98476	59	.19109	.19468	5.1366	.98157	59	.19109	.19468	5.1366	.98157	59
2	.17422	.17693	5.6521	.98471	58	.19138	.19498	5.1286	.98152	58	.19138	.19498	5.1286	.98152	58
3	.17451	.17723	5.6425	.98466	57	.19167	.19529	5.1207	.98146	57	.19167	.19529	5.1207	.98146	57
4	.17479	.17753	5.6329	.98461	56	.19195	.19559	5.1128	.98140	56	.19195	.19559	5.1128	.98140	56
5	.17508	.17783	5.6234	.98455	55	.19224	.19589	5.1049	.98135	55	.19224	.19589	5.1049	.98135	55
6	.17537	.17813	5.6140	.98449	54	.19252	.19619	5.0970	.98129	54	.19252	.19619	5.0970	.98129	54
7	.17565	.17843	5.6045	.98445	53	.19281	.19649	5.0892	.98124	53	.19281	.19649	5.0892	.98124	53
8	.17594	.17873	5.5951	.98440	52	.19309	.19680	5.0814	.98118	52	.19309	.19680	5.0814	.98118	52
9	.17623	.17903	5.5857	.98435	51	.19338	.19710	5.0736	.98112	51	.19338	.19710	5.0736	.98112	51
10	.17651	.17933	5.5764	.98430	50	.19366	.19740	5.0658	.98107	50	.19366	.19740	5.0658	.98107	50
11	.17680	.17963	5.5671	.98425	49	.19395	.19770	5.0581	.98101	49	.19395	.19770	5.0581	.98101	49
12	.17708	.17993	5.5579	.98420	48	.19423	.19801	5.0504	.98096	48	.19423	.19801	5.0504	.98096	48
13	.17737	.18023	5.5483	.98414	47	.19452	.19831	5.0427	.98090	47	.19452	.19831	5.0427	.98090	47
14	.17766	.18053	5.5393	.98409	46	.19481	.19861	5.0350	.98084	46	.19481	.19861	5.0350	.98084	46
15	.17794	.18083	5.5301	.98404	45	.19509	.19891	5.0273	.98079	45	.19509	.19891	5.0273	.98079	45
16	.17823	.18113	5.5209	.98399	44	.19538	.19921	5.0197	.98073	44	.19538	.19921	5.0197	.98073	44
17	.17852	.18143	5.5118	.98394	43	.19566	.19952	5.0121	.98067	43	.19566	.19952	5.0121	.98067	43
18	.17880	.18173	5.5026	.98389	42	.19595	.19982	5.0045	.98061	42	.19595	.19982	5.0045	.98061	42
19	.17909	.18203	5.4936	.98383	41	.19623	.20012	4.9969	.98056	41	.19623	.20012	4.9969	.98056	41
20	.17937	.18233	5.4845	.98378	40	.19652	.20042	4.9894	.98050	40	.19652	.20042	4.9894	.98050	40
21	.17966	.18263	5.4756	.98373	39	.19680	.20073	4.9819	.98044	39	.19680	.20073	4.9819	.98044	39
22	.17995	.18293	5.4665	.98368	38	.19709	.20103	4.9744	.98039	38	.19709	.20103	4.9744	.98039	38
23	.18023	.18323	5.4575	.98362	37	.19737	.20133	4.9669	.98033	37	.19737	.20133	4.9669	.98033	37
24	.18052	.18353	5.4486	.98357	36	.19766	.20164	4.9594	.98027	36	.19766	.20164	4.9594	.98027	36
25	.18081	.18384	5.4397	.98352	35	.19794	.20194	4.9520	.98021	35	.19794	.20194	4.9520	.98021	35
26	.18109	.18414	5.4308	.98347	34	.19823	.20224	4.9446	.98016	34	.19823	.20224	4.9446	.98016	34
27	.18138	.18444	5.4219	.98341	33	.19851	.20254	4.9372	.98010	33	.19851	.20254	4.9372	.98010	33
28	.18166	.18474	5.4131	.98336	32	.19880	.20285	4.9298	.98004	32	.19880	.20285	4.9298	.98004	32
29	.18195	.18504	5.4043	.98331	31	.19908	.20316	4.9225	.97998	31	.19908	.20316	4.9225	.97998	31
30	.18224	.18534	5.3955	.98325	30	.19937	.20346	4.9152	.97992	30	.19937	.20346	4.9152	.97992	30
31	.18252	.18564	5.3868	.98320	29	.19965	.20376	4.9078	.97987	29	.19965	.20376	4.9078	.97987	29
32	.18281	.18594	5.3781	.98315	28	.19994	.20406	4.9006	.97981	28	.19994	.20406	4.9006	.97981	28
33	.18309	.18624	5.3694	.98310	27	.20022	.20436	4.8933	.97975	27	.20022	.20436	4.8933	.97975	27
34	.18338	.18654	5.3607	.98304	26	.20051	.20466	4.8860	.97969	26	.20051	.20466	4.8860	.97969	26
35	.18367	.18684	5.3521	.98299	25	.20079	.20497	4.8788	.97963	25	.20079	.20497	4.8788	.97963	25
36	.18395	.18714	5.3435	.98294	24	.20108	.20527	4.8715	.97958	24	.20108	.20527	4.8715	.97958	24
37	.18424	.18745	5.3349	.98288	23	.20136	.20557	4.8644	.97952	23	.20136	.20557	4.8644	.97952	23
38	.18452	.18775	5.3263	.98283	22	.20165	.20588	4.8573	.97946	22	.20165	.20588	4.8573	.97946	22
39	.18481	.18805	5.3178	.98277	21	.20193	.20618	4.8501	.97940	21	.20193	.20618	4.8501	.97940	21
40	.18509	.18835	5.3093	.98272	20	.20222	.20648	4.8430	.97934	20	.20222	.20648	4.8430	.97934	20
41	.18538	.18865	5.3008	.98267	19	.20250	.20679	4.8359	.97928	19	.20250	.20679	4.8359	.97928	19
42	.18567	.18895	5.2924	.98261	18	.20279	.20709	4.8288	.97922	18	.20279	.20709	4.8288	.97922	18
43	.18595	.18925	5.2839	.98256	17	.20307	.20739	4.8218	.97916	17	.20307	.20739	4.8218	.97916	17
44	.18624	.18955	5.2755	.98250	16	.20336	.20770	4.8147	.97910	16	.20336	.20770	4.8147	.97910	16
45	.18652	.18985	5.2672	.98245	15	.20364	.20800	4.8077	.97905	15	.20364	.20800	4.8077	.97905	15
46	.18681	.19015	5.2588	.98240	14	.20393	.20830	4.8007	.97899	14	.20393	.20830	4.8007	.97899	14
47	.18710	.19046	5.2505	.98234	13	.20421	.20861	4.7937	.97893	13	.20421	.20861	4.7937	.97893	13
48	.18738	.19076	5.2422	.98229	12	.20450	.20891	4.7867	.97887	12	.20450	.20891	4.7867	.97887	12
49	.18767	.19106	5.2339	.98223	11	.20478	.20921	4.7796	.97881	11	.20478	.20921	4.7796	.97881	11
50	.18795	.19136	5.2257	.98218	10	.20507	.20952	4.7726	.97875	10	.20507	.20952	4.7726	.97875	10
51	.18824	.19166	5.2174	.98212	9	.20535	.20982	4.7655	.97869	9	.20535	.20982	4.7655	.97869	9
52	.18852	.19197	5.2092	.98207	8	.20563	.21013	4.7585	.97863	8	.20563	.21013	4.7585	.97863	8
53	.18881	.19227	5.2011	.98201	7	.20592	.21043	4.7512	.97857	7	.20592	.21043	4.7512	.97857	7
54	.18910	.19257	5.1929	.98196	6	.20620	.21073	4.7443	.97851	6	.20620	.21073	4.7443	.97851	6
55	.18938	.19287	5.1848	.98190	5	.20649	.21104	4.7384	.97845	5	.20649	.21104	4.7384	.97845	5
56	.18967	.19317	5.1767	.98185	4	.20677	.21134	4.7317	.97839	4	.20677	.21134	4.7317	.97839	4
57	.18995	.19347	5.1686	.98179	3	.20706	.21164	4.7249	.97833	3	.20706	.21164	4.7249	.97833	3
58	.19024	.19376	5.1605	.98174	2	.20734	.21195	4.7181	.97827	2	.20734	.21195	4.7181	.97827	2
59	.19052	.19406	5.1526	.98168	1	.20763	.21225	4.7114	.97821	1	.20763	.21225	4.7114	.97821	1
60	.19081	.19438	5.1446	.98163	0	.20791	.21256	4.7048	.97815	0	.20791	.21256	4.7048	.97815	0
°	Cos	Ctn	Tan	Sin	°	Cos	Ctn	Tan	Sin	°	Cos	Ctn	Tan	Sin	°

100° (280°)

(259°) 79°

101° (281°)

(258°) 78°

Table 7 NATURAL TRIGONOMETRIC FUNCTIONS (continued)

12° (192°)					(347°) 167°					13° (193°)					(346°) 166°				
'	Sin	Tan	Ctn	Cos	'	Sin	Tan	Ctn	Cos	'	Sin	Tan	Ctn	Cos	'	Sin	Tan	Ctn	Cos
0	.20791	.21256	4.7046	.97815	60	.22495	.23087	4.3315	.97437	60	.22495	.23087	4.3315	.97437	60	.22523	.23117	4.3287	.97430
1	.20820	.21286	4.6979	.97809	59	.22523	.23147	4.3260	.97424	59	.22552	.23148	4.3200	.97417	58	.22580	.23179	4.3143	.97411
2	.20848	.21316	4.6912	.97803	58	.22552	.23179	4.3143	.97411	58	.22580	.23209	4.3086	.97404	57	.22608	.23240	4.3029	.97398
3	.20877	.21347	4.6845	.97797	57	.22580	.23209	4.3029	.97398	57	.22608	.23240	4.2972	.97391	56	.22637	.23271	4.2915	.97385
4	.20905	.21377	4.6779	.97791	56	.22608	.23240	4.2915	.97385	56	.22637	.23271	4.2858	.97378	55	.22665	.23301	4.2803	.97371
5	.20933	.21408	4.6712	.97784	55	.22637	.23271	4.2858	.97378	55	.22665	.23301	4.2803	.97371	54	.22693	.23332	4.2747	.97365
6	.20962	.21438	4.6646	.97778	54	.22665	.23301	4.2747	.97365	54	.22693	.23332	4.2690	.97358	53	.22722	.23363	4.2690	.97351
7	.20990	.21469	4.6580	.97772	53	.22693	.23332	4.2690	.97358	53	.22722	.23363	4.2633	.97351	52	.22750	.23394	4.2574	.97344
8	.21019	.21499	4.6514	.97765	52	.22722	.23363	4.2574	.97344	52	.22750	.23394	4.2517	.97337	51	.22778	.23424	4.2517	.97330
9	.21047	.21529	4.6448	.97760	51	.22750	.23394	4.2517	.97337	51	.22778	.23424	4.2460	.97330	50	.22807	.23455	4.2460	.97323
10	.21076	.21560	4.6382	.97754	50	.22807	.23455	4.2460	.97323	50	.22835	.23485	4.2403	.97316	49	.22835	.23485	4.2347	.97310
11	.21104	.21590	4.6317	.97748	49	.22835	.23485	4.2347	.97310	49	.22863	.23515	4.2347	.97303	48	.22863	.23515	4.2290	.97294
12	.21132	.21621	4.6252	.97742	48	.22863	.23515	4.2290	.97294	48	.22892	.23546	4.2290	.97287	47	.22892	.23546	4.2233	.97287
13	.21161	.21651	4.6187	.97735	47	.22892	.23546	4.2233	.97287	47	.22920	.23577	4.2233	.97280	46	.22920	.23577	4.2176	.97280
14	.21189	.21682	4.6122	.97729	46	.22920	.23577	4.2176	.97280	46	.22949	.23607	4.2176	.97273	45	.22949	.23607	4.2119	.97273
15	.21218	.21712	4.6057	.97723	45	.22949	.23607	4.2119	.97273	45	.22977	.23638	4.2119	.97266	44	.22977	.23638	4.2062	.97266
16	.21246	.21743	4.6033	.97717	44	.22977	.23638	4.2062	.97266	44	.22997	.23668	4.2062	.97259	43	.22997	.23668	4.2005	.97259
17	.21275	.21773	4.5998	.97711	43	.22997	.23668	4.2005	.97259	43	.23005	.23699	4.2005	.97252	42	.23005	.23699	4.1948	.97252
18	.21303	.21804	4.5964	.97705	42	.23005	.23699	4.1948	.97252	42	.23033	.23730	4.1948	.97245	41	.23033	.23730	4.1891	.97245
19	.21331	.21834	4.5900	.97698	41	.23033	.23730	4.1891	.97245	41	.23062	.23760	4.1891	.97238	40	.23062	.23760	4.1834	.97238
20	.21360	.21864	4.5836	.97692	40	.23062	.23760	4.1834	.97238	40	.23090	.23791	4.1834	.97231	39	.23090	.23791	4.1777	.97231
21	.21388	.21895	4.5773	.97686	39	.23090	.23791	4.1777	.97231	39	.23118	.23822	4.1777	.97224	38	.23118	.23822	4.1720	.97224
22	.21417	.21925	4.5699	.97680	38	.23118	.23822	4.1720	.97224	38	.23146	.23853	4.1720	.97217	37	.23146	.23853	4.1663	.97217
23	.21445	.21956	4.5646	.97673	37	.23146	.23853	4.1663	.97217	37	.23175	.23883	4.1663	.97210	36	.23175	.23883	4.1606	.97210
24	.21474	.21986	4.5583	.97667	36	.23175	.23883	4.1606	.97210	36	.23203	.23914	4.1606	.97203	35	.23203	.23914	4.1549	.97203
25	.21502	.22017	4.5520	.97661	35	.23203	.23914	4.1549	.97203	35	.23231	.23945	4.1549	.97196	34	.23231	.23945	4.1492	.97196
26	.21530	.22047	4.5457	.97655	34	.23231	.23945	4.1492	.97196	34	.23260	.23976	4.1492	.97189	33	.23260	.23976	4.1435	.97189
27	.21559	.22078	4.5394	.97648	33	.23260	.23976	4.1435	.97189	33	.23288	.24007	4.1435	.97182	32	.23288	.24007	4.1378	.97182
28	.21587	.22108	4.5332	.97642	32	.23288	.24007	4.1378	.97182	32	.23316	.24037	4.1378	.97175	31	.23316	.24037	4.1321	.97175
29	.21616	.22139	4.5269	.97636	31	.23316	.24037	4.1321	.97175	31	.23345	.24068	4.1321	.97168	30	.23345	.24068	4.1264	.97168
30	.21644	.22169	4.5207	.97630	30	.23345	.24068	4.1264	.97168	30	.23373	.24099	4.1264	.97161	29	.23373	.24099	4.1207	.97161
31	.21672	.22200	4.5145	.97623	29	.23373	.24099	4.1207	.97161	29	.23401	.24130	4.1207	.97154	28	.23401	.24130	4.1150	.97154
32	.21701	.22231	4.5083	.97617	28	.23401	.24130	4.1150	.97154	28	.23429	.24160	4.1150	.97147	27	.23429	.24160	4.1093	.97147
33	.21729	.22261	4.5022	.97611	27	.23429	.24160	4.1093	.97147	27	.23458	.24191	4.1093	.97140	26	.23458	.24191	4.1036	.97140
34	.21758	.22292	4.4960	.97604	26	.23458	.24191	4.1036	.97140	26	.23486	.24222	4.1036	.97133	25	.23486	.24222	4.0979	.97133
35	.21786	.22322	4.4899	.97598	25	.23486	.24222	4.0979	.97133	25	.23514	.24253	4.0979	.97126	24	.23514	.24253	4.0922	.97126
36	.21814	.22353	4.4837	.97592	24	.23514	.24253	4.0922	.97126	24	.23542	.24283	4.0922	.97119	23	.23542	.24283	4.0865	.97119
37	.21843	.22383	4.4776	.97585	23	.23542	.24283	4.0865	.97119	23	.23571	.24314	4.0865	.97112	22	.23571	.24314	4.0808	.97112
38	.21871	.22414	4.4715	.97579	22	.23571	.24314	4.0808	.97112	22	.23600	.24344	4.0808	.97105	21	.23600	.24344	4.0751	.97105
39	.21899	.22444	4.4655	.97573	21	.23600	.24344	4.0751	.97105	21	.23629	.24375	4.0751	.97098	20	.23629	.24375	4.0694	.97098
40	.21928	.22475	4.4594	.97566	20	.23629	.24375	4.0694	.97098	20	.23657	.24406	4.0694	.97091	19	.23657	.24406	4.0637	.97091
41	.21956	.22505	4.4534	.97560	19	.23657	.24406	4.0637	.97091	19	.23686	.24437	4.0637	.97084	18	.23686	.24437	4.0580	.97084
42	.21985	.22536	4.4473	.97553	18	.23686	.24437	4.0580	.97084	18	.23712	.24468	4.0580	.97077	17	.23712	.24468	4.0523	.97077
43	.22013	.22567	4.4413	.97547	17	.23712	.24468	4.0523	.97077	17	.23740	.24499	4.0523	.97070	16	.23740	.24499	4.0466	.97070
44	.22041	.22597	4.4353	.97541	16	.23740	.24499	4.0466	.97070	16	.23769	.24530	4.0466	.97063	15	.23769	.24530	4.0409	.97063
45	.22070	.22628	4.4294	.97534	15	.23769	.24530	4.0409	.97063	15	.23797	.24561	4.0409	.97056	14	.23797	.24561	4.0352	.97056
46	.22098	.22658	4.4234	.97528	14	.23797	.24561	4.0352	.97056	14	.23825	.24592	4.0352	.97049	13	.23825	.24592	4.0295	.97049
47	.22126	.22689	4.4175	.97521	13	.23825	.24592	4.0295	.97049	13	.23853	.24623	4.0295	.97042	12	.23853	.24623	4.0238	.97042
48	.22155	.22719	4.4115	.97515	12	.23853	.24623	4.0238	.97042	12	.23882	.24654	4.0238	.97035	11	.23882	.24654	4.0181	.97035
49	.22183	.22750	4.4056	.97508	11	.23882	.24654	4.0181	.97035	11	.23910	.24685	4.0181	.97028	10	.23910	.24685	4.0124	.97028
50	.22212	.22781	4.4009	.97502	10	.23910	.24685	4.0124	.97028	10	.23938	.24716	4.0124	.97021	9	.23938	.24716	4.0067	.97021
51	.22240	.22811	4.3938	.97496	9	.23938	.24716	4.0067	.97021	9	.23966	.24747	4.0067	.97014	8	.23966	.24747	4.0010	.97014
52	.22268	.22842	4.3879	.97489	8	.23966	.24747	4.0010	.97014	8	.23995	.24778	4.0010	.97007	7	.23995	.24778	3.9953	.97007
53	.22297	.22872	4.3779	.97483	7	.23995	.24778	3.9953	.97007	7	.24023	.24809	3.9953	.97000	6	.24023	.24809	3.9896	.97000
54	.22325	.22903	4.3662	.97476	6	.24023	.24809	3.9896	.97000	6	.24051	.24840	3.9896	.96993	5	.24051	.24840	3.9839	.96993
55	.22353	.22934	4.3604	.97470	5	.24051	.24840	3.9839	.96993	5	.24079	.24871	3.9839	.96986	4	.24079	.24871	3.9782	.96986
56	.22382	.22964	4.3546	.97463	4	.24079	.24871	3.9782	.96986	4	.24108	.24902	3.9782	.96979	3	.24108	.24902	3.9725	.96979
57	.22410	.22995	4.3488	.97457	3	.24108	.24902	3.9725	.96979	3	.24136	.24933	3.9725	.96972	2	.24136	.24933	3.9668	.96972
58	.22438	.23026	4.3430	.97450	2	.24136	.24933	3.9668	.96972	2	.24164	.24964	3.9668	.96965	1	.24164	.24964	3.9611	.96965
59	.22467	.23056	4.3372	.97444	1	.24164	.24964	3.9611	.96965	1	.24192	.24995	3.9611	.96958	0	.24192	.24995	3.9554	.96958
60	.22495	.23087	4.3315	.97437	0	.24192	.24995	3.9554	.96958	0	.24220	.25026	3.9554	.96951		.24220	.25026	3.9497	.96951
	Cos	Ctn	Tan	Sin		Cos	Ctn	Tan											

Table 7 NATURAL TRIGONOMETRIC FUNCTIONS (continued)

14° (194°)				(345°) 165°				15° (195°)				(344°) 164°							
	Sin	Tan	Ctn	Cos		Sin	Tan	Ctn	Cos		Sin	Tan	Ctn	Cos		Sin	Tan	Ctn	Cos
0	.24192	.24933	4.0108	.97030	60	.25882	.26795	3.7321	.96593	60	.25882	.26795	3.7321	.96593	60	.25882	.26795	3.7321	.96593
1	.24220	.24954	4.0058	.97023	59	.25910	.26826	3.7277	.96585	59	.25910	.26826	3.7277	.96585	59	.25910	.26826	3.7277	.96585
2	.24249	.24995	4.0009	.97015	58	.25938	.26857	3.7234	.96578	58	.25938	.26857	3.7234	.96578	58	.25938	.26857	3.7234	.96578
3	.24277	.25026	3.9959	.97008	57	.25966	.26888	3.7191	.96570	57	.25966	.26888	3.7191	.96570	57	.25966	.26888	3.7191	.96570
4	.24305	.25056	3.9910	.97001	56	.25994	.26920	3.7148	.96562	56	.25994	.26920	3.7148	.96562	56	.25994	.26920	3.7148	.96562
5	.24333	.25087	3.9861	.96994	55	.26022	.26951	3.7105	.96555	55	.26022	.26951	3.7105	.96555	55	.26022	.26951	3.7105	.96555
6	.24362	.25118	3.9812	.96987	54	.26050	.26982	3.7062	.96547	54	.26050	.26982	3.7062	.96547	54	.26050	.26982	3.7062	.96547
7	.24390	.25149	3.9763	.96980	53	.26079	.27013	3.7019	.96540	53	.26079	.27013	3.7019	.96540	53	.26079	.27013	3.7019	.96540
8	.24418	.25180	3.9714	.96973	52	.26107	.27044	3.6976	.96532	52	.26107	.27044	3.6976	.96532	52	.26107	.27044	3.6976	.96532
9	.24446	.25211	3.9665	.96966	51	.26135	.27076	3.6933	.96524	51	.26135	.27076	3.6933	.96524	51	.26135	.27076	3.6933	.96524
10	.24474	.25242	3.9617	.96959	50	.26163	.27107	3.6891	.96517	50	.26163	.27107	3.6891	.96517	50	.26163	.27107	3.6891	.96517
11	.24503	.25273	3.9568	.96952	49	.26191	.27138	3.6848	.96509	49	.26191	.27138	3.6848	.96509	49	.26191	.27138	3.6848	.96509
12	.24531	.25304	3.9520	.96945	48	.26219	.27169	3.6805	.96502	48	.26219	.27169	3.6805	.96502	48	.26219	.27169	3.6805	.96502
13	.24559	.25335	3.9471	.96937	47	.26247	.27201	3.6762	.96494	47	.26247	.27201	3.6762	.96494	47	.26247	.27201	3.6762	.96494
14	.24587	.25366	3.9423	.96930	46	.26275	.27232	3.6722	.96486	46	.26275	.27232	3.6722	.96486	46	.26275	.27232	3.6722	.96486
15	.24615	.25397	3.9375	.96923	45	.26303	.27263	3.6680	.96479	45	.26303	.27263	3.6680	.96479	45	.26303	.27263	3.6680	.96479
16	.24644	.25428	3.9327	.96916	44	.26331	.27294	3.6638	.96471	44	.26331	.27294	3.6638	.96471	44	.26331	.27294	3.6638	.96471
17	.24672	.25459	3.9279	.96909	43	.26359	.27326	3.6596	.96463	43	.26359	.27326	3.6596	.96463	43	.26359	.27326	3.6596	.96463
18	.24700	.25490	3.9232	.96902	42	.26387	.27357	3.6554	.96456	42	.26387	.27357	3.6554	.96456	42	.26387	.27357	3.6554	.96456
19	.24728	.25521	3.9184	.96894	41	.26415	.27388	3.6512	.96448	41	.26415	.27388	3.6512	.96448	41	.26415	.27388	3.6512	.96448
20	.24756	.25552	3.9136	.96887	40	.26443	.27419	3.6470	.96440	40	.26443	.27419	3.6470	.96440	40	.26443	.27419	3.6470	.96440
21	.24784	.25583	3.9089	.96880	39	.26471	.27451	3.6429	.96433	39	.26471	.27451	3.6429	.96433	39	.26471	.27451	3.6429	.96433
22	.24813	.25614	3.9042	.96873	38	.26500	.27482	3.6387	.96425	38	.26500	.27482	3.6387	.96425	38	.26500	.27482	3.6387	.96425
23	.24841	.25645	3.8995	.96866	37	.26528	.27513	3.6345	.96417	37	.26528	.27513	3.6345	.96417	37	.26528	.27513	3.6345	.96417
24	.24869	.25676	3.8947	.96858	36	.26556	.27545	3.6305	.96410	36	.26556	.27545	3.6305	.96410	36	.26556	.27545	3.6305	.96410
25	.24897	.25707	3.8900	.96851	35	.26584	.27576	3.6264	.96402	35	.26584	.27576	3.6264	.96402	35	.26584	.27576	3.6264	.96402
26	.24925	.25738	3.8854	.96844	34	.26612	.27607	3.6222	.96394	34	.26612	.27607	3.6222	.96394	34	.26612	.27607	3.6222	.96394
27	.24954	.25769	3.8807	.96837	33	.26640	.27638	3.6181	.96386	33	.26640	.27638	3.6181	.96386	33	.26640	.27638	3.6181	.96386
28	.24982	.25800	3.8760	.96829	32	.26668	.27670	3.6140	.96378	32	.26668	.27670	3.6140	.96378	32	.26668	.27670	3.6140	.96378
29	.25010	.25831	3.8714	.96822	31	.26696	.27701	3.6100	.96371	31	.26696	.27701	3.6100	.96371	31	.26696	.27701	3.6100	.96371
30	.25038	.25862	3.8667	.96815	30	.26724	.27732	3.6059	.96363	30	.26724	.27732	3.6059	.96363	30	.26724	.27732	3.6059	.96363
31	.25066	.25893	3.8621	.96807	29	.26752	.27764	3.6018	.96355	29	.26752	.27764	3.6018	.96355	29	.26752	.27764	3.6018	.96355
32	.25094	.25924	3.8575	.96800	28	.26780	.27795	3.5978	.96347	28	.26780	.27795	3.5978	.96347	28	.26780	.27795	3.5978	.96347
33	.25122	.25955	3.8528	.96793	27	.26808	.27826	3.5937	.96340	27	.26808	.27826	3.5937	.96340	27	.26808	.27826	3.5937	.96340
34	.25151	.25986	3.8482	.96786	26	.26836	.27858	3.5897	.96332	26	.26836	.27858	3.5897	.96332	26	.26836	.27858	3.5897	.96332
35	.25179	.26017	3.8436	.96778	25	.26864	.27889	3.5856	.96324	25	.26864	.27889	3.5856	.96324	25	.26864	.27889	3.5856	.96324
36	.25207	.26048	3.8391	.96771	24	.26892	.27921	3.5815	.96316	24	.26892	.27921	3.5815	.96316	24	.26892	.27921	3.5815	.96316
37	.25235	.26079	3.8345	.96764	23	.26920	.27952	3.5775	.96308	23	.26920	.27952	3.5775	.96308	23	.26920	.27952	3.5775	.96308
38	.25263	.26110	3.8299	.96756	22	.26948	.27983	3.5735	.96301	22	.26948	.27983	3.5735	.96301	22	.26948	.27983	3.5735	.96301
39	.25291	.26141	3.8254	.96749	21	.26976	.28015	3.5695	.96293	21	.26976	.28015	3.5695	.96293	21	.26976	.28015	3.5695	.96293
40	.25320	.26172	3.8208	.96742	20	.27004	.28045	3.5656	.96285	20	.27004	.28045	3.5656	.96285	20	.27004	.28045	3.5656	.96285
41	.25348	.26203	3.8163	.96734	19	.27032	.28077	3.5616	.96277	19	.27032	.28077	3.5616	.96277	19	.27032	.28077	3.5616	.96277
42	.25376	.26235	3.8118	.96727	18	.27060	.28109	3.5576	.96269	18	.27060	.28109	3.5576	.96269	18	.27060	.28109	3.5576	.96269
43	.25404	.26266	3.8073	.96719	17	.27088	.28140	3.5536	.96261	17	.27088	.28140	3.5536	.96261	17	.27088	.28140	3.5536	.96261
44	.25432	.26297	3.8028	.96712	16	.27116	.28172	3.5497	.96253	16	.27116	.28172	3.5497	.96253	16	.27116	.28172	3.5497	.96253
45	.25460	.26328	3.7983	.96705	15	.27144	.28203	3.5457	.96246	15	.27144	.28203	3.5457	.96246	15	.27144	.28203	3.5457	.96246
46	.25488	.26359	3.7938	.96697	14	.27172	.28234	3.5418	.96238	14	.27172	.28234	3.5418	.96238	14	.27172	.28234	3.5418	.96238
47	.25516	.26390	3.7893	.96690	13	.27200	.28265	3.5379	.96230	13	.27200	.28265	3.5379	.96230	13	.27200	.28265	3.5379	.96230
48	.25545	.26421	3.7848	.96682	12	.27228	.28297	3.5339	.96222	12	.27228	.28297	3.5339	.96222	12	.27228	.28297	3.5339	.96222
49	.25573	.26452	3.7804	.96675	11	.27256	.28328	3.5300	.96214	11	.27256	.28328	3.5300	.96214	11	.27256	.28328	3.5300	.96214
50	.25601	.26483	3.7759	.96667	10	.27284	.28360	3.5261	.96206	10	.27284	.28360	3.5261	.96206	10	.27284	.28360	3.5261	.96206
51	.25629	.26515	3.7715	.96660	9	.27312	.28391	3.5222	.96198	9	.27312	.28391	3.5222	.96198	9	.27312	.28391	3.5222	.96198
52	.25657	.26546	3.7671	.96653	8	.27340	.28423	3.5183	.96190	8	.27340	.28423	3.5183	.96190	8	.27340	.28423	3.5183	.96190
53	.25685	.26577	3.7627	.96645	7	.27368	.28454	3.5144	.96182	7	.27368	.28454	3.5144	.96182	7	.27368	.28454	3.5144	.96182
54	.25713	.26608	3.7583	.96638	6	.27395	.28485	3.5105	.96174	6	.27395	.28485	3.5105	.96174	6	.27395	.28485	3.5105	.96174
55	.25741	.26639	3.7539	.96630	5	.27424	.28517	3.5067	.96166	5	.27424	.28517	3.5067	.96166	5	.27424	.28517	3.5067	.96166
56	.25769	.26670	3.7495	.96623	4	.27452	.28549	3.5028	.96158	4	.27452	.28549	3.5028	.96158	4	.27452	.28549	3.5028	.96158
57	.25798	.26701	3.7451	.96615	3	.27480	.28580	3.4989	.96150	3	.27480	.28580	3.4989	.96150	3	.27480	.28580	3.4989	.96150
58	.25826	.26733	3.7408	.96608	2	.27508	.28612	3.4951	.96142	2	.27508	.28612	3.4951	.96142	2	.27508	.28612	3.4951	.96142
59	.25854	.26764	3.7364	.96600	1	.27538	.28643	3.4912	.96134	1	.27538	.28643	3.4912	.96134	1	.27538	.28643	3.4912	.96134
60	.25882	.26795	3.7321	.96593	0	.27564	.28675	3.4874	.96126	0	.27564	.28675	3.4874	.96126	0	.27564	.28675	3.4874	.96126

104° (284°) (255°) 75° 105° (285°) (254°) 74°

Table 7 NATURAL TRIGONOMETRIC FUNCTIONS (continued)

16° (196°)				(343°) 163°				17° (197°)				(342°) 162°					
	Sin	Tan	Ctn	Cos	'		Sin	Tan	Ctn	Cos	'		Sin	Tan	Ctn	Cos	'
0	.27564	.28675	3.4874	.96126	60	0	.29237	.30573	3.2709	.95630	60	0	.29237	.30573	3.2709	.95630	60
1	.27582	.28705	3.4836	.96118	59	1	.29265	.30605	3.2675	.95622	59	1	.29265	.30605	3.2675	.95622	59
2	.27620	.28735	3.4798	.96110	58	2	.29293	.30637	3.2641	.95613	58	2	.29293	.30637	3.2641	.95613	58
3	.27648	.28769	3.4760	.96102	57	3	.29321	.30669	3.2607	.95605	57	3	.29321	.30669	3.2607	.95605	57
4	.27676	.28801	3.4722	.96094	56	4	.29348	.30700	3.2573	.95596	56	4	.29348	.30700	3.2573	.95596	56
5	.27704	.28832	3.4684	.96086	55	5	.29376	.30732	3.2539	.95588	55	5	.29376	.30732	3.2539	.95588	55
6	.27731	.28864	3.4646	.96078	54	6	.29404	.30764	3.2508	.95579	54	6	.29404	.30764	3.2508	.95579	54
7	.27759	.28895	3.4608	.96070	53	7	.29432	.30796	3.2472	.95571	53	7	.29432	.30796	3.2472	.95571	53
8	.27787	.28927	3.4570	.96062	52	8	.29460	.30828	3.2438	.95562	52	8	.29460	.30828	3.2438	.95562	52
9	.27815	.28958	3.4533	.96054	51	9	.29487	.30860	3.2405	.95554	51	9	.29487	.30860	3.2405	.95554	51
10	.27843	.28990	3.4495	.96046	50	10	.29515	.30891	3.2371	.95545	50	10	.29515	.30891	3.2371	.95545	50
11	.27871	.29021	3.4458	.96037	49	11	.29543	.30923	3.2338	.95536	49	11	.29543	.30923	3.2338	.95536	49
12	.27899	.29053	3.4420	.96029	48	12	.29571	.30955	3.2305	.95528	48	12	.29571	.30955	3.2305	.95528	48
13	.27927	.29084	3.4383	.96021	47	13	.29599	.30987	3.2272	.95519	47	13	.29599	.30987	3.2272	.95519	47
14	.27955	.29116	3.4345	.96013	46	14	.29626	.31019	3.2238	.95511	46	14	.29626	.31019	3.2238	.95511	46
15	.27983	.29147	3.4308	.96005	45	15	.29654	.31051	3.2205	.95502	45	15	.29654	.31051	3.2205	.95502	45
16	.28011	.29179	3.4271	.96000	44	16	.29682	.31083	3.2172	.95493	44	16	.29682	.31083	3.2172	.95493	44
17	.28038	.29210	3.4234	.95999	43	17	.29710	.31115	3.2139	.95485	43	17	.29710	.31115	3.2139	.95485	43
18	.28067	.29242	3.4197	.95991	42	18	.29737	.31147	3.2106	.95476	42	18	.29737	.31147	3.2106	.95476	42
19	.28095	.29274	3.4160	.95982	41	19	.29765	.31178	3.2073	.95467	41	19	.29765	.31178	3.2073	.95467	41
20	.28123	.29305	3.4124	.95974	40	20	.29793	.31210	3.2041	.95459	40	20	.29793	.31210	3.2041	.95459	40
21	.28150	.29337	3.4087	.95966	39	21	.29821	.31242	3.2008	.95450	39	21	.29821	.31242	3.2008	.95450	39
22	.28178	.29368	3.4050	.95958	38	22	.29849	.31274	3.1975	.95441	38	22	.29849	.31274	3.1975	.95441	38
23	.28206	.29400	3.4014	.95949	37	23	.29876	.31306	3.1943	.95433	37	23	.29876	.31306	3.1943	.95433	37
24	.28234	.29432	3.3977	.95941	36	24	.29904	.31338	3.1910	.95424	36	24	.29904	.31338	3.1910	.95424	36
25	.28262	.29463	3.3941	.95932	35	25	.29932	.31370	3.1878	.95415	35	25	.29932	.31370	3.1878	.95415	35
26	.28290	.29495	3.3904	.95924	34	26	.29960	.31402	3.1845	.95407	34	26	.29960	.31402	3.1845	.95407	34
27	.28318	.29526	3.3868	.95915	33	27	.29987	.31434	3.1813	.95398	33	27	.29987	.31434	3.1813	.95398	33
28	.28346	.29558	3.3832	.95906	32	28	.30015	.31466	3.1780	.95389	32	28	.30015	.31466	3.1780	.95389	32
29	.28374	.29590	3.3795	.95899	31	29	.30043	.31498	3.1748	.95380	31	29	.30043	.31498	3.1748	.95380	31
30	.28402	.29621	3.3759	.95882	30	30	.30071	.31530	3.1715	.95372	30	30	.30071	.31530	3.1715	.95372	30
31	.28429	.29653	3.3723	.95874	29	31	.30098	.31562	3.1684	.95363	29	31	.30098	.31562	3.1684	.95363	29
32	.28457	.29685	3.3687	.95865	28	32	.30126	.31594	3.1652	.95354	28	32	.30126	.31594	3.1652	.95354	28
33	.28485	.29716	3.3652	.95857	27	33	.30154	.31626	3.1620	.95345	27	33	.30154	.31626	3.1620	.95345	27
34	.28513	.29748	3.3616	.95849	26	34	.30182	.31658	3.1588	.95337	26	34	.30182	.31658	3.1588	.95337	26
35	.28541	.29780	3.3580	.95841	25	35	.30209	.31690	3.1556	.95328	25	35	.30209	.31690	3.1556	.95328	25
36	.28569	.29811	3.3544	.95832	24	36	.30237	.31722	3.1524	.95319	24	36	.30237	.31722	3.1524	.95319	24
37	.28597	.29843	3.3509	.95824	23	37	.30265	.31754	3.1492	.95310	23	37	.30265	.31754	3.1492	.95310	23
38	.28625	.29875	3.3473	.95816	22	38	.30292	.31786	3.1460	.95301	22	38	.30292	.31786	3.1460	.95301	22
39	.28652	.29906	3.3438	.95807	21	39	.30320	.31818	3.1429	.95293	21	39	.30320	.31818	3.1429	.95293	21
40	.28680	.29938	3.3402	.95799	20	40	.30348	.31850	3.1397	.95284	20	40	.30348	.31850	3.1397	.95284	20
41	.28708	.29970	3.3367	.95791	19	41	.30376	.31882	3.1366	.95275	19	41	.30376	.31882	3.1366	.95275	19
42	.28736	.30001	3.3332	.95782	18	42	.30403	.31914	3.1334	.95266	18	42	.30403	.31914	3.1334	.95266	18
43	.28764	.30033	3.3297	.95774	17	43	.30431	.31946	3.1303	.95257	17	43	.30431	.31946	3.1303	.95257	17
44	.28792	.30065	3.3261	.95766	16	44	.30459	.31978	3.1271	.95248	16	44	.30459	.31978	3.1271	.95248	16
45	.28820	.30097	3.3226	.95757	15	45	.30486	.32010	3.1240	.95240	15	45	.30486	.32010	3.1240	.95240	15
46	.28847	.30128	3.3191	.95749	14	46	.30514	.32042	3.1209	.95231	14	46	.30514	.32042	3.1209	.95231	14
47	.28875	.30160	3.3156	.95740	13	47	.30542	.32074	3.1178	.95222	13	47	.30542	.32074	3.1178	.95222	13
48	.28903	.30192	3.3122	.95732	12	48	.30570	.32106	3.1146	.95213	12	48	.30570	.32106	3.1146	.95213	12
49	.28931	.30224	3.3087	.95724	11	49	.30597	.32139	3.1115	.95204	11	49	.30597	.32139	3.1115	.95204	11
50	.28959	.30255	3.3052	.95715	10	50	.30625	.32171	3.1084	.95195	10	50	.30625	.32171	3.1084	.95195	10
51	.28987	.30287	3.3017	.95707	9	51	.30653	.32203	3.1053	.95186	9	51	.30653	.32203	3.1053	.95186	9
52	.29015	.30319	3.2983	.95698	8	52	.30680	.32235	3.1022	.95177	8	52	.30680	.32235	3.1022	.95177	8
53	.29042	.30351	3.2948	.95690	7	53	.30708	.32267	3.0991	.95168	7	53	.30708	.32267	3.0991	.95168	7
54	.29070	.30382	3.2914	.95681	6	54	.30736	.32299	3.0961	.95159	6	54	.30736	.32299	3.0961	.95159	6
55	.29098	.30414	3.2879	.95673	5	55	.30763	.32331	3.0930	.95150	5	55	.30763	.32331	3.0930	.95150	5
56	.29126	.30446	3.2845	.95664	4	56	.30791	.32363	3.0899	.95142	4	56	.30791	.32363	3.0899	.95142	4
57	.29154	.30478	3.2811	.95655	3	57	.30819	.32395	3.0868	.95133	3	57	.30819	.32395	3.0868	.95133	3
58	.29182	.30509	3.2777	.95647	2	58	.30846	.32428	3.0838	.95124	2	58	.30846	.32428	3.0838	.95124	2
59	.29209	.30541	3.2743	.95639	1	59	.30874	.32460	3.0807	.95115	1	59	.30874	.32460	3.0807	.95115	1
60	.29237	.30573	3.2709	.95630	0	60	.30902	.32492	3.0777	.95106	0	60	.30902	.32492	3.0777	.95106	0
'	Cos	Ctn	Tan	Sin	'	'	Cos	Ctn	Tan	Sin	'	'	Cos	Ctn	Tan	Sin	'
166° (286°)				(258°) 78°				107° (287°)				(252°) 72°					

Table 7 NATURAL TRIGONOMETRIC FUNCTIONS (continued)

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18° (198°)				(341°) 161°				19° (199°)				(340°) 160°			
'	Sin	Tan	Sec	Cos	'	Sin	Tan	Sec	Cos	'	Sin	Tan	Sec	Cos	'
0	.36002	.32492	3.0777	.98106	60	.32657	.34433	2.9042	.94552	60					
1	.36929	.32524	3.0746	.95097	59	.32584	.34465	2.9015	.94542	59					
2	.36957	.32556	3.0716	.95088	58	.32512	.34498	2.8987	.94533	58					
3	.36985	.32588	3.0686	.95079	57	.32439	.34530	2.8960	.94523	57					
4	.31012	.32621	3.0656	.95070	66	.32367	.34563	2.8933	.94514	56					
5	.31040	.32653	3.0625	.95061	55	.32294	.34596	2.8905	.94504	55					
6	.31068	.32685	3.0595	.95052	54	.32222	.34628	2.8878	.94495	54					
7	.31095	.32717	3.0565	.95043	53	.32149	.34661	2.8851	.94485	53					
8	.31123	.32749	3.0535	.95033	52	.32077	.34693	2.8824	.94476	52					
9	.31151	.32782	3.0505	.95024	51	.32004	.34726	2.8797	.94466	51					
10	.31178	.32814	3.0475	.95015	50	.31932	.34758	2.8770	.94457	50					
11	.31206	.32846	3.0445	.95006	49	.31859	.34791	2.8743	.94447	49					
12	.31233	.32878	3.0415	.94997	48	.31787	.34824	2.8716	.94438	48					
13	.31261	.32911	3.0385	.94988	47	.31714	.34856	2.8689	.94428	47					
14	.31289	.32943	3.0355	.94979	46	.31642	.34889	2.8662	.94418	46					
15	.31316	.32975	3.0325	.94970	45	.31569	.34922	2.8636	.94409	45					
16	.31344	.33007	3.0295	.94961	44	.31497	.34954	2.8609	.94400	44					
17	.31372	.33040	3.0265	.94952	43	.31424	.34987	2.8582	.94390	43					
18	.31399	.33072	3.0235	.94943	42	.31351	.35020	2.8555	.94380	42					
19	.31427	.33104	3.0205	.94933	41	.31279	.35052	2.8528	.94370	41					
20	.31454	.33136	3.0175	.94924	40	.31206	.35085	2.8502	.94361	40					
21	.31482	.33169	3.0145	.94915	39	.31134	.35118	2.8476	.94351	39					
22	.31510	.33201	3.0115	.94906	38	.31061	.35150	2.8449	.94342	38					
23	.31537	.33233	3.0085	.94897	37	.30989	.35183	2.8423	.94332	37					
24	.31565	.33265	3.0055	.94888	36	.30916	.35216	2.8397	.94322	36					
25	.31593	.33298	3.0025	.94878	35	.30844	.35248	2.8370	.94313	35					
26	.31620	.33330	3.0003	.94869	34	.30771	.35281	2.8344	.94303	34					
27	.31648	.33363	2.9974	.94860	33	.30699	.35314	2.8318	.94293	33					
28	.31675	.33395	2.9944	.94851	32	.30626	.35346	2.8292	.94284	32					
29	.31703	.33427	2.9914	.94842	31	.30554	.35379	2.8265	.94274	31					
30	.31730	.33460	2.9887	.94832	30	.30481	.35412	2.8239	.94264	30					
31	.31758	.33492	2.9858	.94823	29	.30409	.35445	2.8213	.94254	29					
32	.31786	.33524	2.9829	.94814	28	.30336	.35477	2.8187	.94245	28					
33	.31813	.33557	2.9800	.94805	27	.30264	.35510	2.8161	.94235	27					
34	.31841	.33589	2.9772	.94795	26	.30191	.35543	2.8135	.94225	26					
35	.31868	.33621	2.9743	.94786	25	.30119	.35576	2.8109	.94215	25					
36	.31896	.33654	2.9714	.94777	24	.30046	.35608	2.8083	.94206	24					
37	.31923	.33686	2.9686	.94768	23	.29974	.35641	2.8057	.94196	23					
38	.31951	.33718	2.9657	.94758	22	.29901	.35674	2.8031	.94186	22					
39	.31979	.33751	2.9629	.94749	21	.29829	.35707	2.8005	.94176	21					
40	.32006	.33783	2.9600	.94740	20	.29756	.35740	2.7980	.94167	20					
41	.32034	.33815	2.9572	.94730	19	.29684	.35772	2.7955	.94157	19					
42	.32061	.33848	2.9544	.94721	18	.29611	.35805	2.7929	.94147	18					
43	.32089	.33881	2.9515	.94712	17	.29539	.35838	2.7903	.94137	17					
44	.32116	.33913	2.9487	.94702	16	.29466	.35871	2.7878	.94127	16					
45	.32144	.33945	2.9459	.94693	15	.29394	.35904	2.7852	.94118	15					
46	.32171	.33978	2.9431	.94684	14	.29321	.35937	2.7827	.94108	14					
47	.32199	.34010	2.9403	.94674	13	.29249	.35969	2.7801	.94098	13					
48	.32227	.34043	2.9375	.94665	12	.29176	.36002	2.7776	.94088	12					
49	.32254	.34075	2.9347	.94656	11	.29104	.36035	2.7751	.94078	11					
50	.32282	.34108	2.9319	.94646	10	.29031	.36068	2.7725	.94068	10					
51	.32309	.34140	2.9291	.94637	9	.28959	.36101	2.7700	.94058	9					
52	.32337	.34173	2.9263	.94627	8	.28886	.36134	2.7675	.94049	8					
53	.32364	.34205	2.9235	.94618	7	.28814	.36167	2.7650	.94039	7					
54	.32392	.34238	2.9207	.94609	6	.28741	.36199	2.7625	.94029	6					
55	.32419	.34270	2.9180	.94599	5	.28669	.36232	2.7600	.94019	5					
56	.32447	.34303	2.9152	.94590	4	.28596	.36265	2.7575	.94009	4					
57	.32474	.34335	2.9125	.94580	3	.28524	.36298	2.7550	.93999	3					
58	.32502	.34368	2.9097	.94571	2	.28451	.36331	2.7525	.93989	2					
59	.32529	.34400	2.9070	.94561	1	.28379	.36364	2.7500	.93979	1					
60	.32557	.34433	2.9042	.94552	0	.28306	.36397	2.7475	.93969	0					
'	Cos	Ctn	Tan	Sin	'	Ctn	Ctn	Tan	Sin	'					

108° (288°)

(251°) 71°

109° (289°)

(250°) 70°

Table 7 NATURAL TRIGONOMETRIC FUNCTIONS (continued)

22° (202°)					(337°) 157°					23° (203°)					(336°) 156°				
	Sin	Tan	Ctn	Cos		Sin	Tan	Ctn	Cos		Sin	Tan	Ctn	Cos		Sin	Tan	Ctn	Cos
0	.37461	.40403	2.4751	.92718	60	.39073	.42447	2.3559	.92050	60	.39073	.42447	2.3559	.92050	60	.40674	.44523	2.2460	.91355
1	.37488	.40436	2.4730	.92707	59	.39100	.42482	2.3539	.92039	59	.39100	.42482	2.3539	.92039	59	.40587	.44384	2.2531	.91402
2	.37515	.40470	2.4709	.92697	58	.39127	.42516	2.3520	.92028	58	.39127	.42516	2.3520	.92028	58	.40504	.44244	2.2602	.91449
3	.37542	.40504	2.4688	.92686	57	.39153	.42551	2.3501	.92016	57	.39153	.42551	2.3501	.92016	57	.40421	.44105	2.2673	.91496
4	.37569	.40538	2.4668	.92675	56	.39180	.42585	2.3483	.92005	56	.39180	.42585	2.3483	.92005	56	.40338	.43966	2.2745	.91543
5	.37595	.40572	2.4648	.92664	55	.39207	.42619	2.3464	.91994	55	.39207	.42619	2.3464	.91994	55	.40255	.44001	2.2727	.91531
6	.37622	.40606	2.4627	.92653	54	.39234	.42654	2.3445	.91982	54	.39234	.42654	2.3445	.91982	54	.40172	.44036	2.2709	.91519
7	.37649	.40640	2.4606	.92642	53	.39260	.42688	2.3426	.91971	53	.39260	.42688	2.3426	.91971	53	.40089	.44071	2.2691	.91508
8	.37676	.40674	2.4586	.92631	52	.39287	.42722	2.3407	.91959	52	.39287	.42722	2.3407	.91959	52	.40006	.44105	2.2673	.91496
9	.37703	.40707	2.4566	.92620	51	.39314	.42757	2.3388	.91948	51	.39314	.42757	2.3388	.91948	51	.39923	.44140	2.2655	.91484
10	.37730	.40741	2.4545	.92609	50	.39341	.42791	2.3369	.91936	50	.39341	.42791	2.3369	.91936	50	.39840	.44175	2.2637	.91472
11	.37757	.40775	2.4525	.92598	49	.39367	.42826	2.3351	.91925	49	.39367	.42826	2.3351	.91925	49	.39758	.44210	2.2620	.91461
12	.37784	.40809	2.4504	.92587	48	.39394	.42860	2.3332	.91914	48	.39394	.42860	2.3332	.91914	48	.39676	.44244	2.2602	.91449
13	.37811	.40843	2.4484	.92576	47	.39421	.42894	2.3313	.91902	47	.39421	.42894	2.3313	.91902	47	.39594	.44278	2.2584	.91437
14	.37838	.40877	2.4464	.92565	46	.39448	.42929	2.3294	.91891	46	.39448	.42929	2.3294	.91891	46	.39512	.44312	2.2566	.91425
15	.37865	.40911	2.4443	.92554	45	.39474	.42963	2.3275	.91879	45	.39474	.42963	2.3275	.91879	45	.39430	.44346	2.2549	.91414
16	.37892	.40945	2.4423	.92543	44	.39501	.42998	2.3257	.91868	44	.39501	.42998	2.3257	.91868	44	.39348	.44380	2.2531	.91402
17	.37919	.40979	2.4403	.92532	43	.39529	.43032	2.3238	.91856	43	.39529	.43032	2.3238	.91856	43	.39266	.44414	2.2513	.91390
18	.37946	.41013	2.4383	.92521	42	.39555	.43067	2.3220	.91845	42	.39555	.43067	2.3220	.91845	42	.39184	.44448	2.2495	.91378
19	.37973	.41047	2.4362	.92510	41	.39581	.43101	2.3201	.91833	41	.39581	.43101	2.3201	.91833	41	.39102	.44482	2.2478	.91366
20	.37999	.41081	2.4342	.92499	40	.39608	.43136	2.3183	.91822	40	.39608	.43136	2.3183	.91822	40	.39020	.44516	2.2460	.91355
21	.38026	.41115	2.4322	.92488	39	.39635	.43170	2.3164	.91810	39	.39635	.43170	2.3164	.91810	39	.38938	.44550	2.2442	.91343
22	.38053	.41149	2.4302	.92477	38	.39661	.43205	2.3146	.91799	38	.39661	.43205	2.3146	.91799	38	.38856	.44584	2.2424	.91331
23	.38080	.41183	2.4282	.92466	37	.39688	.43239	2.3127	.91787	37	.39688	.43239	2.3127	.91787	37	.38774	.44618	2.2406	.91319
24	.38107	.41217	2.4262	.92455	36	.39715	.43274	2.3109	.91775	36	.39715	.43274	2.3109	.91775	36	.38692	.44652	2.2388	.91307
25	.38134	.41251	2.4242	.92444	35	.39741	.43308	2.3090	.91764	35	.39741	.43308	2.3090	.91764	35	.38610	.44686	2.2370	.91295
26	.38161	.41285	2.4222	.92432	34	.39768	.43343	2.3072	.91752	34	.39768	.43343	2.3072	.91752	34	.38528	.44720	2.2352	.91283
27	.38188	.41319	2.4202	.92421	33	.39795	.43378	2.3053	.91741	33	.39795	.43378	2.3053	.91741	33	.38446	.44754	2.2334	.91271
28	.38215	.41353	2.4182	.92410	32	.39822	.43412	2.3035	.91729	32	.39822	.43412	2.3035	.91729	32	.38364	.44788	2.2316	.91259
29	.38241	.41387	2.4162	.92399	31	.39848	.43447	2.3017	.91718	31	.39848	.43447	2.3017	.91718	31	.38282	.44822	2.2298	.91247
30	.38268	.41421	2.4142	.92388	30	.39875	.43481	2.2998	.91706	30	.39875	.43481	2.2998	.91706	30	.38200	.44856	2.2280	.91235
31	.38295	.41455	2.4122	.92377	29	.39902	.43516	2.2980	.91694	29	.39902	.43516	2.2980	.91694	29	.38118	.44890	2.2262	.91223
32	.38322	.41489	2.4102	.92366	28	.39928	.43550	2.2962	.91683	28	.39928	.43550	2.2962	.91683	28	.38036	.44924	2.2244	.91211
33	.38349	.41524	2.4083	.92355	27	.39955	.43585	2.2944	.91671	27	.39955	.43585	2.2944	.91671	27	.37954	.44958	2.2226	.91199
34	.38376	.41558	2.4063	.92343	26	.39982	.43620	2.2925	.91660	26	.39982	.43620	2.2925	.91660	26	.37872	.44992	2.2208	.91187
35	.38403	.41592	2.4043	.92332	25	.40008	.43654	2.2907	.91648	25	.40008	.43654	2.2907	.91648	25	.37790	.45026	2.2190	.91175
36	.38430	.41626	2.4023	.92321	24	.40035	.43689	2.2889	.91636	24	.40035	.43689	2.2889	.91636	24	.37708	.45060	2.2172	.91163
37	.38456	.41660	2.4004	.92310	23	.40062	.43724	2.2871	.91625	23	.40062	.43724	2.2871	.91625	23	.37626	.45094	2.2154	.91151
38	.38483	.41694	2.3984	.92299	22	.40088	.43758	2.2853	.91613	22	.40088	.43758	2.2853	.91613	22	.37544	.45128	2.2136	.91139
39	.38510	.41728	2.3964	.92287	21	.40115	.43793	2.2835	.91601	21	.40115	.43793	2.2835	.91601	21	.37462	.45162	2.2118	.91127
40	.38537	.41763	2.3945	.92276	20	.40141	.43828	2.2817	.91589	20	.40141	.43828	2.2817	.91589	20	.37380	.45196	2.2100	.91115
41	.38564	.41797	2.3925	.92265	19	.40168	.43862	2.2799	.91578	19	.40168	.43862	2.2799	.91578	19	.37298	.45230	2.2082	.91103
42	.38591	.41831	2.3906	.92254	18	.40195	.43897	2.2781	.91566	18	.40195	.43897	2.2781	.91566	18	.37216	.45264	2.2064	.91091
43	.38618	.41865	2.3886	.92243	17	.40221	.43932	2.2763	.91555	17	.40221	.43932	2.2763	.91555	17	.37134	.45298	2.2046	.91079
44	.38644	.41899	2.3867	.92231	16	.40248	.43966	2.2745	.91543	16	.40248	.43966	2.2745	.91543	16	.37052	.45332	2.2028	.91067
45	.38671	.41933	2.3847	.92220	15	.40275	.44001	2.2727	.91531	15	.40275	.44001	2.2727	.91531	15	.36970	.45366	2.2010	.91055
46	.38698	.41968	2.3828	.92209	14	.40301	.44036	2.2709	.91519	14	.40301	.44036	2.2709	.91519	14	.36888	.45400	2.1992	.91043
47	.38725	.42002	2.3808	.92198	13	.40328	.44071	2.2691	.91508	13	.40328	.44071	2.2691	.91508	13	.36806	.45434	2.1974	.91031
48	.38752	.42036	2.3789	.92186	12	.40355	.44105	2.2673	.91496	12	.40355	.44105	2.2673	.91496	12	.36724	.45468	2.1956	.91019
49	.38778	.42070	2.3770	.92175	11	.40381	.44140	2.2655	.91484	11	.40381	.44140	2.2655	.91484	11	.36642	.45502	2.1938	.91007
50	.38805	.42105	2.3750	.92164	10	.40408	.44175	2.2637	.91472	10	.40408	.44175	2.2637	.91472	10	.36560	.45536	2.1920	.90995
51	.38832	.42139	2.3731	.92152	9	.40434	.44210	2.2620	.91461	9	.40434	.44210	2.2620	.91461	9	.36478	.45570	2.1902	.90983
52	.38859	.42173	2.3712	.92141	8	.40461	.44244	2.2602	.91449	8	.40461	.44244	2.2602	.91449	8	.36396	.45604	2.1884	.90971
53	.38886	.42207	2.3693	.92130	7	.40488	.44278	2.2584	.91437	7	.40488	.44278	2.2584	.91437	7	.36314	.45638	2.1866	.90959
54	.38912	.42242	2.3673	.92119	6	.40514	.44312	2.2566	.91425	6	.40514	.44312	2.2566	.91425	6	.36232	.45672	2.1848	.90947
55	.38939	.42276	2.3654	.92107	5	.40541	.44346	2.2549	.91414	5	.40541	.44346	2.2549	.91414	5	.36150	.45706	2.1830	.90935
56	.38966	.42310	2.3635	.92096	4	.40567	.44380	2.2531	.91402	4	.40567	.44380	2.2531	.91402	4	.36068	.45740	2.1812	.90923
57	.38993	.42345	2.3616	.92085	3	.40594	.44414	2.2513	.91390	3	.40594	.44414	2.2513	.91390	3	.35986	.45774	2.1794	.90911
58	.39020	.42379	2.3597	.92073	2	.40621	.44448	2.2495	.91378	2	.40621	.44448	2.2495	.91378	2	.35904	.45808	2.1776	.90899
59	.39046	.42413	2.3578	.92062	1	.40647	.44482	2.2478	.91366	1	.40647	.44482	2.2478	.91366	1	.35822	.45842	2.1758	.90887
60	.39073	.42447	2.3559	.92050	0	.40674	.44523	2.2460	.91355	0	.40674	.44523	2.2460	.91355	0	.35740	.45880	2.1740	.90875

Table 7 NATURAL TRIGONOMETRIC FUNCTIONS (continued)

24° (204°)					(335°) 155°					25° (205°)					(334°) 154°									
°	Sin	Tan	Ctn	Cos	°	Sin	Tan	Ctn	Cos	°	Sin	Tan	Ctn	Cos	°	Sin	Tan	Ctn	Cos	°	Sin	Tan	Ctn	Cos
0	.40674	.44523	2.2460	.91355	60	.42262	.46631	2.1445	.90631	60	.42262	.46631	2.1445	.90631	60	.42262	.46631	2.1445	.90631	60	.42262	.46631	2.1445	.90631
1	.40700	.44558	2.2443	.91343	59	.42288	.46666	2.1429	.90618	59	.42288	.46666	2.1429	.90618	59	.42288	.46666	2.1429	.90618	59	.42288	.46666	2.1429	.90618
2	.40727	.44593	2.2425	.91331	58	.42315	.46702	2.1413	.90606	58	.42315	.46702	2.1413	.90606	58	.42315	.46702	2.1413	.90606	58	.42315	.46702	2.1413	.90606
3	.40753	.44627	2.2408	.91319	57	.42341	.46737	2.1396	.90594	57	.42341	.46737	2.1396	.90594	57	.42341	.46737	2.1396	.90594	57	.42341	.46737	2.1396	.90594
4	.40780	.44662	2.2390	.91307	56	.42367	.46772	2.1380	.90582	56	.42367	.46772	2.1380	.90582	56	.42367	.46772	2.1380	.90582	56	.42367	.46772	2.1380	.90582
5	.40806	.44697	2.2373	.91295	55	.42394	.46808	2.1364	.90569	55	.42394	.46808	2.1364	.90569	55	.42394	.46808	2.1364	.90569	55	.42394	.46808	2.1364	.90569
6	.40833	.44732	2.2355	.91283	54	.42420	.46843	2.1348	.90557	54	.42420	.46843	2.1348	.90557	54	.42420	.46843	2.1348	.90557	54	.42420	.46843	2.1348	.90557
7	.40860	.44767	2.2338	.91272	53	.42446	.46879	2.1332	.90545	53	.42446	.46879	2.1332	.90545	53	.42446	.46879	2.1332	.90545	53	.42446	.46879	2.1332	.90545
8	.40886	.44802	2.2320	.91260	52	.42473	.46914	2.1315	.90532	52	.42473	.46914	2.1315	.90532	52	.42473	.46914	2.1315	.90532	52	.42473	.46914	2.1315	.90532
9	.40913	.44837	2.2303	.91248	51	.42499	.46950	2.1299	.90520	51	.42499	.46950	2.1299	.90520	51	.42499	.46950	2.1299	.90520	51	.42499	.46950	2.1299	.90520
10	.40939	.44872	2.2286	.91236	50	.42525	.46985	2.1283	.90507	50	.42525	.46985	2.1283	.90507	50	.42525	.46985	2.1283	.90507	50	.42525	.46985	2.1283	.90507
11	.40966	.44907	2.2268	.91224	49	.42552	.47021	2.1267	.90495	49	.42552	.47021	2.1267	.90495	49	.42552	.47021	2.1267	.90495	49	.42552	.47021	2.1267	.90495
12	.40992	.44942	2.2251	.91212	48	.42578	.47056	2.1251	.90483	48	.42578	.47056	2.1251	.90483	48	.42578	.47056	2.1251	.90483	48	.42578	.47056	2.1251	.90483
13	.41019	.44977	2.2234	.91200	47	.42604	.47092	2.1235	.90470	47	.42604	.47092	2.1235	.90470	47	.42604	.47092	2.1235	.90470	47	.42604	.47092	2.1235	.90470
14	.41045	.45012	2.2216	.91188	46	.42631	.47128	2.1219	.90458	46	.42631	.47128	2.1219	.90458	46	.42631	.47128	2.1219	.90458	46	.42631	.47128	2.1219	.90458
15	.41072	.45047	2.2199	.91176	45	.42657	.47163	2.1203	.90446	45	.42657	.47163	2.1203	.90446	45	.42657	.47163	2.1203	.90446	45	.42657	.47163	2.1203	.90446
16	.41098	.45082	2.2182	.91164	44	.42683	.47199	2.1187	.90433	44	.42683	.47199	2.1187	.90433	44	.42683	.47199	2.1187	.90433	44	.42683	.47199	2.1187	.90433
17	.41126	.45117	2.2165	.91152	43	.42709	.47234	2.1171	.90421	43	.42709	.47234	2.1171	.90421	43	.42709	.47234	2.1171	.90421	43	.42709	.47234	2.1171	.90421
18	.41151	.45152	2.2148	.91140	42	.42736	.47270	2.1155	.90408	42	.42736	.47270	2.1155	.90408	42	.42736	.47270	2.1155	.90408	42	.42736	.47270	2.1155	.90408
19	.41178	.45187	2.2130	.91128	41	.42762	.47305	2.1139	.90396	41	.42762	.47305	2.1139	.90396	41	.42762	.47305	2.1139	.90396	41	.42762	.47305	2.1139	.90396
20	.41204	.45222	2.2113	.91116	40	.42788	.47341	2.1123	.90383	40	.42788	.47341	2.1123	.90383	40	.42788	.47341	2.1123	.90383	40	.42788	.47341	2.1123	.90383
21	.41231	.45257	2.2096	.91104	39	.42815	.47377	2.1107	.90371	39	.42815	.47377	2.1107	.90371	39	.42815	.47377	2.1107	.90371	39	.42815	.47377	2.1107	.90371
22	.41257	.45292	2.2079	.91092	38	.42841	.47412	2.1092	.90358	38	.42841	.47412	2.1092	.90358	38	.42841	.47412	2.1092	.90358	38	.42841	.47412	2.1092	.90358
23	.41284	.45327	2.2062	.91080	37	.42867	.47448	2.1076	.90346	37	.42867	.47448	2.1076	.90346	37	.42867	.47448	2.1076	.90346	37	.42867	.47448	2.1076	.90346
24	.41310	.45362	2.2045	.91068	36	.42894	.47483	2.1060	.90334	36	.42894	.47483	2.1060	.90334	36	.42894	.47483	2.1060	.90334	36	.42894	.47483	2.1060	.90334
25	.41337	.45397	2.2028	.91056	35	.42920	.47519	2.1044	.90321	35	.42920	.47519	2.1044	.90321	35	.42920	.47519	2.1044	.90321	35	.42920	.47519	2.1044	.90321
26	.41363	.45432	2.2011	.91044	34	.42946	.47555	2.1028	.90309	34	.42946	.47555	2.1028	.90309	34	.42946	.47555	2.1028	.90309	34	.42946	.47555	2.1028	.90309
27	.41390	.45467	2.1994	.91032	33	.42972	.47590	2.1013	.90296	33	.42972	.47590	2.1013	.90296	33	.42972	.47590	2.1013	.90296	33	.42972	.47590	2.1013	.90296
28	.41416	.45502	2.1977	.91020	32	.43000	.47626	2.0997	.90284	32	.43000	.47626	2.0997	.90284	32	.43000	.47626	2.0997	.90284	32	.43000	.47626	2.0997	.90284
29	.41443	.45538	2.1960	.91008	31	.43025	.47662	2.0981	.90271	31	.43025	.47662	2.0981	.90271	31	.43025	.47662	2.0981	.90271	31	.43025	.47662	2.0981	.90271
30	.41469	.45573	2.1943	.90996	30	.43051	.47698	2.0965	.90259	30	.43051	.47698	2.0965	.90259	30	.43051	.47698	2.0965	.90259	30	.43051	.47698	2.0965	.90259
31	.41495	.45608	2.1926	.90984	29	.43077	.47733	2.0950	.90246	29	.43077	.47733	2.0950	.90246	29	.43077	.47733	2.0950	.90246	29	.43077	.47733	2.0950	.90246
32	.41522	.45643	2.1909	.90972	28	.43104	.47769	2.0934	.90233	28	.43104	.47769	2.0934	.90233	28	.43104	.47769	2.0934	.90233	28	.43104	.47769	2.0934	.90233
33	.41549	.45678	2.1892	.90960	27	.43130	.47805	2.0918	.90221	27	.43130	.47805	2.0918	.90221	27	.43130	.47805	2.0918	.90221	27	.43130	.47805	2.0918	.90221
34	.41575	.45713	2.1876	.90948	26	.43156	.47840	2.0903	.90208	26	.43156	.47840	2.0903	.90208	26	.43156	.47840	2.0903	.90208	26	.43156	.47840	2.0903	.90208
35	.41602	.45748	2.1859	.90936	25	.43182	.47876	2.0887	.90196	25	.43182	.47876	2.0887	.90196	25	.43182	.47876	2.0887	.90196	25	.43182	.47876	2.0887	.90196
36	.41628	.45784	2.1842	.90924	24	.43209	.47912	2.0872	.90183	24	.43209	.47912	2.0872	.90183	24	.43209	.47912	2.0872	.90183	24	.43209	.47912	2.0872	.90183
37	.41655	.45819	2.1825	.90911	23	.43235	.47948	2.0856	.90171	23	.43235	.47948	2.0856	.90171	23	.43235	.47948	2.0856	.90171	23	.43235	.47948	2.0856	.90171
38	.41681	.45854	2.1808	.90898	22	.43261	.47984	2.0840	.90158	22	.43261	.47984	2.0840	.90158	22	.43261	.47984	2.0840	.90158	22	.43261	.47984	2.0840	.90158
39	.41707	.45889	2.1792	.90887	21	.43287	.48019	2.0825	.90146	21	.43287	.48019	2.0825	.90146	21	.43287	.48019	2.0825	.90146	21	.43287	.48019	2.0825	.90146
40	.41734	.45924	2.1775	.90875	20	.43313	.48055	2.0809	.90133	20	.43313	.48055	2.0809	.90133	20	.43313	.48055	2.0809	.90133	20	.43313	.48055	2.0809	.90133
41	.41760	.45959	2.1758	.90863	19	.43340	.48091	2.0794	.90120	19	.43340	.48091	2.0794	.90120	19	.43340	.48091	2.0794	.90120	19	.43340	.48091	2.0794	.90120
42	.41787	.45995	2.1742	.90851	18	.43366	.48127	2.0778	.90108	18	.43366	.48127	2.0778	.90108	18	.43366	.48127	2.0778	.90108	18	.43366	.48127	2.0778	.90108
43	.41813	.46030	2.1725	.90839	17	.43392	.48163	2.0763	.90095	17	.43392	.48163	2.0763	.90095	17	.43392	.48163	2.0763	.90095	17	.43392	.48163	2.0763	.90095
44	.41840	.46065	2.1708	.90826	16	.43418	.48198	2.0748	.90082	16	.43418	.48198	2.0748	.90082	16	.43418	.48198	2.0748	.90082	16	.43418	.48198	2.0748	.90082
45	.41866	.46101	2.1692	.90814	15	.43445	.48234	2.0732	.90070	15	.43445	.48234	2.0732	.90070	15	.43445	.48234	2.0732	.90070	15	.43445	.48234	2.0732	.90070
46	.41892	.46136	2.1675	.90802	14	.43471	.48270	2.0717	.90057	14	.43471	.48270	2.0717	.90057	14	.43471	.48270	2.0717	.90057	14	.43471	.48270	2.0717	.90057
47	.41919	.46171	2.1659	.90790	13	.43497	.48306	2.0701	.90045	13	.43497	.48306	2.0701	.90045	13	.43497	.48306	2.0701	.90045	13	.43497	.48306	2.0701	.90045
48	.41945	.46206	2.1642	.90778	12	.43523	.48342	2.0686	.90032	12	.43523	.48342	2.0686	.90032	12	.43523	.48342	2.0686	.90032	12	.43523	.48342	2.0686	.

Table 7 NATURAL TRIGONOMETRIC FUNCTIONS (continued)

26° (206°)				(333°) 153°	27° (207°)				(332°) 152°	
	Sin	Tan	Ctn	Cos		Sin	Tan	Ctn	Cos	
0	.43837	.49773	2.0503	.89879	60	.45399	.50953	1.9626	.80101	60
1	.43863	.49809	2.0488	.89867	59	.45425	.50989	1.9612	.80087	59
2	.43889	.49845	2.0473	.89854	58	.45451	.51025	1.9598	.80074	58
3	.43916	.49881	2.0458	.89841	57	.45477	.51061	1.9584	.80061	57
4	.43942	.49917	2.0443	.89828	56	.45503	.51099	1.9570	.80048	56
5	.43968	.49953	2.0428	.89816	55	.45529	.51136	1.9556	.80035	55
6	.43994	.49989	2.0413	.89803	54	.45554	.51173	1.9542	.80021	54
7	.44020	.49926	2.0398	.89790	53	.45580	.51209	1.9528	.80008	53
8	.44046	.49962	2.0383	.89777	52	.45606	.51246	1.9514	.80095	52
9	.44072	.49998	2.0368	.89764	51	.45632	.51283	1.9500	.80081	51
10	.44098	.49134	2.0353	.89752	50	.45658	.51319	1.9486	.80068	50
11	.44124	.49170	2.0338	.89739	49	.45684	.51355	1.9472	.80055	49
12	.44151	.49206	2.0323	.89726	48	.45710	.51391	1.9458	.80042	48
13	.44177	.49242	2.0308	.89713	47	.45736	.51427	1.9444	.80028	47
14	.44203	.49278	2.0293	.89700	46	.45762	.51463	1.9430	.80015	46
15	.44229	.49315	2.0278	.89687	45	.45787	.51503	1.9416	.80002	45
16	.44255	.49351	2.0263	.89674	44	.45813	.51540	1.9402	.80088	44
17	.44281	.49387	2.0248	.89662	43	.45839	.51577	1.9388	.80075	43
18	.44307	.49423	2.0233	.89649	42	.45865	.51614	1.9375	.80062	42
19	.44333	.49459	2.0219	.89636	41	.45891	.51651	1.9361	.80048	41
20	.44359	.49495	2.0204	.89623	40	.45917	.51688	1.9347	.80035	40
21	.44385	.49532	2.0189	.89610	39	.45942	.51724	1.9333	.80022	39
22	.44411	.49568	2.0174	.89597	38	.45968	.51761	1.9319	.80008	38
23	.44437	.49604	2.0160	.89584	37	.45994	.51798	1.9306	.80095	37
24	.44464	.49640	2.0145	.89571	36	.46020	.51835	1.9292	.80082	36
25	.44490	.49677	2.0130	.89558	35	.46046	.51872	1.9278	.80068	35
26	.44516	.49713	2.0115	.89545	34	.46072	.51909	1.9265	.80055	34
27	.44542	.49749	2.0101	.89532	33	.46097	.51946	1.9251	.80041	33
28	.44568	.49786	2.0086	.89519	32	.46123	.51983	1.9237	.80028	32
29	.44594	.49822	2.0072	.89506	31	.46149	.52020	1.9223	.80015	31
30	.44620	.49858	2.0057	.89493	30	.46175	.52057	1.9210	.80001	30
31	.44646	.49894	2.0042	.89480	29	.46201	.52094	1.9196	.80088	29
32	.44672	.49931	2.0028	.89467	28	.46226	.52131	1.9183	.80074	28
33	.44698	.49967	2.0013	.89454	27	.46252	.52168	1.9169	.80061	27
34	.44724	.50004	1.9999	.89441	26	.46278	.52205	1.9155	.80047	26
35	.44750	.50040	1.9984	.89428	25	.46304	.52242	1.9142	.80034	25
36	.44776	.50076	1.9970	.89415	24	.46330	.52279	1.9128	.80020	24
37	.44802	.50113	1.9955	.89402	23	.46355	.52316	1.9115	.80007	23
38	.44828	.50149	1.9941	.89389	22	.46381	.52353	1.9101	.80093	22
39	.44854	.50185	1.9926	.89376	21	.46407	.52390	1.9088	.80080	21
40	.44880	.50222	1.9912	.89363	20	.46433	.52427	1.9074	.80066	20
41	.44906	.50258	1.9897	.89350	19	.46458	.52464	1.9061	.80053	19
42	.44932	.50295	1.9883	.89337	18	.46484	.52501	1.9047	.80039	18
43	.44958	.50331	1.9868	.89324	17	.46510	.52538	1.9034	.80026	17
44	.44984	.50368	1.9854	.89311	16	.46536	.52575	1.9020	.80012	16
45	.45010	.50404	1.9840	.89298	15	.46561	.52613	1.9007	.80099	15
46	.45036	.50441	1.9825	.89285	14	.46587	.52650	1.8993	.80085	14
47	.45062	.50477	1.9811	.89272	13	.46613	.52687	1.8980	.80072	13
48	.45088	.50514	1.9797	.89259	12	.46639	.52724	1.8967	.80058	12
49	.45114	.50550	1.9782	.89245	11	.46664	.52761	1.8953	.80045	11
50	.45140	.50587	1.9768	.89232	10	.46690	.52798	1.8940	.80031	10
51	.45166	.50623	1.9754	.89219	9	.46716	.52836	1.8927	.80017	9
52	.45192	.50660	1.9740	.89206	8	.46742	.52873	1.8913	.80004	8
53	.45218	.50696	1.9725	.89193	7	.46767	.52910	1.8900	.80090	7
54	.45243	.50733	1.9711	.89180	6	.46793	.52947	1.8887	.80077	6
55	.45269	.50769	1.9697	.89167	5	.46819	.52985	1.8873	.80063	5
56	.45295	.50806	1.9683	.89153	4	.46844	.53022	1.8860	.80049	4
57	.45321	.50843	1.9669	.89140	3	.46870	.53059	1.8847	.80035	3
58	.45347	.50879	1.9654	.89127	2	.46896	.53096	1.8834	.80022	2
59	.45373	.50916	1.9640	.89114	1	.46921	.53134	1.8820	.80008	1
60	.45399	.50953	1.9626	.89101	0	.46947	.53171	1.8807	.80095	0
	Cos	Ctn	Tan	Sin		Cos	Ctn	Tan	Sin	

116° (296°)

(243°) 63°

117° (297°)

(242°) 62°

Table 7 NATURAL TRIGONOMETRIC FUNCTIONS (continued)

29° (208°)				(331°) 151°				29° (209°)				(330°) 150°			
	Sin	Tan	Ctn	Cos		Sin	Tan	Ctn	Cos		Sin	Tan	Ctn	Cos	
0	.46947	.53171	1.8807	.88295	60	.48481	.55431	1.8040	.87462	60	.48481	.55431	1.8040	.87462	
1	.46973	.53203	1.8794	.88281	59	.48506	.55469	1.8028	.87448	59	.48506	.55469	1.8028	.87448	
2	.47024	.53283	1.8758	.88254	58	.48532	.55507	1.8016	.87434	58	.48532	.55507	1.8016	.87434	
3	.47050	.53320	1.8755	.88240	57	.48557	.55545	1.8003	.87420	57	.48557	.55545	1.8003	.87420	
4	.47076	.53358	1.8741	.88226	56	.48583	.55583	1.7991	.87406	56	.48583	.55583	1.7991	.87406	
5	.47101	.53395	1.8728	.88213	55	.48608	.55621	1.7979	.87391	55	.48608	.55621	1.7979	.87391	
6	.47127	.53432	1.8715	.88199	54	.48634	.55659	1.7966	.87377	54	.48634	.55659	1.7966	.87377	
7	.47153	.53470	1.8702	.88185	53	.48659	.55697	1.7954	.87363	53	.48659	.55697	1.7954	.87363	
8	.47178	.53507	1.8689	.88172	52	.48684	.55736	1.7942	.87349	52	.48684	.55736	1.7942	.87349	
9	.47204	.53545	1.8676	.88158	51	.48710	.55774	1.7930	.87335	51	.48710	.55774	1.7930	.87335	
10	.47229	.53582	1.8663	.88144	50	.48735	.55812	1.7917	.87321	50	.48735	.55812	1.7917	.87321	
11	.47255	.53620	1.8650	.88130	49	.48761	.55850	1.7905	.87306	49	.48761	.55850	1.7905	.87306	
12	.47281	.53657	1.8637	.88117	48	.48786	.55888	1.7893	.87292	48	.48786	.55888	1.7893	.87292	
13	.47306	.53694	1.8624	.88103	47	.48811	.55926	1.7881	.87278	47	.48811	.55926	1.7881	.87278	
14	.47332	.53732	1.8611	.88089	46	.48837	.55964	1.7868	.87264	46	.48837	.55964	1.7868	.87264	
15	.47358	.53769	1.8598	.88075	45	.48862	.56003	1.7856	.87250	45	.48862	.56003	1.7856	.87250	
16	.47383	.53807	1.8585	.88062	44	.48888	.56041	1.7844	.87235	44	.48888	.56041	1.7844	.87235	
17	.47409	.53844	1.8572	.88048	43	.48913	.56079	1.7832	.87221	43	.48913	.56079	1.7832	.87221	
18	.47434	.53882	1.8559	.88034	42	.48938	.56117	1.7820	.87207	42	.48938	.56117	1.7820	.87207	
19	.47460	.53920	1.8546	.88020	41	.48964	.56156	1.7808	.87193	41	.48964	.56156	1.7808	.87193	
20	.47486	.53957	1.8533	.88006	40	.48989	.56194	1.7796	.87178	40	.48989	.56194	1.7796	.87178	
21	.47511	.53995	1.8520	.87993	39	.49014	.56232	1.7783	.87164	39	.49014	.56232	1.7783	.87164	
22	.47537	.54032	1.8507	.87979	38	.49040	.56270	1.7771	.87150	38	.49040	.56270	1.7771	.87150	
23	.47562	.54070	1.8495	.87965	37	.49065	.56309	1.7759	.87136	37	.49065	.56309	1.7759	.87136	
24	.47588	.54107	1.8482	.87951	36	.49090	.56347	1.7747	.87121	36	.49090	.56347	1.7747	.87121	
25	.47614	.54145	1.8469	.87937	35	.49116	.56385	1.7735	.87107	35	.49116	.56385	1.7735	.87107	
26	.47639	.54183	1.8456	.87923	34	.49141	.56424	1.7723	.87093	34	.49141	.56424	1.7723	.87093	
27	.47665	.54220	1.8443	.87909	33	.49166	.56462	1.7711	.87079	33	.49166	.56462	1.7711	.87079	
28	.47690	.54258	1.8430	.87895	32	.49192	.56500	1.7699	.87064	32	.49192	.56500	1.7699	.87064	
29	.47716	.54296	1.8418	.87882	31	.49217	.56539	1.7687	.87050	31	.49217	.56539	1.7687	.87050	
30	.47741	.54333	1.8405	.87868	30	.49242	.56577	1.7675	.87036	30	.49242	.56577	1.7675	.87036	
31	.47767	.54371	1.8392	.87854	29	.49268	.56616	1.7663	.87021	29	.49268	.56616	1.7663	.87021	
32	.47793	.54409	1.8379	.87840	28	.49293	.56654	1.7651	.87007	28	.49293	.56654	1.7651	.87007	
33	.47818	.54446	1.8367	.87826	27	.49318	.56693	1.7639	.86993	27	.49318	.56693	1.7639	.86993	
34	.47844	.54484	1.8354	.87812	26	.49344	.56731	1.7627	.86978	26	.49344	.56731	1.7627	.86978	
35	.47869	.54522	1.8341	.87798	25	.49369	.56769	1.7615	.86964	25	.49369	.56769	1.7615	.86964	
36	.47895	.54560	1.8329	.87784	24	.49394	.56808	1.7603	.86949	24	.49394	.56808	1.7603	.86949	
37	.47920	.54597	1.8316	.87770	23	.49419	.56846	1.7591	.86935	23	.49419	.56846	1.7591	.86935	
38	.47946	.54635	1.8303	.87756	22	.49445	.56885	1.7579	.86921	22	.49445	.56885	1.7579	.86921	
39	.47971	.54673	1.8291	.87743	21	.49470	.56923	1.7567	.86906	21	.49470	.56923	1.7567	.86906	
40	.47997	.54711	1.8278	.87729	20	.49495	.56962	1.7556	.86892	20	.49495	.56962	1.7556	.86892	
41	.48022	.54748	1.8265	.87715	19	.49521	.57000	1.7544	.86878	19	.49521	.57000	1.7544	.86878	
42	.48048	.54786	1.8253	.87701	18	.49546	.57039	1.7532	.86863	18	.49546	.57039	1.7532	.86863	
43	.48073	.54824	1.8240	.87687	17	.49571	.57078	1.7520	.86849	17	.49571	.57078	1.7520	.86849	
44	.48099	.54862	1.8228	.87673	16	.49596	.57116	1.7508	.86834	16	.49596	.57116	1.7508	.86834	
45	.48124	.54900	1.8215	.87659	15	.49622	.57155	1.7496	.86820	15	.49622	.57155	1.7496	.86820	
46	.48150	.54938	1.8202	.87646	14	.49647	.57193	1.7486	.86805	14	.49647	.57193	1.7486	.86805	
47	.48175	.54976	1.8190	.87631	13	.49672	.57232	1.7473	.86791	13	.49672	.57232	1.7473	.86791	
48	.48201	.55013	1.8177	.87617	12	.49697	.57271	1.7461	.86777	12	.49697	.57271	1.7461	.86777	
49	.48226	.55051	1.8165	.87603	11	.49723	.57309	1.7449	.86762	11	.49723	.57309	1.7449	.86762	
50	.48252	.55089	1.8152	.87589	10	.49748	.57348	1.7437	.86748	10	.49748	.57348	1.7437	.86748	
51	.48277	.55127	1.8140	.87575	9	.49773	.57386	1.7426	.86733	9	.49773	.57386	1.7426	.86733	
52	.48303	.55165	1.8127	.87561	8	.49798	.57425	1.7414	.86719	8	.49798	.57425	1.7414	.86719	
53	.48328	.55203	1.8115	.87546	7	.49824	.57464	1.7402	.86704	7	.49824	.57464	1.7402	.86704	
54	.48354	.55241	1.8103	.87532	6	.49849	.57503	1.7391	.86690	6	.49849	.57503	1.7391	.86690	
55	.48379	.55279	1.8090	.87518	5	.49874	.57541	1.7379	.86675	5	.49874	.57541	1.7379	.86675	
56	.48405	.55317	1.8078	.87504	4	.49899	.57580	1.7367	.86661	4	.49899	.57580	1.7367	.86661	
57	.48430	.55355	1.8065	.87490	3	.49924	.57619	1.7355	.86646	3	.49924	.57619	1.7355	.86646	
58	.48456	.55393	1.8053	.87476	2	.49950	.57657	1.7344	.86632	2	.49950	.57657	1.7344	.86632	
59	.48481	.55431	1.8040	.87462	1	.49975	.57696	1.7332	.86617	1	.49975	.57696	1.7332	.86617	
60	.48481	.55431	1.8040	.87462	0	.50000	.57735	1.7321	.86603	0	.50000	.57735	1.7321	.86603	

118° (298°)

(241°) 61°

119° (299°)

(240°) 60°

Table 7 NATURAL TRIGONOMETRIC FUNCTIONS (continued)

30° (210°)					(329°) 149°					31° (211°)					(328°) 148°				
'	Sin	Tan	Ctn	Cos	'	Sin	Tan	Ctn	Cos	'	Sin	Tan	Ctn	Cos	'	Sin	Tan	Ctn	Cos
0	.50000	.57735	1.7321	.86603	50	.51504	.60086	1.6643	.85717	60	.51504	.60086	1.6643	.85717	60	.51504	.60086	1.6643	.85717
1	.50025	.57774	1.7309	.86588	59	.51529	.60126	1.6632	.85702	59	.51529	.60126	1.6632	.85702	59	.51529	.60126	1.6632	.85702
2	.50050	.57813	1.7297	.86573	58	.51554	.60165	1.6621	.85687	58	.51554	.60165	1.6621	.85687	58	.51554	.60165	1.6621	.85687
3	.50076	.57851	1.7286	.86559	57	.51579	.60205	1.6610	.85672	57	.51579	.60205	1.6610	.85672	57	.51579	.60205	1.6610	.85672
4	.50101	.57880	1.7274	.86544	56	.51604	.60245	1.6599	.85657	56	.51604	.60245	1.6599	.85657	56	.51604	.60245	1.6599	.85657
5	.50126	.57920	1.7262	.86530	55	.51628	.60284	1.6588	.85642	55	.51628	.60284	1.6588	.85642	55	.51628	.60284	1.6588	.85642
6	.50151	.57968	1.7251	.86515	54	.51653	.60324	1.6577	.85627	54	.51653	.60324	1.6577	.85627	54	.51653	.60324	1.6577	.85627
7	.50176	.58007	1.7239	.86501	53	.51678	.60364	1.6566	.85612	53	.51678	.60364	1.6566	.85612	53	.51678	.60364	1.6566	.85612
8	.50201	.58046	1.7228	.86486	52	.51703	.60403	1.6555	.85597	52	.51703	.60403	1.6555	.85597	52	.51703	.60403	1.6555	.85597
9	.50227	.58086	1.7216	.86471	51	.51728	.60443	1.6545	.85582	51	.51728	.60443	1.6545	.85582	51	.51728	.60443	1.6545	.85582
10	.50252	.58124	1.7205	.86457	50	.51753	.60483	1.6534	.85567	50	.51753	.60483	1.6534	.85567	50	.51753	.60483	1.6534	.85567
11	.50277	.58162	1.7193	.86442	49	.51778	.60522	1.6523	.85551	49	.51778	.60522	1.6523	.85551	49	.51778	.60522	1.6523	.85551
12	.50302	.58201	1.7182	.86427	48	.51803	.60562	1.6512	.85536	48	.51803	.60562	1.6512	.85536	48	.51803	.60562	1.6512	.85536
13	.50327	.58240	1.7170	.86413	47	.51828	.60602	1.6501	.85521	47	.51828	.60602	1.6501	.85521	47	.51828	.60602	1.6501	.85521
14	.50352	.58279	1.7159	.86398	46	.51852	.60642	1.6490	.85506	46	.51852	.60642	1.6490	.85506	46	.51852	.60642	1.6490	.85506
15	.50377	.58318	1.7147	.86384	45	.51877	.60681	1.6479	.85491	45	.51877	.60681	1.6479	.85491	45	.51877	.60681	1.6479	.85491
16	.50403	.58357	1.7136	.86369	44	.51902	.60721	1.6468	.85476	44	.51902	.60721	1.6468	.85476	44	.51902	.60721	1.6468	.85476
17	.50428	.58396	1.7124	.86354	43	.51927	.60761	1.6458	.85461	43	.51927	.60761	1.6458	.85461	43	.51927	.60761	1.6458	.85461
18	.50453	.58435	1.7113	.86340	42	.51952	.60801	1.6447	.85446	42	.51952	.60801	1.6447	.85446	42	.51952	.60801	1.6447	.85446
19	.50478	.58474	1.7102	.86325	41	.51977	.60841	1.6436	.85431	41	.51977	.60841	1.6436	.85431	41	.51977	.60841	1.6436	.85431
20	.50503	.58513	1.7090	.86310	40	.52002	.60881	1.6426	.85416	40	.52002	.60881	1.6426	.85416	40	.52002	.60881	1.6426	.85416
21	.50528	.58552	1.7079	.86295	39	.52026	.60921	1.6415	.85401	39	.52026	.60921	1.6415	.85401	39	.52026	.60921	1.6415	.85401
22	.50553	.58591	1.7067	.86281	38	.52051	.60960	1.6404	.85385	38	.52051	.60960	1.6404	.85385	38	.52051	.60960	1.6404	.85385
23	.50578	.58630	1.7056	.86266	37	.52076	.61000	1.6393	.85370	37	.52076	.61000	1.6393	.85370	37	.52076	.61000	1.6393	.85370
24	.50603	.58670	1.7045	.86251	36	.52101	.61040	1.6383	.85355	36	.52101	.61040	1.6383	.85355	36	.52101	.61040	1.6383	.85355
25	.50628	.58709	1.7033	.86237	35	.52126	.61080	1.6372	.85340	35	.52126	.61080	1.6372	.85340	35	.52126	.61080	1.6372	.85340
26	.50654	.58748	1.7022	.86222	34	.52151	.61120	1.6361	.85325	34	.52151	.61120	1.6361	.85325	34	.52151	.61120	1.6361	.85325
27	.50679	.58787	1.7011	.86207	33	.52175	.61160	1.6351	.85310	33	.52175	.61160	1.6351	.85310	33	.52175	.61160	1.6351	.85310
28	.50704	.58826	1.6999	.86193	32	.52200	.61200	1.6340	.85294	32	.52200	.61200	1.6340	.85294	32	.52200	.61200	1.6340	.85294
29	.50729	.58865	1.6988	.86178	31	.52225	.61240	1.6329	.85279	31	.52225	.61240	1.6329	.85279	31	.52225	.61240	1.6329	.85279
30	.50754	.58905	1.6977	.86163	30	.52250	.61280	1.6319	.85264	30	.52250	.61280	1.6319	.85264	30	.52250	.61280	1.6319	.85264
31	.50779	.58944	1.6965	.86148	29	.52275	.61320	1.6308	.85249	29	.52275	.61320	1.6308	.85249	29	.52275	.61320	1.6308	.85249
32	.50804	.58983	1.6954	.86133	28	.52299	.61360	1.6298	.85234	28	.52299	.61360	1.6298	.85234	28	.52299	.61360	1.6298	.85234
33	.50829	.59022	1.6943	.86119	27	.52324	.61400	1.6287	.85218	27	.52324	.61400	1.6287	.85218	27	.52324	.61400	1.6287	.85218
34	.50854	.59061	1.6932	.86104	26	.52349	.61440	1.6276	.85203	26	.52349	.61440	1.6276	.85203	26	.52349	.61440	1.6276	.85203
35	.50879	.59101	1.6920	.86089	25	.52374	.61480	1.6265	.85188	25	.52374	.61480	1.6265	.85188	25	.52374	.61480	1.6265	.85188
36	.50904	.59140	1.6909	.86074	24	.52399	.61520	1.6255	.85173	24	.52399	.61520	1.6255	.85173	24	.52399	.61520	1.6255	.85173
37	.50929	.59179	1.6898	.86059	23	.52423	.61560	1.6244	.85157	23	.52423	.61560	1.6244	.85157	23	.52423	.61560	1.6244	.85157
38	.50954	.59218	1.6887	.86045	22	.52448	.61600	1.6234	.85142	22	.52448	.61600	1.6234	.85142	22	.52448	.61600	1.6234	.85142
39	.50979	.59258	1.6875	.86030	21	.52473	.61640	1.6223	.85127	21	.52473	.61640	1.6223	.85127	21	.52473	.61640	1.6223	.85127
40	.51004	.59297	1.6864	.86015	20	.52498	.61680	1.6212	.85112	20	.52498	.61680	1.6212	.85112	20	.52498	.61680	1.6212	.85112
41	.51029	.59336	1.6853	.86000	19	.52522	.61720	1.6202	.85096	19	.52522	.61720	1.6202	.85096	19	.52522	.61720	1.6202	.85096
42	.51054	.59376	1.6842	.85985	18	.52547	.61760	1.6191	.85081	18	.52547	.61760	1.6191	.85081	18	.52547	.61760	1.6191	.85081
43	.51079	.59415	1.6831	.85970	17	.52572	.61800	1.6181	.85066	17	.52572	.61800	1.6181	.85066	17	.52572	.61800	1.6181	.85066
44	.51104	.59454	1.6820	.85955	16	.52597	.61840	1.6170	.85051	16	.52597	.61840	1.6170	.85051	16	.52597	.61840	1.6170	.85051
45	.51129	.59494	1.6808	.85941	15	.52621	.61880	1.6160	.85035	15	.52621	.61880	1.6160	.85035	15	.52621	.61880	1.6160	.85035
46	.51154	.59533	1.6797	.85926	14	.52646	.61920	1.6149	.85020	14	.52646	.61920	1.6149	.85020	14	.52646	.61920	1.6149	.85020
47	.51179	.59573	1.6786	.85911	13	.52671	.61960	1.6139	.85005	13	.52671	.61960	1.6139	.85005	13	.52671	.61960	1.6139	.85005
48	.51204	.59612	1.6775	.85896	12	.52696	.62000	1.6128	.84989	12	.52696	.62000	1.6128	.84989	12	.52696	.62000	1.6128	.84989
49	.51229	.59651	1.6764	.85881	11	.52720	.62040	1.6118	.84974	11	.52720	.62040	1.6118	.84974	11	.52720	.62040	1.6118	.84974
50	.51254	.59691	1.6753	.85866	10	.52745	.62080	1.6107	.84959	10	.52745	.62080	1.6107	.84959	10	.52745	.62080	1.6107	.84959
51	.51279	.59730	1.6742	.85851	9	.52770	.62120	1.6097	.84943	9	.52770	.62120	1.6097	.84943	9	.52770	.62120	1.6097	.84943
52	.51304	.59770	1.6731	.85835	8	.52794	.62160	1.6087	.84928	8	.52794	.62160	1.6087	.84928	8	.52794	.62160	1.6087	.84928
53	.51329	.59809	1.6720	.85820	7	.52819	.62200	1.6077	.84913	7	.52819	.62200	1.6077	.84913	7	.52819	.62200	1.6077	.84913
54	.51354	.59849	1.6709	.85805	6	.52844	.62240	1.6066	.84897	6	.52844	.62240	1.6066	.84897	6	.52844	.62240	1.6066	.84897
55	.51379	.59888	1.6698	.85790	5	.52868	.62280	1.6055	.84882	5	.52868	.62280	1.6055	.84882	5	.52868	.62280	1.6055	.84882
56	.51404	.59928	1.6687	.85777	4	.52893	.62320	1.6045	.84866	4	.52893	.62320	1.6045	.84866	4	.52893	.62320	1.6045	.84866
57	.51429	.59967	1.6676	.85762	3	.52918	.62360	1.6034	.84851	3	.52918	.62360	1.6034	.84851	3	.52918	.62360	1.6034	.84851
58	.51454	.60007	1.6665	.85747	2	.52943	.62400	1.6024	.84836	2	.52943	.62400	1.6024	.84836	2	.52943	.62400	1.6024	.84836
59	.51479	.60046	1.6654	.85732	1	.52967	.62440	1.6014	.84820	1	.52967	.62440	1.6014	.84820	1	.52967	.62440	1.6014	.84820
60	.51504	.60086	1.6643	.85717	0	.52992	.62480	1.6003	.84805	0	.52992	.62480	1.6003	.84805	0	.52992	.62480	1.6003	.84805
	Cos	Ctn	Tan	Sin		Cos	Ctn	Tan	Sin	</									

Table 7 NATURAL TRIGONOMETRIC FUNCTIONS (continued)

32° (212°)					(327°) 147°					33° (213°)					(326°) 146°				
°	Sin	Tan	Ctn	Cos	°	Sin	Tan	Ctn	Cos	°	Sin	Tan	Ctn	Cos	°	Sin	Tan	Ctn	Cos
0	.52992	.62487	1.6003	.84836	60	.54464	.64941	1.5399	.83867	60	.54464	.64941	1.5399	.83867	60	.54464	.64941	1.5399	.83867
1	.53017	.62527	1.5993	.84789	59	.54488	.64982	1.5389	.83861	59	.54488	.64982	1.5389	.83861	59	.54488	.64982	1.5389	.83861
2	.53041	.62566	1.5983	.84744	58	.54513	.65024	1.5379	.83855	58	.54513	.65024	1.5379	.83855	58	.54513	.65024	1.5379	.83855
3	.53066	.62608	1.5972	.84759	57	.54537	.65065	1.5369	.83849	57	.54537	.65065	1.5369	.83849	57	.54537	.65065	1.5369	.83849
4	.53091	.62649	1.5962	.84743	56	.54561	.65106	1.5359	.83844	56	.54561	.65106	1.5359	.83844	56	.54561	.65106	1.5359	.83844
5	.53115	.62689	1.5952	.84728	55	.54586	.65148	1.5350	.83788	55	.54586	.65148	1.5350	.83788	55	.54586	.65148	1.5350	.83788
6	.53140	.62730	1.5941	.84712	54	.54610	.65189	1.5340	.83772	54	.54610	.65189	1.5340	.83772	54	.54610	.65189	1.5340	.83772
7	.53164	.62770	1.5931	.84697	53	.54635	.65231	1.5330	.83766	53	.54635	.65231	1.5330	.83766	53	.54635	.65231	1.5330	.83766
8	.53189	.62811	1.5921	.84681	52	.54659	.65272	1.5320	.83740	52	.54659	.65272	1.5320	.83740	52	.54659	.65272	1.5320	.83740
9	.53214	.62852	1.5911	.84666	51	.54683	.65314	1.5311	.83724	51	.54683	.65314	1.5311	.83724	51	.54683	.65314	1.5311	.83724
10	.53239	.62892	1.5900	.84650	50	.54708	.65356	1.5301	.83708	50	.54708	.65356	1.5301	.83708	50	.54708	.65356	1.5301	.83708
11	.53263	.62933	1.5890	.84636	49	.54732	.65397	1.5291	.83692	49	.54732	.65397	1.5291	.83692	49	.54732	.65397	1.5291	.83692
12	.53288	.62973	1.5880	.84619	48	.54756	.65438	1.5282	.83676	48	.54756	.65438	1.5282	.83676	48	.54756	.65438	1.5282	.83676
13	.53312	.63014	1.5869	.84604	47	.54781	.65480	1.5272	.83660	47	.54781	.65480	1.5272	.83660	47	.54781	.65480	1.5272	.83660
14	.53337	.63055	1.5859	.84588	46	.54805	.65521	1.5262	.83645	46	.54805	.65521	1.5262	.83645	46	.54805	.65521	1.5262	.83645
15	.53361	.63096	1.5849	.84573	45	.54829	.65563	1.5253	.83629	45	.54829	.65563	1.5253	.83629	45	.54829	.65563	1.5253	.83629
16	.53385	.63136	1.5839	.84557	44	.54853	.65604	1.5243	.83613	44	.54853	.65604	1.5243	.83613	44	.54853	.65604	1.5243	.83613
17	.53411	.63177	1.5829	.84542	43	.54878	.65646	1.5233	.83597	43	.54878	.65646	1.5233	.83597	43	.54878	.65646	1.5233	.83597
18	.53435	.63217	1.5818	.84526	42	.54902	.65688	1.5224	.83581	42	.54902	.65688	1.5224	.83581	42	.54902	.65688	1.5224	.83581
19	.53460	.63258	1.5808	.84511	41	.54927	.65729	1.5214	.83565	41	.54927	.65729	1.5214	.83565	41	.54927	.65729	1.5214	.83565
20	.53484	.63299	1.5798	.84495	40	.54951	.65771	1.5204	.83549	40	.54951	.65771	1.5204	.83549	40	.54951	.65771	1.5204	.83549
21	.53509	.63340	1.5788	.84480	39	.54975	.65813	1.5195	.83533	39	.54975	.65813	1.5195	.83533	39	.54975	.65813	1.5195	.83533
22	.53534	.63380	1.5778	.84464	38	.54999	.65854	1.5185	.83517	38	.54999	.65854	1.5185	.83517	38	.54999	.65854	1.5185	.83517
23	.53558	.63421	1.5768	.84448	37	.55024	.65896	1.5175	.83501	37	.55024	.65896	1.5175	.83501	37	.55024	.65896	1.5175	.83501
24	.53583	.63462	1.5757	.84433	36	.55048	.65938	1.5166	.83485	36	.55048	.65938	1.5166	.83485	36	.55048	.65938	1.5166	.83485
25	.53607	.63503	1.5747	.84417	35	.55072	.65980	1.5156	.83469	35	.55072	.65980	1.5156	.83469	35	.55072	.65980	1.5156	.83469
26	.53632	.63544	1.5737	.84402	34	.55097	.66021	1.5147	.83453	34	.55097	.66021	1.5147	.83453	34	.55097	.66021	1.5147	.83453
27	.53656	.63584	1.5727	.84386	33	.55121	.66063	1.5137	.83437	33	.55121	.66063	1.5137	.83437	33	.55121	.66063	1.5137	.83437
28	.53681	.63625	1.5717	.84370	32	.55145	.66105	1.5127	.83421	32	.55145	.66105	1.5127	.83421	32	.55145	.66105	1.5127	.83421
29	.53705	.63666	1.5707	.84355	31	.55169	.66147	1.5118	.83405	31	.55169	.66147	1.5118	.83405	31	.55169	.66147	1.5118	.83405
30	.53730	.63707	1.5697	.84339	30	.55194	.66189	1.5108	.83389	30	.55194	.66189	1.5108	.83389	30	.55194	.66189	1.5108	.83389
31	.53754	.63748	1.5687	.84324	29	.55218	.66230	1.5099	.83373	29	.55218	.66230	1.5099	.83373	29	.55218	.66230	1.5099	.83373
32	.53779	.63789	1.5677	.84308	28	.55242	.66272	1.5089	.83357	28	.55242	.66272	1.5089	.83357	28	.55242	.66272	1.5089	.83357
33	.53804	.63830	1.5667	.84292	27	.55266	.66314	1.5080	.83340	27	.55266	.66314	1.5080	.83340	27	.55266	.66314	1.5080	.83340
34	.53828	.63871	1.5657	.84277	26	.55291	.66356	1.5070	.83324	26	.55291	.66356	1.5070	.83324	26	.55291	.66356	1.5070	.83324
35	.53853	.63912	1.5647	.84261	25	.55315	.66398	1.5061	.83308	25	.55315	.66398	1.5061	.83308	25	.55315	.66398	1.5061	.83308
36	.53877	.63953	1.5637	.84245	24	.55339	.66440	1.5051	.83292	24	.55339	.66440	1.5051	.83292	24	.55339	.66440	1.5051	.83292
37	.53902	.63994	1.5627	.84230	23	.55363	.66482	1.5042	.83276	23	.55363	.66482	1.5042	.83276	23	.55363	.66482	1.5042	.83276
38	.53926	.64035	1.5617	.84214	22	.55388	.66524	1.5032	.83260	22	.55388	.66524	1.5032	.83260	22	.55388	.66524	1.5032	.83260
39	.53951	.64076	1.5607	.84198	21	.55412	.66566	1.5023	.83244	21	.55412	.66566	1.5023	.83244	21	.55412	.66566	1.5023	.83244
40	.53975	.64117	1.5597	.84182	20	.55436	.66608	1.5013	.83228	20	.55436	.66608	1.5013	.83228	20	.55436	.66608	1.5013	.83228
41	.54000	.64158	1.5587	.84167	19	.55460	.66650	1.5004	.83212	19	.55460	.66650	1.5004	.83212	19	.55460	.66650	1.5004	.83212
42	.54024	.64199	1.5577	.84151	18	.55484	.66692	1.4994	.83195	18	.55484	.66692	1.4994	.83195	18	.55484	.66692	1.4994	.83195
43	.54049	.64240	1.5567	.84135	17	.55509	.66734	1.4985	.83179	17	.55509	.66734	1.4985	.83179	17	.55509	.66734	1.4985	.83179
44	.54073	.64281	1.5557	.84120	16	.55533	.66776	1.4975	.83163	16	.55533	.66776	1.4975	.83163	16	.55533	.66776	1.4975	.83163
45	.54097	.64322	1.5547	.84104	15	.55557	.66818	1.4966	.83147	15	.55557	.66818	1.4966	.83147	15	.55557	.66818	1.4966	.83147
46	.54122	.64363	1.5537	.84088	14	.55581	.66860	1.4957	.83131	14	.55581	.66860	1.4957	.83131	14	.55581	.66860	1.4957	.83131
47	.54146	.64404	1.5527	.84072	13	.55605	.66902	1.4947	.83115	13	.55605	.66902	1.4947	.83115	13	.55605	.66902	1.4947	.83115
48	.54171	.64446	1.5517	.84057	12	.55630	.66944	1.4938	.83098	12	.55630	.66944	1.4938	.83098	12	.55630	.66944	1.4938	.83098
49	.54195	.64487	1.5507	.84041	11	.55654	.66986	1.4928	.83082	11	.55654	.66986	1.4928	.83082	11	.55654	.66986	1.4928	.83082
50	.54220	.64528	1.5497	.84025	10	.55678	.67028	1.4919	.83065	10	.55678	.67028	1.4919	.83065	10	.55678	.67028	1.4919	.83065
51	.54244	.64569	1.5487	.84009	9	.55702	.67071	1.4910	.83050	9	.55702	.67071	1.4910	.83050	9	.55702	.67071	1.4910	.83050
52	.54269	.64610	1.5477	.83994	8	.55726	.67113	1.4900	.83034	8	.55726	.67113	1.4900	.83034	8	.55726	.67113	1.4900	.83034
53	.54293	.64652	1.5468	.83978	7	.55750	.67155	1.4891	.83017	7	.55750	.67155	1.4891	.83017	7	.55750	.67155	1.4891	.83017
54	.54317	.64693	1.5458	.83962	6	.55775	.67197	1.4882	.83001	6	.55775	.67197	1.4882	.83001	6	.55775	.67197	1.4882	.83001
55	.54342	.64734	1.5448	.83946	5	.55799	.67239	1.4872	.82985	5	.55799	.67239	1.4872	.82985	5	.55799	.67239	1.4872	.82985
56	.54366	.64775	1.5438	.83930	4	.55823	.67282	1.4863	.82969	4	.55823	.67282	1.4863	.82969	4	.55823	.67282	1.4863	.82969
57	.54391	.64817	1.5428	.83915	3	.55847	.67324	1.4854	.82953	3	.55847	.67324	1.4854	.82953	3	.55847	.67324	1.4854	.82953
58	.54415	.64858	1.5418	.83899	2	.55871	.67366	1.4844	.82935	2	.55871	.67366	1.4844	.82935	2	.55871	.67366	1.4844	.82935
59	.54440	.64899	1.5408	.83883	1	.55895	.67409	1.4835	.82920	1	.55895	.67409	1.4835	.82920	1	.55895	.67409	1.4835	.82920
60	.54464	.64941	1.5399	.83867	0	.55919	.67451	1.4826	.82904	0	.55919	.67451	1.4826	.82904	0	.55919	.67451	1.4826	.82904
°	Cos	Ctn	Tan	Sin	°	Cos	Ctn	Tan											

Table 7 NATURAL TRIGONOMETRIC FUNCTIONS (continued)

114° (214°)				(325°) 145°				35° (215°)				(324°) 144°			
	Sin	Tan	Ctn	Cos		Sin	Tan	Ctn	Cos		Sin	Tan	Ctn	Cos	
0	.55919	.67451	1.4826	.82904	60	.57586	.70921	1.4281	.81915	60	.57586	.70921	1.4281	.81915	
1	.55943	.67493	1.4816	.82887	59	.57381	.70964	1.4273	.81889	59	.57381	.70964	1.4273	.81889	
2	.55968	.67536	1.4807	.82871	58	.57180	.71007	1.4264	.81862	58	.57180	.71007	1.4264	.81862	
3	.55992	.67578	1.4798	.82855	57	.57019	.71050	1.4255	.81835	57	.57019	.71050	1.4255	.81835	
4	.56016	.67620	1.4788	.82839	56	.56863	.71094	1.4246	.81808	56	.56863	.71094	1.4246	.81808	
5	.56040	.67663	1.4779	.82822	55	.56712	.71138	1.4237	.81782	55	.56712	.71138	1.4237	.81782	
6	.56064	.67705	1.4770	.82806	54	.56564	.71182	1.4228	.81756	54	.56564	.71182	1.4228	.81756	
7	.56088	.67748	1.4761	.82790	53	.56419	.71226	1.4219	.81730	53	.56419	.71226	1.4219	.81730	
8	.56112	.67790	1.4751	.82773	52	.56276	.71270	1.4210	.81704	52	.56276	.71270	1.4210	.81704	
9	.56136	.67832	1.4742	.82757	51	.56136	.71314	1.4202	.81678	51	.56136	.71314	1.4202	.81678	
10	.56160	.67875	1.4733	.82741	50	.56000	.71358	1.4193	.81652	50	.56000	.71358	1.4193	.81652	
11	.56184	.67917	1.4724	.82724	49	.55863	.71402	1.4185	.81626	49	.55863	.71402	1.4185	.81626	
12	.56208	.67960	1.4715	.82708	48	.55728	.71446	1.4176	.81600	48	.55728	.71446	1.4176	.81600	
13	.56232	.68002	1.4705	.82692	47	.55595	.71490	1.4167	.81574	47	.55595	.71490	1.4167	.81574	
14	.56256	.68045	1.4696	.82675	46	.55463	.71534	1.4158	.81548	46	.55463	.71534	1.4158	.81548	
15	.56280	.68088	1.4687	.82659	45	.55333	.71578	1.4150	.81522	45	.55333	.71578	1.4150	.81522	
16	.56305	.68130	1.4678	.82643	44	.55204	.71622	1.4141	.81496	44	.55204	.71622	1.4141	.81496	
17	.56329	.68173	1.4669	.82626	43	.55076	.71666	1.4132	.81470	43	.55076	.71666	1.4132	.81470	
18	.56353	.68215	1.4660	.82610	42	.54949	.71710	1.4123	.81444	42	.54949	.71710	1.4123	.81444	
19	.56377	.68258	1.4650	.82593	41	.54823	.71754	1.4115	.81418	41	.54823	.71754	1.4115	.81418	
20	.56401	.68301	1.4641	.82577	40	.54700	.71798	1.4106	.81392	40	.54700	.71798	1.4106	.81392	
21	.56425	.68343	1.4632	.82561	39	.54576	.71842	1.4097	.81366	39	.54576	.71842	1.4097	.81366	
22	.56449	.68386	1.4623	.82544	38	.54454	.71886	1.4088	.81340	38	.54454	.71886	1.4088	.81340	
23	.56473	.68429	1.4614	.82528	37	.54333	.71930	1.4079	.81314	37	.54333	.71930	1.4079	.81314	
24	.56497	.68471	1.4605	.82511	36	.54214	.71974	1.4071	.81288	36	.54214	.71974	1.4071	.81288	
25	.56521	.68514	1.4596	.82495	35	.54096	.72018	1.4063	.81262	35	.54096	.72018	1.4063	.81262	
26	.56545	.68557	1.4586	.82478	34	.53979	.72062	1.4054	.81236	34	.53979	.72062	1.4054	.81236	
27	.56569	.68600	1.4577	.82462	33	.53863	.72106	1.4045	.81210	33	.53863	.72106	1.4045	.81210	
28	.56593	.68642	1.4568	.82446	32	.53748	.72150	1.4036	.81184	32	.53748	.72150	1.4036	.81184	
29	.56617	.68685	1.4559	.82429	31	.53633	.72194	1.4028	.81158	31	.53633	.72194	1.4028	.81158	
30	.56641	.68728	1.4550	.82413	30	.53519	.72238	1.4019	.81132	30	.53519	.72238	1.4019	.81132	
31	.56665	.68771	1.4541	.82396	29	.53406	.72282	1.4011	.81106	29	.53406	.72282	1.4011	.81106	
32	.56689	.68814	1.4532	.82380	28	.53294	.72326	1.4002	.81080	28	.53294	.72326	1.4002	.81080	
33	.56713	.68857	1.4523	.82363	27	.53183	.72370	1.3994	.81054	27	.53183	.72370	1.3994	.81054	
34	.56736	.68900	1.4514	.82347	26	.53073	.72414	1.3986	.81028	26	.53073	.72414	1.3986	.81028	
35	.56760	.68942	1.4505	.82330	25	.52964	.72458	1.3977	.81002	25	.52964	.72458	1.3977	.81002	
36	.56784	.68985	1.4496	.82314	24	.52856	.72502	1.3968	.80976	24	.52856	.72502	1.3968	.80976	
37	.56808	.69028	1.4487	.82297	23	.52749	.72546	1.3959	.80950	23	.52749	.72546	1.3959	.80950	
38	.56832	.69071	1.4478	.82281	22	.52643	.72590	1.3951	.80924	22	.52643	.72590	1.3951	.80924	
39	.56856	.69114	1.4469	.82264	21	.52538	.72634	1.3942	.80898	21	.52538	.72634	1.3942	.80898	
40	.56880	.69157	1.4460	.82248	20	.52434	.72678	1.3934	.80872	20	.52434	.72678	1.3934	.80872	
41	.56904	.69200	1.4451	.82231	19	.52331	.72722	1.3925	.80846	19	.52331	.72722	1.3925	.80846	
42	.56928	.69243	1.4442	.82214	18	.52229	.72766	1.3916	.80820	18	.52229	.72766	1.3916	.80820	
43	.56952	.69286	1.4433	.82198	17	.52128	.72810	1.3908	.80794	17	.52128	.72810	1.3908	.80794	
44	.56976	.69329	1.4424	.82181	16	.52028	.72854	1.3899	.80768	16	.52028	.72854	1.3899	.80768	
45	.57000	.69372	1.4415	.82165	15	.51929	.72898	1.3891	.80742	15	.51929	.72898	1.3891	.80742	
46	.57024	.69416	1.4405	.82148	14	.51831	.72942	1.3882	.80716	14	.51831	.72942	1.3882	.80716	
47	.57047	.69459	1.4397	.82132	13	.51734	.72986	1.3874	.80690	13	.51734	.72986	1.3874	.80690	
48	.57071	.69502	1.4388	.82115	12	.51638	.73030	1.3865	.80664	12	.51638	.73030	1.3865	.80664	
49	.57095	.69545	1.4379	.82098	11	.51543	.73074	1.3857	.80638	11	.51543	.73074	1.3857	.80638	
50	.57119	.69588	1.4370	.82082	10	.51449	.73118	1.3848	.80612	10	.51449	.73118	1.3848	.80612	
51	.57143	.69631	1.4361	.82065	9	.51356	.73162	1.3840	.80586	9	.51356	.73162	1.3840	.80586	
52	.57167	.69675	1.4352	.82048	8	.51264	.73206	1.3831	.80560	8	.51264	.73206	1.3831	.80560	
53	.57191	.69718	1.4344	.82032	7	.51173	.73250	1.3823	.80534	7	.51173	.73250	1.3823	.80534	
54	.57215	.69761	1.4335	.82016	6	.51083	.73294	1.3814	.80508	6	.51083	.73294	1.3814	.80508	
55	.57238	.69804	1.4326	.81999	5	.51038	.73338	1.3806	.80482	5	.51038	.73338	1.3806	.80482	
56	.57262	.69847	1.4317	.81982	4	.50994	.73382	1.3798	.80456	4	.50994	.73382	1.3798	.80456	
57	.57286	.69891	1.4308	.81965	3	.50951	.73426	1.3789	.80430	3	.50951	.73426	1.3789	.80430	
58	.57310	.69934	1.4299	.81948	2	.50909	.73470	1.3781	.80404	2	.50909	.73470	1.3781	.80404	
59	.57334	.69977	1.4290	.81932	1	.50868	.73514	1.3772	.80378	1	.50868	.73514	1.3772	.80378	
60	.57358	.70021	1.4281	.81915	0	.50828	.73558	1.3764	.80352	0	.50828	.73558	1.3764	.80352	
	Cos	Ctn	Tan	Sin		Cos	Ctn	Tan	Sin		Cos	Ctn	Tan	Sin	

124° (304°)

(235°) 55°

125° (305°)

(234°) 54°

Table 7 NATURAL TRIGONOMETRIC FUNCTIONS (continued)

36° (216°)				(323°) 143°				37° (217°)				(322°) 142°			
	Sin	Tan	Ctn	Cos		Sin	Tan	Ctn	Cos		Sin	Tan	Ctn	Cos	
0	.58779	.72854	1.3764	.80902	50	.60182	.75355	1.3270	.79864	90					
1	.58802	.72899	1.3755	.80885	59	.60205	.75401	1.3262	.79846	59					
2	.58825	.72943	1.3747	.80867	58	.60228	.75447	1.3254	.79829	58					
3	.58849	.72988	1.3739	.80850	57	.60251	.75492	1.3246	.79811	57					
4	.58873	.73032	1.3730	.80833	56	.60274	.75538	1.3238	.79793	56					
5	.58896	.73077	1.3722	.80816	55	.60298	.75584	1.3230	.79776	55					
6	.58920	.73121	1.3713	.80799	54	.60321	.75629	1.3222	.79758	54					
7	.58943	.73166	1.3705	.80782	53	.60344	.75675	1.3214	.79741	53					
8	.58967	.73210	1.3697	.80765	52	.60367	.75721	1.3206	.79723	52					
9	.58990	.73255	1.3688	.80748	51	.60390	.75767	1.3198	.79706	51					
10	.59014	.73300	1.3680	.80730	50	.60414	.75812	1.3190	.79688	50					
11	.59037	.73344	1.3672	.80713	49	.60437	.75858	1.3182	.79671	49					
12	.59061	.73389	1.3663	.80696	48	.60460	.75904	1.3175	.79653	48					
13	.59084	.73434	1.3655	.80679	47	.60483	.75950	1.3167	.79635	47					
14	.59108	.73478	1.3647	.80662	46	.60506	.75996	1.3159	.79618	46					
15	.59131	.73523	1.3638	.80644	45	.60529	.76042	1.3151	.79600	45					
16	.59154	.73568	1.3630	.80627	44	.60552	.76088	1.3143	.79582	44					
17	.59178	.73613	1.3622	.80610	43	.60576	.76134	1.3135	.79565	43					
18	.59201	.73657	1.3613	.80593	42	.60599	.76180	1.3127	.79547	42					
19	.59225	.73702	1.3605	.80576	41	.60622	.76226	1.3119	.79530	41					
20	.59248	.73747	1.3597	.80558	40	.60645	.76272	1.3111	.79512	40					
21	.59272	.73792	1.3588	.80541	39	.60668	.76318	1.3103	.79494	39					
22	.59295	.73837	1.3580	.80524	38	.60691	.76364	1.3095	.79477	38					
23	.59318	.73881	1.3572	.80507	37	.60714	.76410	1.3087	.79459	37					
24	.59342	.73926	1.3564	.80489	36	.60738	.76455	1.3079	.79441	36					
25	.59365	.73971	1.3556	.80472	35	.60761	.76502	1.3072	.79424	35					
26	.59389	.74016	1.3547	.80455	34	.60784	.76548	1.3064	.79406	34					
27	.59412	.74061	1.3539	.80438	33	.60807	.76594	1.3056	.79388	33					
28	.59436	.74106	1.3531	.80420	32	.60830	.76640	1.3048	.79371	32					
29	.59459	.74151	1.3522	.80403	31	.60853	.76686	1.3040	.79353	31					
30	.59482	.74196	1.3514	.80386	30	.60876	.76733	1.3032	.79335	30					
31	.59506	.74241	1.3506	.80368	29	.60899	.76779	1.3024	.79318	29					
32	.59529	.74286	1.3498	.80351	28	.60922	.76825	1.3017	.79300	28					
33	.59552	.74331	1.3490	.80334	27	.60945	.76871	1.3009	.79282	27					
34	.59576	.74376	1.3481	.80316	26	.60968	.76918	1.3001	.79264	26					
35	.59599	.74421	1.3473	.80299	25	.60991	.76964	1.2993	.79247	25					
36	.59622	.74466	1.3465	.80282	24	.61015	.77010	1.2985	.79229	24					
37	.59646	.74512	1.3457	.80264	23	.61038	.77057	1.2977	.79211	23					
38	.59669	.74557	1.3449	.80247	22	.61061	.77103	1.2970	.79193	22					
39	.59693	.74602	1.3440	.80230	21	.61084	.77149	1.2962	.79175	21					
40	.59716	.74647	1.3432	.80212	20	.61107	.77196	1.2954	.79158	20					
41	.59739	.74692	1.3424	.80195	19	.61130	.77242	1.2946	.79140	19					
42	.59763	.74738	1.3416	.80178	18	.61153	.77289	1.2938	.79122	18					
43	.59786	.74783	1.3408	.80160	17	.61176	.77335	1.2931	.79105	17					
44	.59809	.74828	1.3400	.80143	16	.61199	.77382	1.2923	.79087	16					
45	.59832	.74874	1.3392	.80125	15	.61222	.77428	1.2915	.79069	15					
46	.59856	.74919	1.3384	.80108	14	.61245	.77475	1.2907	.79051	14					
47	.59879	.74964	1.3375	.80091	13	.61268	.77521	1.2900	.79033	13					
48	.59902	.75010	1.3367	.80073	12	.61291	.77568	1.2892	.79016	12					
49	.59926	.75055	1.3359	.80055	11	.61314	.77615	1.2884	.78998	11					
50	.59949	.75100	1.3351	.80038	10	.61337	.77661	1.2876	.78980	10					
51	.59972	.75146	1.3343	.80021	9	.61360	.77708	1.2869	.78962	9					
52	.59995	.75191	1.3335	.80003	8	.61383	.77754	1.2861	.78944	8					
53	.60019	.75237	1.3327	.79986	7	.61406	.77801	1.2853	.78926	7					
54	.60042	.75282	1.3319	.79968	6	.61429	.77848	1.2846	.78908	6					
55	.60065	.75328	1.3311	.79951	5	.61451	.77895	1.2838	.78891	5					
56	.60089	.75373	1.3303	.79934	4	.61474	.77941	1.2830	.78873	4					
57	.60112	.75419	1.3295	.79916	3	.61497	.77988	1.2822	.78855	3					
58	.60135	.75464	1.3287	.79899	2	.61520	.78035	1.2815	.78837	2					
59	.60158	.75510	1.3278	.79881	1	.61543	.78082	1.2807	.78819	1					
60	.60182	.75555	1.3270	.79864	0	.61566	.78129	1.2799	.78801	0					

126° (306°)

(223°) 53°

127° (307°)

(232°) 52°

Table 7 NATURAL TRIGONOMETRIC FUNCTIONS (continued)

Table with columns for angles (38°, 39°, 40°), functions (Sin, Tan, Ctn, Cos), and degrees (128°, 129°, 130°).

Table 7 NATURAL TRIGONOMETRIC FUNCTIONS (continued)

40° (220°)				(319°) 139°				41° (221°)				(318°) 138°			
'	Sin	Tan	Ctn	Cos	'	Sin	Tan	Ctn	Cos	'	Sin	Tan	Ctn	Cos	'
0	.64279	.83910	1.1918	.78604	60	.65606	.86929	1.1504	.75471	60	.65606	.86929	1.1504	.75471	60
1	.64301	.83960	1.1910	.78586	59	.65628	.86980	1.1497	.75452	59	.65628	.86980	1.1497	.75452	59
2	.64323	.84009	1.1903	.78567	58	.65650	.87031	1.1490	.75433	58	.65650	.87031	1.1490	.75433	58
3	.64346	.84059	1.1896	.78548	57	.65672	.87082	1.1483	.75414	57	.65672	.87082	1.1483	.75414	57
4	.64368	.84108	1.1889	.78530	56	.65694	.87133	1.1477	.75395	56	.65694	.87133	1.1477	.75395	56
5	.64390	.84158	1.1882	.78511	55	.65716	.87184	1.1470	.75375	55	.65716	.87184	1.1470	.75375	55
6	.64412	.84208	1.1875	.78492	54	.65738	.87235	1.1463	.75356	54	.65738	.87235	1.1463	.75356	54
7	.64435	.84258	1.1868	.78473	53	.65759	.87287	1.1456	.75337	53	.65759	.87287	1.1456	.75337	53
8	.64457	.84307	1.1861	.78455	52	.65781	.87338	1.1450	.75318	52	.65781	.87338	1.1450	.75318	52
9	.64479	.84357	1.1854	.78436	51	.65803	.87389	1.1443	.75299	51	.65803	.87389	1.1443	.75299	51
10	.64501	.84407	1.1847	.78417	50	.65825	.87441	1.1436	.75280	50	.65825	.87441	1.1436	.75280	50
11	.64524	.84457	1.1840	.78398	49	.65847	.87492	1.1430	.75261	49	.65847	.87492	1.1430	.75261	49
12	.64546	.84507	1.1833	.78380	48	.65869	.87543	1.1423	.75241	48	.65869	.87543	1.1423	.75241	48
13	.64568	.84556	1.1826	.78361	47	.65891	.87595	1.1416	.75222	47	.65891	.87595	1.1416	.75222	47
14	.64590	.84606	1.1819	.78342	46	.65913	.87646	1.1410	.75203	46	.65913	.87646	1.1410	.75203	46
15	.64612	.84656	1.1812	.78323	45	.65935	.87698	1.1403	.75184	45	.65935	.87698	1.1403	.75184	45
16	.64635	.84706	1.1806	.78304	44	.65956	.87749	1.1396	.75165	44	.65956	.87749	1.1396	.75165	44
17	.64657	.84756	1.1799	.78286	43	.65978	.87801	1.1389	.75146	43	.65978	.87801	1.1389	.75146	43
18	.64679	.84806	1.1792	.78267	42	.66000	.87852	1.1383	.75127	42	.66000	.87852	1.1383	.75127	42
19	.64701	.84856	1.1785	.78248	41	.66022	.87904	1.1376	.75107	41	.66022	.87904	1.1376	.75107	41
20	.64723	.84906	1.1778	.78229	40	.66044	.87955	1.1369	.75088	40	.66044	.87955	1.1369	.75088	40
21	.64745	.84956	1.1771	.78210	39	.66066	.88007	1.1363	.75069	39	.66066	.88007	1.1363	.75069	39
22	.64768	.85006	1.1764	.78192	38	.66088	.88059	1.1356	.75050	38	.66088	.88059	1.1356	.75050	38
23	.64790	.85057	1.1757	.78173	37	.66109	.88110	1.1349	.75030	37	.66109	.88110	1.1349	.75030	37
24	.64812	.85107	1.1750	.78154	36	.66131	.88162	1.1343	.75011	36	.66131	.88162	1.1343	.75011	36
25	.64834	.85157	1.1743	.78135	35	.66153	.88214	1.1336	.74992	35	.66153	.88214	1.1336	.74992	35
26	.64856	.85207	1.1736	.78116	34	.66175	.88265	1.1329	.74973	34	.66175	.88265	1.1329	.74973	34
27	.64878	.85257	1.1729	.78097	33	.66197	.88317	1.1323	.74953	33	.66197	.88317	1.1323	.74953	33
28	.64901	.85308	1.1722	.78078	32	.66218	.88368	1.1316	.74934	32	.66218	.88368	1.1316	.74934	32
29	.64923	.85358	1.1715	.78059	31	.66240	.88420	1.1310	.74915	31	.66240	.88420	1.1310	.74915	31
30	.64945	.85408	1.1708	.78041	30	.66262	.88471	1.1303	.74896	30	.66262	.88471	1.1303	.74896	30
31	.64967	.85458	1.1702	.78022	29	.66284	.88524	1.1296	.74876	29	.66284	.88524	1.1296	.74876	29
32	.64989	.85509	1.1695	.78003	28	.66306	.88576	1.1290	.74857	28	.66306	.88576	1.1290	.74857	28
33	.65011	.85559	1.1688	.77984	27	.66327	.88628	1.1283	.74838	27	.66327	.88628	1.1283	.74838	27
34	.65033	.85609	1.1681	.77965	26	.66349	.88680	1.1276	.74818	26	.66349	.88680	1.1276	.74818	26
35	.65055	.85660	1.1674	.77946	25	.66371	.88732	1.1270	.74799	25	.66371	.88732	1.1270	.74799	25
36	.65077	.85710	1.1667	.77927	24	.66393	.88784	1.1263	.74780	24	.66393	.88784	1.1263	.74780	24
37	.65100	.85761	1.1660	.77908	23	.66414	.88836	1.1257	.74760	23	.66414	.88836	1.1257	.74760	23
38	.65122	.85811	1.1653	.77889	22	.66436	.88888	1.1250	.74741	22	.66436	.88888	1.1250	.74741	22
39	.65144	.85862	1.1647	.77870	21	.66458	.88940	1.1243	.74722	21	.66458	.88940	1.1243	.74722	21
40	.65166	.85912	1.1640	.77851	20	.66480	.88992	1.1237	.74703	20	.66480	.88992	1.1237	.74703	20
41	.65188	.85963	1.1633	.77832	19	.66501	.89045	1.1230	.74683	19	.66501	.89045	1.1230	.74683	19
42	.65210	.86014	1.1626	.77813	18	.66523	.89097	1.1224	.74664	18	.66523	.89097	1.1224	.74664	18
43	.65232	.86064	1.1619	.77794	17	.66545	.89149	1.1217	.74644	17	.66545	.89149	1.1217	.74644	17
44	.65254	.86115	1.1612	.77775	16	.66566	.89201	1.1211	.74625	16	.66566	.89201	1.1211	.74625	16
45	.65276	.86166	1.1606	.77756	15	.66588	.89253	1.1204	.74606	15	.66588	.89253	1.1204	.74606	15
46	.65298	.86216	1.1599	.77737	14	.66610	.89306	1.1197	.74586	14	.66610	.89306	1.1197	.74586	14
47	.65320	.86267	1.1592	.77718	13	.66632	.89358	1.1191	.74567	13	.66632	.89358	1.1191	.74567	13
48	.65342	.86318	1.1585	.77700	12	.66653	.89410	1.1184	.74548	12	.66653	.89410	1.1184	.74548	12
49	.65364	.86368	1.1578	.77680	11	.66675	.89463	1.1176	.74529	11	.66675	.89463	1.1176	.74529	11
50	.65386	.86419	1.1571	.77661	10	.66697	.89515	1.1171	.74509	10	.66697	.89515	1.1171	.74509	10
51	.65408	.86470	1.1565	.77642	9	.66718	.89567	1.1165	.74489	9	.66718	.89567	1.1165	.74489	9
52	.65430	.86521	1.1558	.77623	8	.66740	.89620	1.1159	.74470	8	.66740	.89620	1.1159	.74470	8
53	.65452	.86572	1.1551	.77604	7	.66762	.89672	1.1152	.74451	7	.66762	.89672	1.1152	.74451	7
54	.65474	.86623	1.1544	.77585	6	.66783	.89725	1.1146	.74431	6	.66783	.89725	1.1146	.74431	6
55	.65496	.86674	1.1538	.77566	5	.66805	.89777	1.1139	.74412	5	.66805	.89777	1.1139	.74412	5
56	.65518	.86725	1.1531	.77547	4	.66827	.89830	1.1132	.74392	4	.66827	.89830	1.1132	.74392	4
57	.65540	.86776	1.1524	.77528	3	.66848	.89883	1.1126	.74373	3	.66848	.89883	1.1126	.74373	3
58	.65562	.86827	1.1517	.77509	2	.66870	.89935	1.1119	.74353	2	.66870	.89935	1.1119	.74353	2
59	.65584	.86878	1.1510	.77490	1	.66891	.89988	1.1113	.74334	1	.66891	.89988	1.1113	.74334	1
60	.65606	.86929	1.1504	.77471	0	.66913	.90040	1.1106	.74314	0	.66913	.90040	1.1106	.74314	0
'	Cos	Ctn	Tan	Sin	'	Cos	Ctn	Tan	Sin	'	Cos	Ctn	Tan	Sin	'

120° (210°)

(220°) 49°

131° (311°)

(228°) 48°

Table 7 NATURAL TRIGONOMETRIC FUNCTIONS (continued)

42° (222°)					(317°) 137°					43° (223°)					(316°) 136°				
	Sin	Tan	Ctn	Cos			Sin	Tan	Ctn	Cos			Sin	Tan	Ctn	Cos			
0	.66913	.90040	1.1108	.74314	60	0	.68200	.93252	1.0724	.73135	60	0	.68200	.93252	1.0724	.73135	60		
1	.66935	.90093	1.1109	.74295	59	1	.68221	.93306	1.0717	.73115	59	1	.68221	.93306	1.0717	.73115	59		
2	.66956	.90146	1.1093	.74276	58	2	.68242	.93360	1.0711	.73096	58	2	.68242	.93360	1.0711	.73096	58		
3	.66978	.90199	1.1087	.74256	57	3	.68264	.93415	1.0705	.73076	57	3	.68264	.93415	1.0705	.73076	57		
4	.66999	.90251	1.1080	.74237	56	4	.68285	.93469	1.0699	.73056	56	4	.68285	.93469	1.0699	.73056	56		
5	.67021	.90304	1.1074	.74217	55	5	.68306	.93524	1.0692	.73036	55	5	.68306	.93524	1.0692	.73036	55		
6	.67043	.90357	1.1067	.74198	54	6	.68327	.93578	1.0686	.73016	54	6	.68327	.93578	1.0686	.73016	54		
7	.67064	.90410	1.1061	.74178	53	7	.68349	.93633	1.0680	.72996	53	7	.68349	.93633	1.0680	.72996	53		
8	.67086	.90463	1.1054	.74159	52	8	.68370	.93688	1.0674	.72976	52	8	.68370	.93688	1.0674	.72976	52		
9	.67107	.90516	1.1048	.74139	51	9	.68391	.93742	1.0668	.72957	51	9	.68391	.93742	1.0668	.72957	51		
10	.67129	.90569	1.1041	.74120	50	10	.68412	.93797	1.0661	.72937	50	10	.68412	.93797	1.0661	.72937	50		
11	.67151	.90621	1.1035	.74100	49	11	.68434	.93852	1.0655	.72917	49	11	.68434	.93852	1.0655	.72917	49		
12	.67172	.90674	1.1028	.74080	48	12	.68455	.93906	1.0649	.72897	48	12	.68455	.93906	1.0649	.72897	48		
13	.67194	.90727	1.1022	.74061	47	13	.68476	.93961	1.0643	.72877	47	13	.68476	.93961	1.0643	.72877	47		
14	.67215	.90781	1.1016	.74041	46	14	.68497	.94016	1.0637	.72857	46	14	.68497	.94016	1.0637	.72857	46		
15	.67237	.90834	1.1009	.74022	45	15	.68518	.94071	1.0630	.72837	45	15	.68518	.94071	1.0630	.72837	45		
16	.67258	.90887	1.1003	.74002	44	16	.68539	.94125	1.0624	.72817	44	16	.68539	.94125	1.0624	.72817	44		
17	.67280	.90940	1.0996	.73983	43	17	.68561	.94180	1.0618	.72797	43	17	.68561	.94180	1.0618	.72797	43		
18	.67301	.90993	1.0990	.73963	42	18	.68582	.94235	1.0612	.72777	42	18	.68582	.94235	1.0612	.72777	42		
19	.67323	.91046	1.0983	.73944	41	19	.68603	.94290	1.0606	.72757	41	19	.68603	.94290	1.0606	.72757	41		
20	.67344	.91099	1.0977	.73924	40	20	.68624	.94345	1.0599	.72737	40	20	.68624	.94345	1.0599	.72737	40		
21	.67366	.91153	1.0971	.73904	39	21	.68645	.94400	1.0593	.72717	39	21	.68645	.94400	1.0593	.72717	39		
22	.67387	.91206	1.0964	.73885	38	22	.68666	.94455	1.0587	.72697	38	22	.68666	.94455	1.0587	.72697	38		
23	.67409	.91259	1.0958	.73865	37	23	.68688	.94510	1.0581	.72677	37	23	.68688	.94510	1.0581	.72677	37		
24	.67430	.91313	1.0951	.73846	36	24	.68709	.94565	1.0575	.72657	36	24	.68709	.94565	1.0575	.72657	36		
25	.67452	.91366	1.0945	.73826	35	25	.68730	.94620	1.0569	.72637	35	25	.68730	.94620	1.0569	.72637	35		
26	.67473	.91419	1.0939	.73806	34	26	.68751	.94675	1.0562	.72617	34	26	.68751	.94675	1.0562	.72617	34		
27	.67495	.91473	1.0932	.73787	33	27	.68772	.94731	1.0556	.72597	33	27	.68772	.94731	1.0556	.72597	33		
28	.67516	.91526	1.0926	.73767	32	28	.68793	.94786	1.0550	.72577	32	28	.68793	.94786	1.0550	.72577	32		
29	.67538	.91580	1.0919	.73747	31	29	.68814	.94841	1.0544	.72557	31	29	.68814	.94841	1.0544	.72557	31		
30	.67559	.91633	1.0913	.73728	30	30	.68835	.94896	1.0538	.72537	30	30	.68835	.94896	1.0538	.72537	30		
31	.67580	.91687	1.0907	.73708	29	31	.68857	.94952	1.0532	.72517	29	31	.68857	.94952	1.0532	.72517	29		
32	.67602	.91740	1.0900	.73688	28	32	.68878	.95007	1.0526	.72497	28	32	.68878	.95007	1.0526	.72497	28		
33	.67623	.91794	1.0894	.73669	27	33	.68899	.95062	1.0519	.72477	27	33	.68899	.95062	1.0519	.72477	27		
34	.67645	.91847	1.0888	.73649	26	34	.68920	.95118	1.0513	.72457	26	34	.68920	.95118	1.0513	.72457	26		
35	.67666	.91901	1.0881	.73629	25	35	.68941	.95173	1.0507	.72437	25	35	.68941	.95173	1.0507	.72437	25		
36	.67688	.91955	1.0875	.73610	24	36	.68962	.95229	1.0501	.72417	24	36	.68962	.95229	1.0501	.72417	24		
37	.67709	.92008	1.0869	.73590	23	37	.68983	.95284	1.0495	.72397	23	37	.68983	.95284	1.0495	.72397	23		
38	.67730	.92062	1.0862	.73570	22	38	.69004	.95340	1.0489	.72377	22	38	.69004	.95340	1.0489	.72377	22		
39	.67752	.92116	1.0856	.73551	21	39	.69025	.95395	1.0483	.72357	21	39	.69025	.95395	1.0483	.72357	21		
40	.67773	.92170	1.0850	.73531	20	40	.69046	.95451	1.0477	.72337	20	40	.69046	.95451	1.0477	.72337	20		
41	.67795	.92224	1.0843	.73511	19	41	.69067	.95506	1.0470	.72317	19	41	.69067	.95506	1.0470	.72317	19		
42	.67816	.92277	1.0837	.73491	18	42	.69088	.95562	1.0464	.72297	18	42	.69088	.95562	1.0464	.72297	18		
43	.67837	.92331	1.0831	.73472	17	43	.69109	.95618	1.0458	.72277	17	43	.69109	.95618	1.0458	.72277	17		
44	.67859	.92385	1.0824	.73452	16	44	.69130	.95673	1.0452	.72257	16	44	.69130	.95673	1.0452	.72257	16		
45	.67880	.92439	1.0818	.73432	15	45	.69151	.95729	1.0446	.72236	15	45	.69151	.95729	1.0446	.72236	15		
46	.67901	.92493	1.0812	.73413	14	46	.69172	.95785	1.0440	.72216	14	46	.69172	.95785	1.0440	.72216	14		
47	.67923	.92547	1.0805	.73393	13	47	.69193	.95841	1.0434	.72196	13	47	.69193	.95841	1.0434	.72196	13		
48	.67944	.92601	1.0799	.73373	12	48	.69214	.95897	1.0428	.72176	12	48	.69214	.95897	1.0428	.72176	12		
49	.67965	.92655	1.0793	.73353	11	49	.69235	.95952	1.0422	.72156	11	49	.69235	.95952	1.0422	.72156	11		
50	.67987	.92709	1.0786	.73333	10	50	.69256	.96008	1.0416	.72136	10	50	.69256	.96008	1.0416	.72136	10		
51	.68008	.92763	1.0780	.73314	9	51	.69277	.96064	1.0410	.72116	9	51	.69277	.96064	1.0410	.72116	9		
52	.68029	.92817	1.0774	.73294	8	52	.69298	.96120	1.0404	.72095	8	52	.69298	.96120	1.0404	.72095	8		
53	.68051	.92872	1.0768	.73274	7	53	.69319	.96175	1.0398	.72075	7	53	.69319	.96175	1.0398	.72075	7		
54	.68072	.92926	1.0761	.73254	6	54	.69340	.96232	1.0392	.72055	6	54	.69340	.96232	1.0392	.72055	6		
55	.68093	.92980	1.0755	.73234	5	55	.69361	.96288	1.0386	.72035	5	55	.69361	.96288	1.0386	.72035	5		
56	.68115	.93034	1.0749	.73215	4	56	.69382	.96344	1.0379	.72015	4	56	.69382	.96344	1.0379	.72015	4		
57	.68136	.93088	1.0742	.73195	3	57	.69403	.96400	1.0373	.71995	3	57	.69403	.96400	1.0373	.71995	3		
58	.68157	.93143	1.0736	.73175	2	58	.69424	.96457	1.0367	.71974	2	58	.69424	.96457	1.0367	.71974	2		
59	.68179	.93197	1.0730	.73155	1	59	.69445	.96513	1.0361	.71954	1	59	.69445	.96513	1.0361	.71954	1		
60	.68200	.93252	1.0724	.73135	0	60	.69466	.96569	1.0355	.71934	0	60	.69466	.96569	1.0355	.71934	0		

132° (312°)

(227°) 47°

133° (313°)

(226°) 46°

Table 7 NATURAL TRIGONOMETRIC FUNCTIONS (continued)

44° (224°)		(315°) 135°			
	Sin	Tan	Ctn	Cos	
0	.69466	.95569	1.0355	.71934	60
1	.69487	.95625	1.0349	.71914	59
2	.69508	.95681	1.0343	.71894	58
3	.69529	.95738	1.0337	.71873	57
4	.69549	.95794	1.0331	.71853	56
5	.69570	.95850	1.0325	.71833	55
6	.69591	.95907	1.0319	.71813	54
7	.69612	.95963	1.0313	.71792	53
8	.69633	.96020	1.0307	.71772	52
9	.69654	.96076	1.0301	.71752	51
10	.69675	.97133	1.0295	.71732	50
11	.69696	.97189	1.0289	.71711	49
12	.69717	.97246	1.0283	.71691	48
13	.69737	.97302	1.0277	.71671	47
14	.69758	.97359	1.0271	.71650	46
15	.69779	.97415	1.0265	.71630	45
16	.69800	.97472	1.0259	.71610	44
17	.69821	.97528	1.0253	.71590	43
18	.69842	.97586	1.0247	.71569	42
19	.69862	.97643	1.0241	.71549	41
20	.69883	.97700	1.0235	.71529	40
21	.69904	.97756	1.0230	.71508	39
22	.69925	.97813	1.0224	.71488	38
23	.69946	.97870	1.0218	.71468	37
24	.69966	.97927	1.0212	.71447	36
25	.69987	.97984	1.0206	.71427	35
26	.70008	.98041	1.0200	.71407	34
27	.70029	.98098	1.0194	.71386	33
28	.70049	.98155	1.0188	.71366	32
29	.70070	.98213	1.0182	.71345	31
30	.70091	.98270	1.0176	.71325	30
31	.70112	.98327	1.0170	.71305	29
32	.70132	.98384	1.0164	.71284	28
33	.70153	.98441	1.0158	.71264	27
34	.70174	.98499	1.0152	.71243	26
35	.70195	.98556	1.0147	.71223	25
36	.70215	.98613	1.0141	.71203	24
37	.70236	.98671	1.0135	.71182	23
38	.70257	.98728	1.0129	.71162	22
39	.70277	.98786	1.0123	.71141	21
40	.70298	.98843	1.0117	.71121	20
41	.70319	.98901	1.0111	.71100	19
42	.70339	.98958	1.0105	.71080	18
43	.70360	.99016	1.0099	.71059	17
44	.70381	.99073	1.0094	.71039	16
45	.70401	.99131	1.0088	.71019	15
46	.70422	.99189	1.0082	.70998	14
47	.70443	.99247	1.0076	.70978	13
48	.70463	.99304	1.0070	.70957	12
49	.70484	.99362	1.0064	.70937	11
50	.70505	.99420	1.0058	.70916	10
51	.70525	.99478	1.0052	.70896	9
52	.70546	.99536	1.0047	.70875	8
53	.70567	.99594	1.0041	.70855	7
54	.70587	.99652	1.0035	.70834	6
55	.70608	.99710	1.0029	.70813	5
56	.70628	.99768	1.0023	.70793	4
57	.70649	.99826	1.0017	.70772	3
58	.70670	.99884	1.0012	.70752	2
59	.70690	.99942	1.0006	.70731	1
60	.70711	1.0000	1.0000	.70711	0
	Cos	Ctn	Tan	Sin	

134° (314°)

(225°) 45°

Table 8 NATURAL SECANTS AND COSECANTS

0° (180°)			(359°) 179°			1° (181°)			(358°) 178°			2° (182°)			(357°) 177°		
'	Sec	Csc	'	Sec	Csc	'	Sec	Csc	'	Sec	Csc	'	Sec	Csc	'	Sec	Csc
0	1.0000		80			0	1.0002	97.299	60			0	1.0006	28.654	80		
1	1.0000	3437.7	59			1	1.0002	95.359	59			1	1.0006	28.417	59		
2	1.0000	1718.9	58			2	1.0002	95.451	58			2	1.0006	28.184	58		
3	1.0000	1145.9	57			3	1.0002	94.570	57			3	1.0006	27.955	57		
4	1.0000	859.44	56			4	1.0002	93.718	56			4	1.0007	27.730	56		
5	1.0000	687.55	55			5	1.0002	92.892	55			5	1.0007	27.508	55		
6	1.0000	572.96	54			6	1.0002	92.090	54			6	1.0007	27.290	54		
7	1.0000	491.11	53			7	1.0002	91.313	53			7	1.0007	27.075	53		
8	1.0000	429.72	52			8	1.0002	90.558	52			8	1.0007	26.864	52		
9	1.0000	381.97	51			9	1.0002	49.826	51			9	1.0007	26.655	51		
10	1.0000	343.79	50			10	1.0002	49.114	50			10	1.0007	26.451	50		
11	1.0000	312.92	49			11	1.0002	48.422	49			11	1.0007	26.246	49		
12	1.0000	285.48	48			12	1.0002	47.750	48			12	1.0007	26.050	48		
13	1.0000	264.44	47			13	1.0002	47.096	47			13	1.0007	25.854	47		
14	1.0000	245.55	46			14	1.0002	46.460	46			14	1.0008	25.651	46		
15	1.0000	229.18	45			15	1.0002	45.840	45			15	1.0008	25.471	45		
16	1.0000	214.95	44			16	1.0002	45.237	44			16	1.0008	25.284	44		
17	1.0000	202.23	43			17	1.0002	44.650	43			17	1.0008	25.100	43		
18	1.0000	190.56	42			18	1.0002	44.077	42			18	1.0008	24.918	42		
19	1.0000	180.93	41			19	1.0002	43.520	41			19	1.0008	24.739	41		
20	1.0000	171.89	40			20	1.0003	42.976	40			20	1.0008	24.562	40		
21	1.0000	163.70	39			21	1.0003	42.445	39			21	1.0008	24.388	39		
22	1.0000	156.26	38			22	1.0003	41.928	38			22	1.0009	24.216	38		
23	1.0000	149.47	37			23	1.0003	41.423	37			23	1.0009	24.047	37		
24	1.0000	143.24	36			24	1.0003	40.930	36			24	1.0009	23.880	36		
25	1.0000	137.51	35			25	1.0003	40.448	35			25	1.0009	23.716	35		
26	1.0000	132.22	34			26	1.0003	39.978	34			26	1.0009	23.553	34		
27	1.0000	127.33	33			27	1.0003	39.519	33			27	1.0009	23.393	33		
28	1.0000	122.78	32			28	1.0003	39.070	32			28	1.0009	23.235	32		
29	1.0000	118.54	31			29	1.0003	38.631	31			29	1.0009	23.079	31		
30	1.0000	114.59	30			30	1.0003	38.202	30			30	1.0010	22.926	30		
31	1.0000	110.90	29			31	1.0004	37.782	29			31	1.0010	22.774	29		
32	1.0000	107.43	28			32	1.0004	37.371	28			32	1.0010	22.624	28		
33	1.0000	104.18	27			33	1.0004	36.970	27			33	1.0010	22.476	27		
34	1.0000	101.11	26			34	1.0004	36.576	26			34	1.0010	22.330	26		
35	1.0001	98.222	25			35	1.0004	36.191	25			35	1.0010	22.187	25		
36	1.0001	95.495	24			36	1.0004	35.815	24			36	1.0010	22.044	24		
37	1.0001	92.914	23			37	1.0004	35.445	23			37	1.0010	21.904	23		
38	1.0001	90.469	22			38	1.0004	35.084	22			38	1.0011	21.766	22		
39	1.0001	88.149	21			39	1.0004	34.730	21			39	1.0011	21.629	21		
40	1.0001	85.946	20			40	1.0004	34.382	20			40	1.0011	21.494	20		
41	1.0001	83.849	19			41	1.0004	34.042	19			41	1.0011	21.360	19		
42	1.0001	81.853	18			42	1.0004	33.708	18			42	1.0011	21.229	18		
43	1.0001	79.950	17			43	1.0004	33.381	17			43	1.0011	21.098	17		
44	1.0001	78.133	16			44	1.0005	33.060	16			44	1.0011	20.970	16		
45	1.0001	76.397	15			45	1.0005	32.746	15			45	1.0012	20.843	15		
46	1.0001	74.736	14			46	1.0005	32.437	14			46	1.0012	20.717	14		
47	1.0001	73.146	13			47	1.0005	32.134	13			47	1.0012	20.593	13		
48	1.0001	71.622	12			48	1.0005	31.836	12			48	1.0012	20.471	12		
49	1.0001	70.160	11			49	1.0006	31.544	11			49	1.0012	20.350	11		
50	1.0001	68.757	10			50	1.0006	31.258	10			50	1.0012	20.230	10		
51	1.0001	67.409	9			51	1.0006	30.976	9			51	1.0012	20.112	9		
52	1.0001	66.113	8			52	1.0006	30.700	8			52	1.0013	19.995	8		
53	1.0001	64.868	7			53	1.0006	30.428	7			53	1.0013	19.880	7		
54	1.0001	63.665	6			54	1.0006	30.161	6			54	1.0013	19.766	6		
55	1.0001	62.507	5			55	1.0006	29.899	5			55	1.0013	19.653	5		
56	1.0001	61.391	4			56	1.0006	29.641	4			56	1.0013	19.541	4		
57	1.0001	60.314	3			57	1.0006	29.388	3			57	1.0013	19.431	3		
58	1.0001	59.274	2			58	1.0006	29.139	2			58	1.0013	19.322	2		
59	1.0001	58.270	1			59	1.0006	28.894	1			59	1.0014	19.214	1		
60	1.0002	57.299	0			60	1.0006	28.654	0			60	1.0014	19.107	0		
'	Csc	Sec	'	'	'	'	Csc	Sec	'	'	'	'	Csc	Sec	'	'	'

Table 8 NATURAL SECANTS AND COSECANTS (continued)

3° (183°)			(356°) 176°			4° (184°)			(355°) 175°			5° (185°)			(354°) 174°		
/'	Sec	Csc	/'	Sec	Csc	/'	Sec	Csc	/'	Sec	Csc	/'	Sec	Csc	/'	Sec	Csc
0	1.0014	19.107	60	1.0024	14.336	60	1.0038	11.474	60	1.0038	11.474	60	1.0038	11.474	60	1.0038	11.474
1	1.0014	19.002	59	1.0025	14.278	59	1.0038	11.436	59	1.0038	11.436	59	1.0038	11.436	59	1.0038	11.436
2	1.0014	18.898	58	1.0025	14.217	58	1.0039	11.398	58	1.0039	11.398	58	1.0039	11.398	58	1.0039	11.398
3	1.0014	18.794	57	1.0025	14.159	57	1.0039	11.360	57	1.0039	11.360	57	1.0039	11.360	57	1.0039	11.360
4	1.0014	18.692	56	1.0025	14.101	56	1.0039	11.323	56	1.0039	11.323	56	1.0039	11.323	56	1.0039	11.323
5	1.0014	18.591	55	1.0025	14.044	55	1.0039	11.286	55	1.0039	11.286	55	1.0039	11.286	55	1.0039	11.286
6	1.0015	18.492	54	1.0026	13.987	54	1.0040	11.249	54	1.0040	11.249	54	1.0040	11.249	54	1.0040	11.249
7	1.0015	18.393	53	1.0026	13.930	53	1.0040	11.213	53	1.0040	11.213	53	1.0040	11.213	53	1.0040	11.213
8	1.0015	18.295	52	1.0026	13.874	52	1.0040	11.178	52	1.0040	11.178	52	1.0040	11.178	52	1.0040	11.178
9	1.0015	18.198	51	1.0026	13.818	51	1.0041	11.140	51	1.0041	11.140	51	1.0041	11.140	51	1.0041	11.140
10	1.0015	18.103	50	1.0027	13.763	50	1.0041	11.105	50	1.0041	11.105	50	1.0041	11.105	50	1.0041	11.105
11	1.0015	18.008	49	1.0027	13.708	49	1.0041	11.069	49	1.0041	11.069	49	1.0041	11.069	49	1.0041	11.069
12	1.0016	17.914	48	1.0027	13.654	48	1.0041	11.034	48	1.0041	11.034	48	1.0041	11.034	48	1.0041	11.034
13	1.0016	17.822	47	1.0027	13.600	47	1.0042	10.998	47	1.0042	10.998	47	1.0042	10.998	47	1.0042	10.998
14	1.0016	17.730	46	1.0027	13.547	46	1.0042	10.963	46	1.0042	10.963	46	1.0042	10.963	46	1.0042	10.963
15	1.0016	17.639	45	1.0028	13.494	45	1.0042	10.929	45	1.0042	10.929	45	1.0042	10.929	45	1.0042	10.929
16	1.0016	17.549	44	1.0028	13.441	44	1.0042	10.894	44	1.0042	10.894	44	1.0042	10.894	44	1.0042	10.894
17	1.0016	17.460	43	1.0028	13.389	43	1.0043	10.860	43	1.0043	10.860	43	1.0043	10.860	43	1.0043	10.860
18	1.0017	17.372	42	1.0028	13.337	42	1.0043	10.826	42	1.0043	10.826	42	1.0043	10.826	42	1.0043	10.826
19	1.0017	17.285	41	1.0028	13.286	41	1.0043	10.792	41	1.0043	10.792	41	1.0043	10.792	41	1.0043	10.792
20	1.0017	17.198	40	1.0029	13.235	40	1.0043	10.758	40	1.0043	10.758	40	1.0043	10.758	40	1.0043	10.758
21	1.0017	17.113	39	1.0029	13.184	39	1.0044	10.725	39	1.0044	10.725	39	1.0044	10.725	39	1.0044	10.725
22	1.0017	17.028	38	1.0029	13.134	38	1.0044	10.692	38	1.0044	10.692	38	1.0044	10.692	38	1.0044	10.692
23	1.0017	16.945	37	1.0029	13.084	37	1.0044	10.659	37	1.0044	10.659	37	1.0044	10.659	37	1.0044	10.659
24	1.0018	16.862	36	1.0030	13.035	36	1.0045	10.626	36	1.0045	10.626	36	1.0045	10.626	36	1.0045	10.626
25	1.0018	16.779	35	1.0030	12.985	35	1.0045	10.593	35	1.0045	10.593	35	1.0045	10.593	35	1.0045	10.593
26	1.0018	16.698	34	1.0030	12.937	34	1.0045	10.561	34	1.0045	10.561	34	1.0045	10.561	34	1.0045	10.561
27	1.0018	16.618	33	1.0030	12.888	33	1.0045	10.529	33	1.0045	10.529	33	1.0045	10.529	33	1.0045	10.529
28	1.0018	16.538	32	1.0030	12.840	32	1.0045	10.497	32	1.0045	10.497	32	1.0045	10.497	32	1.0045	10.497
29	1.0019	16.459	31	1.0031	12.793	31	1.0045	10.465	31	1.0045	10.465	31	1.0045	10.465	31	1.0045	10.465
30	1.0019	16.380	30	1.0031	12.745	30	1.0046	10.433	30	1.0046	10.433	30	1.0046	10.433	30	1.0046	10.433
31	1.0019	16.303	29	1.0031	12.699	29	1.0047	10.402	29	1.0047	10.402	29	1.0047	10.402	29	1.0047	10.402
32	1.0019	16.226	28	1.0031	12.652	28	1.0047	10.371	28	1.0047	10.371	28	1.0047	10.371	28	1.0047	10.371
33	1.0019	16.150	27	1.0032	12.606	27	1.0047	10.340	27	1.0047	10.340	27	1.0047	10.340	27	1.0047	10.340
34	1.0019	16.075	26	1.0032	12.560	26	1.0047	10.309	26	1.0047	10.309	26	1.0047	10.309	26	1.0047	10.309
35	1.0020	16.000	25	1.0032	12.514	25	1.0048	10.278	25	1.0048	10.278	25	1.0048	10.278	25	1.0048	10.278
36	1.0020	15.926	24	1.0032	12.469	24	1.0048	10.248	24	1.0048	10.248	24	1.0048	10.248	24	1.0048	10.248
37	1.0020	15.853	23	1.0033	12.424	23	1.0048	10.217	23	1.0048	10.217	23	1.0048	10.217	23	1.0048	10.217
38	1.0020	15.780	22	1.0033	12.379	22	1.0049	10.187	22	1.0049	10.187	22	1.0049	10.187	22	1.0049	10.187
39	1.0020	15.708	21	1.0033	12.335	21	1.0049	10.157	21	1.0049	10.157	21	1.0049	10.157	21	1.0049	10.157
40	1.0021	15.637	20	1.0033	12.291	20	1.0049	10.128	20	1.0049	10.128	20	1.0049	10.128	20	1.0049	10.128
41	1.0021	15.566	19	1.0034	12.248	19	1.0049	10.098	19	1.0049	10.098	19	1.0049	10.098	19	1.0049	10.098
42	1.0021	15.496	18	1.0034	12.204	18	1.0050	10.068	18	1.0050	10.068	18	1.0050	10.068	18	1.0050	10.068
43	1.0021	15.427	17	1.0034	12.161	17	1.0050	10.039	17	1.0050	10.039	17	1.0050	10.039	17	1.0050	10.039
44	1.0021	15.358	16	1.0034	12.119	16	1.0050	10.010	16	1.0050	10.010	16	1.0050	10.010	16	1.0050	10.010
45	1.0021	15.290	15	1.0034	12.076	15	1.0051	9.9812	15	1.0051	9.9812	15	1.0051	9.9812	15	1.0051	9.9812
46	1.0022	15.222	14	1.0035	12.034	14	1.0051	9.9525	14	1.0051	9.9525	14	1.0051	9.9525	14	1.0051	9.9525
47	1.0022	15.155	13	1.0035	11.992	13	1.0051	9.9239	13	1.0051	9.9239	13	1.0051	9.9239	13	1.0051	9.9239
48	1.0022	15.089	12	1.0035	11.951	12	1.0051	9.8955	12	1.0051	9.8955	12	1.0051	9.8955	12	1.0051	9.8955
49	1.0022	15.023	11	1.0035	11.909	11	1.0052	9.8672	11	1.0052	9.8672	11	1.0052	9.8672	11	1.0052	9.8672
50	1.0022	14.958	10	1.0036	11.868	10	1.0052	9.8391	10	1.0052	9.8391	10	1.0052	9.8391	10	1.0052	9.8391
51	1.0023	14.893	9	1.0036	11.826	9	1.0052	9.8112	9	1.0052	9.8112	9	1.0052	9.8112	9	1.0052	9.8112
52	1.0023	14.829	8	1.0036	11.787	8	1.0053	9.7834	8	1.0053	9.7834	8	1.0053	9.7834	8	1.0053	9.7834
53	1.0023	14.766	7	1.0036	11.747	7	1.0053	9.7558	7	1.0053	9.7558	7	1.0053	9.7558	7	1.0053	9.7558
54	1.0023	14.703	6	1.0037	11.707	6	1.0053	9.7283	6	1.0053	9.7283	6	1.0053	9.7283	6	1.0053	9.7283
55	1.0023	14.640	5	1.0037	11.668	5	1.0054	9.7010	5	1.0054	9.7010	5	1.0054	9.7010	5	1.0054	9.7010
56	1.0024	14.578	4	1.0037	11.628	4	1.0054	9.6739	4	1.0054	9.6739	4	1.0054	9.6739	4	1.0054	9.6739
57	1.0024	14.517	3	1.0037	11.589	3	1.0054	9.6469	3	1.0054	9.6469	3	1.0054	9.6469	3	1.0054	9.6469
58	1.0024	14.456	2	1.0038	11.551	2	1.0054	9.6200	2	1.0054	9.6200	2	1.0054	9.6200	2	1.0054	9.6200
59	1.0024	14.395	1	1.0038	11.512	1	1.0055	9.5933	1	1.0055	9.5933	1	1.0055	9.5933	1	1.0055	9.5933
60	1.0024	14.336	0	1.0038	11.474	0	1.0055	9.5668	0	1.0055	9.5668	0	1.0055	9.5668	0	1.0055	9.5668
/'	Csc	Sec	/'	Csc	Sec	/'	Csc	Sec	/'	Csc	Sec	/'	Csc	Sec	/'	Csc	Sec

93° (273°) (266°) 86° 94° (274°) (265°) 85° 95° (275°) (264°) 84°

Table 8 NATURAL SECANTS AND COSECANTS (continued)

6° (186°)			(353°) 173°			7° (187°)			(352°) 172°			8° (188°)			(351°) 171°		
	Sec	Csc		Sec	Csc		Sec	Csc		Sec	Csc		Sec	Csc		Sec	Csc
0	1.0055	9.5568	60	1.0075	8.2055	60	1.0098	7.1853	60	1.0098	7.1853	60	1.0098	7.1853	60	1.0098	7.1853
1	1.0056	9.5404	59	1.0076	8.1861	59	1.0099	7.1705	59	1.0099	7.1705	59	1.0099	7.1705	59	1.0099	7.1705
2	1.0056	9.5141	58	1.0076	8.1658	58	1.0099	7.1557	58	1.0099	7.1557	58	1.0099	7.1557	58	1.0099	7.1557
3	1.0056	9.4880	57	1.0076	8.1476	57	1.0100	7.1410	57	1.0100	7.1410	57	1.0100	7.1410	57	1.0100	7.1410
4	1.0056	9.4620	56	1.0077	8.1285	56	1.0100	7.1263	56	1.0100	7.1263	56	1.0100	7.1263	56	1.0100	7.1263
5	1.0057	9.4362	55	1.0077	8.1095	55	1.0100	7.1117	55	1.0100	7.1117	55	1.0100	7.1117	55	1.0100	7.1117
6	1.0057	9.4105	54	1.0077	8.0906	54	1.0101	7.0972	54	1.0101	7.0972	54	1.0101	7.0972	54	1.0101	7.0972
7	1.0057	9.3850	53	1.0078	8.0717	53	1.0101	7.0827	53	1.0101	7.0827	53	1.0101	7.0827	53	1.0101	7.0827
8	1.0058	9.3596	52	1.0078	8.0529	52	1.0102	7.0683	52	1.0102	7.0683	52	1.0102	7.0683	52	1.0102	7.0683
9	1.0058	9.3343	51	1.0078	8.0342	51	1.0102	7.0539	51	1.0102	7.0539	51	1.0102	7.0539	51	1.0102	7.0539
10	1.0058	9.3092	50	1.0079	8.0156	50	1.0102	7.0396	50	1.0102	7.0396	50	1.0102	7.0396	50	1.0102	7.0396
11	1.0059	9.2842	49	1.0079	7.9971	49	1.0103	7.0254	49	1.0103	7.0254	49	1.0103	7.0254	49	1.0103	7.0254
12	1.0059	9.2593	48	1.0079	7.9787	48	1.0103	7.0112	48	1.0103	7.0112	48	1.0103	7.0112	48	1.0103	7.0112
13	1.0059	9.2346	47	1.0080	7.9604	47	1.0104	6.9971	47	1.0104	6.9971	47	1.0104	6.9971	47	1.0104	6.9971
14	1.0059	9.2100	46	1.0080	7.9422	46	1.0104	6.9830	46	1.0104	6.9830	46	1.0104	6.9830	46	1.0104	6.9830
15	1.0060	9.1855	45	1.0081	7.9240	45	1.0105	6.9690	45	1.0105	6.9690	45	1.0105	6.9690	45	1.0105	6.9690
16	1.0060	9.1612	44	1.0081	7.9059	44	1.0105	6.9550	44	1.0105	6.9550	44	1.0105	6.9550	44	1.0105	6.9550
17	1.0060	9.1370	43	1.0081	7.8879	43	1.0105	6.9411	43	1.0105	6.9411	43	1.0105	6.9411	43	1.0105	6.9411
18	1.0061	9.1129	42	1.0082	7.8700	42	1.0106	6.9273	42	1.0106	6.9273	42	1.0106	6.9273	42	1.0106	6.9273
19	1.0061	9.0890	41	1.0082	7.8522	41	1.0106	6.9135	41	1.0106	6.9135	41	1.0106	6.9135	41	1.0106	6.9135
20	1.0061	9.0652	40	1.0082	7.8344	40	1.0107	6.8998	40	1.0107	6.8998	40	1.0107	6.8998	40	1.0107	6.8998
21	1.0062	9.0415	39	1.0083	7.8166	39	1.0107	6.8861	39	1.0107	6.8861	39	1.0107	6.8861	39	1.0107	6.8861
22	1.0062	9.0179	38	1.0083	7.7992	38	1.0108	6.8725	38	1.0108	6.8725	38	1.0108	6.8725	38	1.0108	6.8725
23	1.0062	8.9944	37	1.0084	7.7817	37	1.0108	6.8589	37	1.0108	6.8589	37	1.0108	6.8589	37	1.0108	6.8589
24	1.0063	8.9711	36	1.0084	7.7642	36	1.0108	6.8454	36	1.0108	6.8454	36	1.0108	6.8454	36	1.0108	6.8454
25	1.0063	8.9479	35	1.0084	7.7468	35	1.0109	6.8320	35	1.0109	6.8320	35	1.0109	6.8320	35	1.0109	6.8320
26	1.0063	8.9248	34	1.0085	7.7296	34	1.0109	6.8186	34	1.0109	6.8186	34	1.0109	6.8186	34	1.0109	6.8186
27	1.0064	8.9019	33	1.0085	7.7124	33	1.0110	6.8053	33	1.0110	6.8053	33	1.0110	6.8053	33	1.0110	6.8053
28	1.0064	8.8790	32	1.0086	7.6953	32	1.0110	6.7919	32	1.0110	6.7919	32	1.0110	6.7919	32	1.0110	6.7919
29	1.0064	8.8563	31	1.0086	7.6783	31	1.0111	6.7787	31	1.0111	6.7787	31	1.0111	6.7787	31	1.0111	6.7787
30	1.0065	8.8337	30	1.0086	7.6613	30	1.0111	6.7655	30	1.0111	6.7655	30	1.0111	6.7655	30	1.0111	6.7655
31	1.0065	8.8112	29	1.0087	7.6444	29	1.0112	6.7523	29	1.0112	6.7523	29	1.0112	6.7523	29	1.0112	6.7523
32	1.0065	8.7888	28	1.0087	7.6276	28	1.0112	6.7392	28	1.0112	6.7392	28	1.0112	6.7392	28	1.0112	6.7392
33	1.0066	8.7665	27	1.0087	7.6109	27	1.0112	6.7262	27	1.0112	6.7262	27	1.0112	6.7262	27	1.0112	6.7262
34	1.0066	8.7444	26	1.0088	7.5942	26	1.0113	6.7132	26	1.0113	6.7132	26	1.0113	6.7132	26	1.0113	6.7132
35	1.0066	8.7223	25	1.0088	7.5776	25	1.0113	6.7003	25	1.0113	6.7003	25	1.0113	6.7003	25	1.0113	6.7003
36	1.0067	8.7004	24	1.0089	7.5611	24	1.0114	6.6874	24	1.0114	6.6874	24	1.0114	6.6874	24	1.0114	6.6874
37	1.0067	8.6786	23	1.0089	7.5446	23	1.0114	6.6745	23	1.0114	6.6745	23	1.0114	6.6745	23	1.0114	6.6745
38	1.0067	8.6569	22	1.0089	7.5282	22	1.0115	6.6616	22	1.0115	6.6616	22	1.0115	6.6616	22	1.0115	6.6616
39	1.0068	8.6353	21	1.0090	7.5119	21	1.0115	6.6490	21	1.0115	6.6490	21	1.0115	6.6490	21	1.0115	6.6490
40	1.0068	8.6138	20	1.0090	7.4957	20	1.0116	6.6363	20	1.0116	6.6363	20	1.0116	6.6363	20	1.0116	6.6363
41	1.0068	8.5924	19	1.0091	7.4795	19	1.0116	6.6237	19	1.0116	6.6237	19	1.0116	6.6237	19	1.0116	6.6237
42	1.0069	8.5711	18	1.0091	7.4635	18	1.0116	6.6111	18	1.0116	6.6111	18	1.0116	6.6111	18	1.0116	6.6111
43	1.0069	8.5500	17	1.0091	7.4474	17	1.0117	6.5986	17	1.0117	6.5986	17	1.0117	6.5986	17	1.0117	6.5986
44	1.0069	8.5289	16	1.0092	7.4315	16	1.0117	6.5861	16	1.0117	6.5861	16	1.0117	6.5861	16	1.0117	6.5861
45	1.0070	8.5079	15	1.0092	7.4156	15	1.0118	6.5736	15	1.0118	6.5736	15	1.0118	6.5736	15	1.0118	6.5736
46	1.0070	8.4871	14	1.0092	7.3998	14	1.0118	6.5612	14	1.0118	6.5612	14	1.0118	6.5612	14	1.0118	6.5612
47	1.0070	8.4663	13	1.0093	7.3840	13	1.0119	6.5489	13	1.0119	6.5489	13	1.0119	6.5489	13	1.0119	6.5489
48	1.0071	8.4457	12	1.0093	7.3684	12	1.0119	6.5366	12	1.0119	6.5366	12	1.0119	6.5366	12	1.0119	6.5366
49	1.0071	8.4251	11	1.0094	7.3527	11	1.0120	6.5243	11	1.0120	6.5243	11	1.0120	6.5243	11	1.0120	6.5243
50	1.0072	8.4047	10	1.0094	7.3372	10	1.0120	6.5121	10	1.0120	6.5121	10	1.0120	6.5121	10	1.0120	6.5121
51	1.0072	8.3843	9	1.0095	7.3217	9	1.0120	6.4999	9	1.0120	6.4999	9	1.0120	6.4999	9	1.0120	6.4999
52	1.0072	8.3641	8	1.0095	7.3063	8	1.0121	6.4878	8	1.0121	6.4878	8	1.0121	6.4878	8	1.0121	6.4878
53	1.0073	8.3439	7	1.0095	7.2909	7	1.0121	6.4757	7	1.0121	6.4757	7	1.0121	6.4757	7	1.0121	6.4757
54	1.0073	8.3238	6	1.0096	7.2757	6	1.0122	6.4637	6	1.0122	6.4637	6	1.0122	6.4637	6	1.0122	6.4637
55	1.0073	8.3039	5	1.0096	7.2604	5	1.0122	6.4517	5	1.0122	6.4517	5	1.0122	6.4517	5	1.0122	6.4517
56	1.0074	8.2840	4	1.0097	7.2453	4	1.0123	6.4398	4	1.0123	6.4398	4	1.0123	6.4398	4	1.0123	6.4398
57	1.0074	8.2642	3	1.0097	7.2302	3	1.0123	6.4279	3	1.0123	6.4279	3	1.0123	6.4279	3	1.0123	6.4279
58	1.0074	8.2446	2	1.0097	7.2152	2	1.0124	6.4160	2	1.0124	6.4160	2	1.0124	6.4160	2	1.0124	6.4160
59	1.0075	8.2250	1	1.0098	7.2002	1	1.0124	6.4042	1	1.0124	6.4042	1	1.0124	6.4042	1	1.0124	6.4042
60	1.0075	8.2055	0	1.0098	7.1853	0	1.0125	6.3925	0	1.0125	6.3925	0	1.0125	6.3925	0	1.0125	6.3925

96° (276°)

(263°) 83°

97° (277°)

(262°) 82°

98° (278°)

(261°) 81°

Table 3 NATURAL SECANTS AND COSECANTS (continued)

12° (192°)			13° (193°)			14° (194°)		
	Sec	Csc		Sec	Csc		Sec	Csc
0	1.0223	4.8097	0	1.0263	4.4454	0	1.0306	4.1336
1	1.0224	4.8032	1	1.0264	4.4398	1	1.0307	4.1287
2	1.0225	4.7966	2	1.0264	4.4342	2	1.0308	4.1239
3	1.0225	4.7901	3	1.0265	4.4287	3	1.0308	4.1191
4	1.0226	4.7836	4	1.0266	4.4231	4	1.0309	4.1144
5	1.0227	4.7771	5	1.0266	4.4176	5	1.0310	4.1096
6	1.0227	4.7706	6	1.0267	4.4121	6	1.0311	4.1048
7	1.0228	4.7641	7	1.0268	4.4066	7	1.0311	4.1001
8	1.0228	4.7577	8	1.0269	4.4011	8	1.0312	4.0954
9	1.0229	4.7512	9	1.0269	4.3956	9	1.0313	4.0906
10	1.0230	4.7448	10	1.0270	4.3901	10	1.0314	4.0859
11	1.0230	4.7384	11	1.0271	4.3847	11	1.0314	4.0812
12	1.0231	4.7321	12	1.0271	4.3792	12	1.0315	4.0765
13	1.0232	4.7257	13	1.0272	4.3738	13	1.0316	4.0718
14	1.0232	4.7194	14	1.0273	4.3684	14	1.0317	4.0672
15	1.0233	4.7130	15	1.0273	4.3630	15	1.0317	4.0625
16	1.0234	4.7067	16	1.0274	4.3576	16	1.0318	4.0579
17	1.0234	4.7004	17	1.0275	4.3522	17	1.0319	4.0532
18	1.0235	4.6942	18	1.0276	4.3469	18	1.0320	4.0486
19	1.0236	4.6879	19	1.0276	4.3415	19	1.0321	4.0440
20	1.0236	4.6817	20	1.0277	4.3362	20	1.0321	4.0394
21	1.0237	4.6755	21	1.0278	4.3309	21	1.0322	4.0348
22	1.0238	4.6693	22	1.0278	4.3256	22	1.0323	4.0302
23	1.0238	4.6631	23	1.0279	4.3203	23	1.0324	4.0256
24	1.0239	4.6569	24	1.0280	4.3150	24	1.0324	4.0211
25	1.0240	4.6507	25	1.0281	4.3098	25	1.0325	4.0165
26	1.0240	4.6446	26	1.0281	4.3045	26	1.0326	4.0120
27	1.0241	4.6385	27	1.0282	4.2993	27	1.0327	4.0075
28	1.0241	4.6324	28	1.0283	4.2941	28	1.0327	4.0029
29	1.0242	4.6263	29	1.0283	4.2889	29	1.0328	3.9984
30	1.0243	4.6202	30	1.0284	4.2837	30	1.0329	3.9939
31	1.0243	4.6142	31	1.0285	4.2785	31	1.0330	3.9894
32	1.0244	4.6081	32	1.0286	4.2733	32	1.0331	3.9850
33	1.0245	4.6021	33	1.0286	4.2681	33	1.0331	3.9805
34	1.0245	4.5961	34	1.0287	4.2630	34	1.0332	3.9760
35	1.0246	4.5901	35	1.0288	4.2579	35	1.0333	3.9716
36	1.0247	4.5841	36	1.0288	4.2527	36	1.0334	3.9672
37	1.0247	4.5782	37	1.0289	4.2476	37	1.0334	3.9627
38	1.0248	4.5722	38	1.0290	4.2425	38	1.0335	3.9583
39	1.0249	4.5663	39	1.0291	4.2375	39	1.0336	3.9539
40	1.0249	4.5604	40	1.0291	4.2324	40	1.0337	3.9495
41	1.0250	4.5545	41	1.0292	4.2273	41	1.0338	3.9451
42	1.0251	4.5486	42	1.0293	4.2223	42	1.0338	3.9408
43	1.0251	4.5428	43	1.0294	4.2173	43	1.0339	3.9364
44	1.0252	4.5369	44	1.0294	4.2122	44	1.0340	3.9320
45	1.0253	4.5311	45	1.0295	4.2072	45	1.0341	3.9277
46	1.0253	4.5253	46	1.0296	4.2022	46	1.0342	3.9234
47	1.0254	4.5195	47	1.0297	4.1973	47	1.0342	3.9190
48	1.0255	4.5137	48	1.0297	4.1923	48	1.0343	3.9147
49	1.0256	4.5079	49	1.0298	4.1873	49	1.0344	3.9104
50	1.0256	4.5022	50	1.0299	4.1824	50	1.0345	3.9061
51	1.0257	4.4964	51	1.0299	4.1774	51	1.0346	3.9018
52	1.0258	4.4907	52	1.0300	4.1725	52	1.0346	3.8976
53	1.0258	4.4850	53	1.0301	4.1676	53	1.0347	3.8933
54	1.0259	4.4793	54	1.0302	4.1627	54	1.0348	3.8890
55	1.0260	4.4736	55	1.0302	4.1578	55	1.0349	3.8848
56	1.0260	4.4679	56	1.0303	4.1529	56	1.0350	3.8806
57	1.0261	4.4623	57	1.0304	4.1481	57	1.0350	3.8763
58	1.0262	4.4566	58	1.0306	4.1432	58	1.0351	3.8721
59	1.0262	4.4510	59	1.0306	4.1384	59	1.0352	3.8679
60	1.0263	4.4454	60	1.0306	4.1336	60	1.0353	3.8637
	Csc	Sec		Csc	Sec		Csc	Sec

102° (282°)

(257°) 77°

103° (283°)

(256°) 76°

104° (284°)

(255°) 75°

Table 8 NATURAL SECANTS AND COSECANTS (continued)

15° (195°)			(344°) 164°			16° (196°)			(343°) 163°			17° (197°) - (342°) 162°			
'	Sec	Csc	'	Sec	Csc	'	Sec	Csc	'	Sec	Csc	'	Sec	Csc	'
0	1.0353	3.8637	60	1.0403	3.8280	60	1.0457	3.4203	60	1.0457	3.4203	60	1.0457	3.4203	60
1	1.0354	3.8595	59	1.0404	3.8243	59	1.0458	3.4171	59	1.0458	3.4171	59	1.0458	3.4171	59
2	1.0354	3.8553	58	1.0405	3.8206	58	1.0459	3.4138	58	1.0459	3.4138	58	1.0459	3.4138	58
3	1.0355	3.8512	57	1.0406	3.8169	57	1.0460	3.4106	57	1.0460	3.4106	57	1.0460	3.4106	57
4	1.0356	3.8470	56	1.0406	3.8133	56	1.0461	3.4073	56	1.0461	3.4073	56	1.0461	3.4073	56
5	1.0357	3.8428	55	1.0407	3.8097	55	1.0462	3.4041	55	1.0462	3.4041	55	1.0462	3.4041	55
6	1.0358	3.8387	54	1.0408	3.8060	54	1.0463	3.4009	54	1.0463	3.4009	54	1.0463	3.4009	54
7	1.0358	3.8346	53	1.0409	3.8024	53	1.0463	3.3977	53	1.0463	3.3977	53	1.0463	3.3977	53
8	1.0359	3.8304	52	1.0410	3.7988	52	1.0464	3.3945	52	1.0464	3.3945	52	1.0464	3.3945	52
9	1.0360	3.8263	51	1.0411	3.7951	51	1.0465	3.3913	51	1.0465	3.3913	51	1.0465	3.3913	51
10	1.0361	3.8222	50	1.0412	3.7915	50	1.0466	3.3881	50	1.0466	3.3881	50	1.0466	3.3881	50
11	1.0362	3.8181	49	1.0413	3.7879	49	1.0467	3.3849	49	1.0467	3.3849	49	1.0467	3.3849	49
12	1.0363	3.8140	48	1.0413	3.7843	48	1.0468	3.3817	48	1.0468	3.3817	48	1.0468	3.3817	48
13	1.0363	3.8100	47	1.0414	3.7808	47	1.0469	3.3785	47	1.0469	3.3785	47	1.0469	3.3785	47
14	1.0364	3.8059	46	1.0415	3.7772	46	1.0470	3.3754	46	1.0470	3.3754	46	1.0470	3.3754	46
15	1.0365	3.8018	45	1.0415	3.7736	45	1.0471	3.3722	45	1.0471	3.3722	45	1.0471	3.3722	45
16	1.0365	3.7978	44	1.0417	3.7700	44	1.0472	3.3691	44	1.0472	3.3691	44	1.0472	3.3691	44
17	1.0367	3.7937	43	1.0417	3.7664	43	1.0472	3.3659	43	1.0472	3.3659	43	1.0472	3.3659	43
18	1.0367	3.7897	42	1.0419	3.7629	42	1.0474	3.3628	42	1.0474	3.3628	42	1.0474	3.3628	42
19	1.0368	3.7857	41	1.0420	3.7594	41	1.0475	3.3596	41	1.0475	3.3596	41	1.0475	3.3596	41
20	1.0369	3.7817	40	1.0421	3.7559	40	1.0476	3.3565	40	1.0476	3.3565	40	1.0476	3.3565	40
21	1.0370	3.7777	39	1.0421	3.7523	39	1.0477	3.3534	39	1.0477	3.3534	39	1.0477	3.3534	39
22	1.0371	3.7737	38	1.0422	3.7488	38	1.0478	3.3502	38	1.0478	3.3502	38	1.0478	3.3502	38
23	1.0372	3.7697	37	1.0423	3.7453	37	1.0479	3.3471	37	1.0479	3.3471	37	1.0479	3.3471	37
24	1.0372	3.7657	36	1.0424	3.7418	36	1.0480	3.3440	36	1.0480	3.3440	36	1.0480	3.3440	36
25	1.0373	3.7617	35	1.0425	3.7383	35	1.0480	3.3409	35	1.0480	3.3409	35	1.0480	3.3409	35
26	1.0374	3.7577	34	1.0426	3.7348	34	1.0481	3.3378	34	1.0481	3.3378	34	1.0481	3.3378	34
27	1.0375	3.7538	33	1.0427	3.7313	33	1.0482	3.3347	33	1.0482	3.3347	33	1.0482	3.3347	33
28	1.0376	3.7498	32	1.0428	3.7279	32	1.0483	3.3317	32	1.0483	3.3317	32	1.0483	3.3317	32
29	1.0377	3.7459	31	1.0429	3.7244	31	1.0484	3.3286	31	1.0484	3.3286	31	1.0484	3.3286	31
30	1.0377	3.7420	30	1.0429	3.7209	30	1.0485	3.3255	30	1.0485	3.3255	30	1.0485	3.3255	30
31	1.0378	3.7381	29	1.0430	3.7175	29	1.0486	3.3224	29	1.0486	3.3224	29	1.0486	3.3224	29
32	1.0379	3.7341	28	1.0431	3.7140	28	1.0487	3.3194	28	1.0487	3.3194	28	1.0487	3.3194	28
33	1.0380	3.7302	27	1.0432	3.7106	27	1.0488	3.3163	27	1.0488	3.3163	27	1.0488	3.3163	27
34	1.0381	3.7263	26	1.0433	3.7072	26	1.0489	3.3133	26	1.0489	3.3133	26	1.0489	3.3133	26
35	1.0382	3.7225	25	1.0434	3.7037	25	1.0490	3.3102	25	1.0490	3.3102	25	1.0490	3.3102	25
36	1.0382	3.7186	24	1.0435	3.7003	24	1.0491	3.3072	24	1.0491	3.3072	24	1.0491	3.3072	24
37	1.0383	3.7147	23	1.0436	3.6969	23	1.0492	3.3042	23	1.0492	3.3042	23	1.0492	3.3042	23
38	1.0384	3.7108	22	1.0437	3.6935	22	1.0493	3.3012	22	1.0493	3.3012	22	1.0493	3.3012	22
39	1.0385	3.7070	21	1.0438	3.6901	21	1.0494	3.2981	21	1.0494	3.2981	21	1.0494	3.2981	21
40	1.0386	3.7032	20	1.0439	3.6867	20	1.0495	3.2951	20	1.0495	3.2951	20	1.0495	3.2951	20
41	1.0387	3.6992	19	1.0439	3.6833	19	1.0496	3.2921	19	1.0496	3.2921	19	1.0496	3.2921	19
42	1.0388	3.6955	18	1.0440	3.6799	18	1.0497	3.2891	18	1.0497	3.2891	18	1.0497	3.2891	18
43	1.0388	3.6917	17	1.0441	3.6766	17	1.0498	3.2861	17	1.0498	3.2861	17	1.0498	3.2861	17
44	1.0389	3.6879	16	1.0442	3.6732	16	1.0499	3.2831	16	1.0499	3.2831	16	1.0499	3.2831	16
45	1.0390	3.6840	15	1.0443	3.6699	15	1.0500	3.2801	15	1.0500	3.2801	15	1.0500	3.2801	15
46	1.0391	3.6803	14	1.0444	3.6665	14	1.0501	3.2772	14	1.0501	3.2772	14	1.0501	3.2772	14
47	1.0392	3.6765	13	1.0445	3.6632	13	1.0502	3.2742	13	1.0502	3.2742	13	1.0502	3.2742	13
48	1.0393	3.6727	12	1.0446	3.6598	12	1.0503	3.2712	12	1.0503	3.2712	12	1.0503	3.2712	12
49	1.0394	3.6689	11	1.0447	3.6565	11	1.0504	3.2683	11	1.0504	3.2683	11	1.0504	3.2683	11
50	1.0394	3.6652	10	1.0448	3.6532	10	1.0505	3.2653	10	1.0505	3.2653	10	1.0505	3.2653	10
51	1.0395	3.6614	9	1.0449	3.6499	9	1.0506	3.2624	9	1.0506	3.2624	9	1.0506	3.2624	9
52	1.0396	3.6578	8	1.0450	3.6465	8	1.0507	3.2594	8	1.0507	3.2594	8	1.0507	3.2594	8
53	1.0397	3.6539	7	1.0451	3.6432	7	1.0508	3.2565	7	1.0508	3.2565	7	1.0508	3.2565	7
54	1.0398	3.6502	6	1.0451	3.6399	6	1.0509	3.2535	6	1.0509	3.2535	6	1.0509	3.2535	6
55	1.0399	3.6465	5	1.0452	3.6367	5	1.0510	3.2506	5	1.0510	3.2506	5	1.0510	3.2506	5
56	1.0400	3.6427	4	1.0453	3.6334	4	1.0511	3.2477	4	1.0511	3.2477	4	1.0511	3.2477	4
57	1.0400	3.6390	3	1.0454	3.6301	3	1.0512	3.2448	3	1.0512	3.2448	3	1.0512	3.2448	3
58	1.0401	3.6353	2	1.0455	3.6268	2	1.0513	3.2419	2	1.0513	3.2419	2	1.0513	3.2419	2
59	1.0402	3.6316	1	1.0456	3.6236	1	1.0514	3.2390	1	1.0514	3.2390	1	1.0514	3.2390	1
60	1.0403	3.6280	0	1.0457	3.6203	0	1.0515	3.2361	0	1.0515	3.2361	0	1.0515	3.2361	0
	Csc	Sec		Csc	Sec		Csc	Sec		Csc	Sec		Csc	Sec	
105° (285°)	(254°)	74°	106° (286°)	(253°)	73°	107° (287°)	(252°)	72°							

Table B NATURAL SECANTS AND COSECANTS (continued)

18° (198°)			(341°) 161°			19° (199°)			(340°) 160°			20° (200°)			(339°) 159°		
°	Sec	Csc	°	Sec	Csc	°	Sec	Csc	°	Sec	Csc	°	Sec	Csc	°	Sec	Csc
0	1.0015	3.2361	60	1.0676	3.0716	60	1.0642	2.9238	60	1.0642	2.9238	60	1.0642	2.9238	60	1.0642	2.9238
1	1.0516	3.2332	59	1.0577	3.0590	59	1.0643	2.9215	59	1.0643	2.9215	59	1.0643	2.9215	59	1.0643	2.9215
2	1.0517	3.2303	58	1.0578	3.0564	58	1.0644	2.9191	58	1.0644	2.9191	58	1.0644	2.9191	58	1.0644	2.9191
3	1.0518	3.2274	57	1.0579	3.0538	57	1.0645	2.9168	57	1.0645	2.9168	57	1.0645	2.9168	57	1.0645	2.9168
4	1.0519	3.2245	56	1.0580	3.0512	56	1.0646	2.9145	56	1.0646	2.9145	56	1.0646	2.9145	56	1.0646	2.9145
5	1.0520	3.2217	55	1.0582	3.0555	55	1.0647	2.9122	55	1.0647	2.9122	55	1.0647	2.9122	55	1.0647	2.9122
6	1.0521	3.2188	54	1.0583	3.0541	54	1.0649	2.9099	54	1.0649	2.9099	54	1.0649	2.9099	54	1.0649	2.9099
7	1.0522	3.2159	53	1.0584	3.0535	53	1.0650	2.9075	53	1.0650	2.9075	53	1.0650	2.9075	53	1.0650	2.9075
8	1.0523	3.2131	52	1.0585	3.0509	52	1.0651	2.9052	52	1.0651	2.9052	52	1.0651	2.9052	52	1.0651	2.9052
9	1.0524	3.2102	51	1.0586	3.0484	51	1.0652	2.9029	51	1.0652	2.9029	51	1.0652	2.9029	51	1.0652	2.9029
10	1.0525	3.2074	50	1.0587	3.0458	50	1.0653	2.9006	50	1.0653	2.9006	50	1.0653	2.9006	50	1.0653	2.9006
11	1.0526	3.2045	49	1.0588	3.0433	49	1.0654	2.8983	49	1.0654	2.8983	49	1.0654	2.8983	49	1.0654	2.8983
12	1.0527	3.2017	48	1.0589	3.0407	48	1.0655	2.8960	48	1.0655	2.8960	48	1.0655	2.8960	48	1.0655	2.8960
13	1.0528	3.1989	47	1.0590	3.0382	47	1.0657	2.8936	47	1.0657	2.8936	47	1.0657	2.8936	47	1.0657	2.8936
14	1.0529	3.1960	46	1.0591	3.0357	46	1.0658	2.8915	46	1.0658	2.8915	46	1.0658	2.8915	46	1.0658	2.8915
15	1.0530	3.1932	45	1.0592	3.0331	45	1.0659	2.8892	45	1.0659	2.8892	45	1.0659	2.8892	45	1.0659	2.8892
16	1.0531	3.1904	44	1.0593	3.0305	44	1.0660	2.8869	44	1.0660	2.8869	44	1.0660	2.8869	44	1.0660	2.8869
17	1.0532	3.1876	43	1.0594	3.0281	43	1.0661	2.8845	43	1.0661	2.8845	43	1.0661	2.8845	43	1.0661	2.8845
18	1.0533	3.1848	42	1.0595	3.0256	42	1.0662	2.8824	42	1.0662	2.8824	42	1.0662	2.8824	42	1.0662	2.8824
19	1.0534	3.1820	41	1.0597	3.0231	41	1.0663	2.8801	41	1.0663	2.8801	41	1.0663	2.8801	41	1.0663	2.8801
20	1.0535	3.1792	40	1.0598	3.0208	40	1.0665	2.8779	40	1.0665	2.8779	40	1.0665	2.8779	40	1.0665	2.8779
21	1.0536	3.1764	39	1.0599	3.0181	39	1.0666	2.8756	39	1.0666	2.8756	39	1.0666	2.8756	39	1.0666	2.8756
22	1.0537	3.1736	38	1.0600	3.0156	38	1.0667	2.8733	38	1.0667	2.8733	38	1.0667	2.8733	38	1.0667	2.8733
23	1.0538	3.1708	37	1.0601	3.0131	37	1.0668	2.8711	37	1.0668	2.8711	37	1.0668	2.8711	37	1.0668	2.8711
24	1.0539	3.1681	36	1.0602	3.0106	36	1.0669	2.8688	36	1.0669	2.8688	36	1.0669	2.8688	36	1.0669	2.8688
25	1.0540	3.1653	35	1.0603	3.0081	35	1.0670	2.8666	35	1.0670	2.8666	35	1.0670	2.8666	35	1.0670	2.8666
26	1.0541	3.1625	34	1.0604	3.0056	34	1.0671	2.8644	34	1.0671	2.8644	34	1.0671	2.8644	34	1.0671	2.8644
27	1.0542	3.1598	33	1.0605	3.0031	33	1.0673	2.8621	33	1.0673	2.8621	33	1.0673	2.8621	33	1.0673	2.8621
28	1.0543	3.1570	32	1.0606	3.0007	32	1.0674	2.8598	32	1.0674	2.8598	32	1.0674	2.8598	32	1.0674	2.8598
29	1.0544	3.1543	31	1.0607	2.9982	31	1.0675	2.8577	31	1.0675	2.8577	31	1.0675	2.8577	31	1.0675	2.8577
30	1.0545	3.1515	30	1.0608	2.9957	30	1.0676	2.8555	30	1.0676	2.8555	30	1.0676	2.8555	30	1.0676	2.8555
31	1.0546	3.1488	29	1.0610	2.9933	29	1.0677	2.8532	29	1.0677	2.8532	29	1.0677	2.8532	29	1.0677	2.8532
32	1.0547	3.1461	28	1.0611	2.9908	28	1.0678	2.8510	28	1.0678	2.8510	28	1.0678	2.8510	28	1.0678	2.8510
33	1.0548	3.1433	27	1.0612	2.9884	27	1.0680	2.8488	27	1.0680	2.8488	27	1.0680	2.8488	27	1.0680	2.8488
34	1.0549	3.1405	26	1.0613	2.9859	26	1.0681	2.8465	26	1.0681	2.8465	26	1.0681	2.8465	26	1.0681	2.8465
35	1.0550	3.1379	25	1.0614	2.9835	25	1.0682	2.8444	25	1.0682	2.8444	25	1.0682	2.8444	25	1.0682	2.8444
36	1.0551	3.1352	24	1.0615	2.9811	24	1.0683	2.8422	24	1.0683	2.8422	24	1.0683	2.8422	24	1.0683	2.8422
37	1.0552	3.1325	23	1.0616	2.9786	23	1.0684	2.8400	23	1.0684	2.8400	23	1.0684	2.8400	23	1.0684	2.8400
38	1.0553	3.1298	22	1.0617	2.9762	22	1.0685	2.8378	22	1.0685	2.8378	22	1.0685	2.8378	22	1.0685	2.8378
39	1.0554	3.1271	21	1.0618	2.9738	21	1.0687	2.8355	21	1.0687	2.8355	21	1.0687	2.8355	21	1.0687	2.8355
40	1.0555	3.1244	20	1.0619	2.9713	20	1.0688	2.8334	20	1.0688	2.8334	20	1.0688	2.8334	20	1.0688	2.8334
41	1.0556	3.1217	19	1.0621	2.9689	19	1.0689	2.8312	19	1.0689	2.8312	19	1.0689	2.8312	19	1.0689	2.8312
42	1.0557	3.1190	18	1.0622	2.9665	18	1.0690	2.8291	18	1.0690	2.8291	18	1.0690	2.8291	18	1.0690	2.8291
43	1.0558	3.1163	17	1.0623	2.9641	17	1.0691	2.8269	17	1.0691	2.8269	17	1.0691	2.8269	17	1.0691	2.8269
44	1.0559	3.1137	16	1.0624	2.9617	16	1.0692	2.8247	16	1.0692	2.8247	16	1.0692	2.8247	16	1.0692	2.8247
45	1.0560	3.1110	15	1.0625	2.9593	15	1.0694	2.8225	15	1.0694	2.8225	15	1.0694	2.8225	15	1.0694	2.8225
46	1.0561	3.1083	14	1.0626	2.9569	14	1.0695	2.8204	14	1.0695	2.8204	14	1.0695	2.8204	14	1.0695	2.8204
47	1.0563	3.1057	13	1.0627	2.9545	13	1.0696	2.8182	13	1.0696	2.8182	13	1.0696	2.8182	13	1.0696	2.8182
48	1.0564	3.1030	12	1.0628	2.9521	12	1.0697	2.8161	12	1.0697	2.8161	12	1.0697	2.8161	12	1.0697	2.8161
49	1.0565	3.1004	11	1.0629	2.9498	11	1.0698	2.8139	11	1.0698	2.8139	11	1.0698	2.8139	11	1.0698	2.8139
50	1.0566	3.0977	10	1.0631	2.9474	10	1.0700	2.8117	10	1.0700	2.8117	10	1.0700	2.8117	10	1.0700	2.8117
51	1.0567	3.0951	9	1.0632	2.9450	9	1.0701	2.8096	9	1.0701	2.8096	9	1.0701	2.8096	9	1.0701	2.8096
52	1.0568	3.0925	8	1.0633	2.9426	8	1.0702	2.8075	8	1.0702	2.8075	8	1.0702	2.8075	8	1.0702	2.8075
53	1.0569	3.0898	7	1.0634	2.9403	7	1.0703	2.8053	7	1.0703	2.8053	7	1.0703	2.8053	7	1.0703	2.8053
54	1.0570	3.0872	6	1.0635	2.9379	6	1.0704	2.8032	6	1.0704	2.8032	6	1.0704	2.8032	6	1.0704	2.8032
55	1.0571	3.0846	5	1.0636	2.9355	5	1.0705	2.8010	5	1.0705	2.8010	5	1.0705	2.8010	5	1.0705	2.8010
56	1.0572	3.0820	4	1.0637	2.9332	4	1.0707	2.7989	4	1.0707	2.7989	4	1.0707	2.7989	4	1.0707	2.7989
57	1.0573	3.0794	3	1.0638	2.9308	3	1.0708	2.7968	3	1.0708	2.7968	3	1.0708	2.7968	3	1.0708	2.7968
58	1.0574	3.0768	2	1.0640	2.9285	2	1.0709	2.7947	2	1.0709	2.7947	2	1.0709	2.7947	2	1.0709	2.7947
59	1.0575	3.0742	1	1.0641	2.9261	1	1.0710	2.7925	1	1.0710	2.7925	1	1.0710	2.7925	1	1.0710	2.7925
60	1.0576	3.0716	0	1.0642	2.9238	0	1.0711	2.7904	0	1.0711	2.7904	0	1.0711	2.7904	0	1.0711	2.7904

108° (288°)

(251°) 71°

109° (289°)

(250°) 70°

110° (290°)

(249°) 69°

Table 8 NATURAL SECANTS AND COSECANTS (continued)

21° (201°)			(338°) 158°			22° (202°)			(337°) 157°			23° (203°)			(336°) 156°		
'	Sec	Csc	'			'	Sec	Csc	'			'	Sec	Csc	'		
0	1.0711	2.7904	60			0	1.0785	2.6696	60			0	1.0864	2.5593	60		
1	1.0713	2.7883	59			1	1.0787	2.6675	59			1	1.0866	2.5576	59		
2	1.0714	2.7862	58			2	1.0788	2.6656	58			2	1.0866	2.5558	58		
3	1.0715	2.7841	57			3	1.0789	2.6637	57			3	1.0868	2.5541	57		
4	1.0716	2.7820	56			4	1.0790	2.6618	56			4	1.0869	2.5523	56		
5	1.0717	2.7799	55			5	1.0792	2.6599	55			5	1.0870	2.5506	55		
6	1.0719	2.7778	54			6	1.0793	2.6580	54			6	1.0872	2.5488	54		
7	1.0720	2.7757	53			7	1.0794	2.6561	53			7	1.0873	2.5471	53		
8	1.0721	2.7736	52			8	1.0796	2.6542	52			8	1.0874	2.5454	52		
9	1.0722	2.7715	51			9	1.0797	2.6523	51			9	1.0876	2.5436	51		
10	1.0723	2.7695	50			10	1.0798	2.6504	50			10	1.0877	2.5419	50		
11	1.0725	2.7674	49			11	1.0799	2.6485	49			11	1.0878	2.5402	49		
12	1.0726	2.7653	48			12	1.0801	2.6466	48			12	1.0880	2.5384	48		
13	1.0727	2.7632	47			13	1.0802	2.6447	47			13	1.0881	2.5367	47		
14	1.0728	2.7612	46			14	1.0803	2.6429	46			14	1.0883	2.5350	46		
15	1.0730	2.7591	45			15	1.0804	2.6410	45			15	1.0884	2.5333	45		
16	1.0731	2.7570	44			16	1.0806	2.6391	44			16	1.0885	2.5316	44		
17	1.0732	2.7550	43			17	1.0807	2.6372	43			17	1.0887	2.5298	43		
18	1.0733	2.7529	42			18	1.0808	2.6354	42			18	1.0888	2.5282	42		
19	1.0734	2.7509	41			19	1.0810	2.6335	41			19	1.0889	2.5264	41		
20	1.0736	2.7488	40			20	1.0811	2.6316	40			20	1.0891	2.5247	40		
21	1.0737	2.7468	39			21	1.0812	2.6298	39			21	1.0892	2.5230	39		
22	1.0738	2.7447	38			22	1.0814	2.6279	38			22	1.0893	2.5213	38		
23	1.0739	2.7427	37			23	1.0815	2.6260	37			23	1.0895	2.5196	37		
24	1.0740	2.7407	36			24	1.0816	2.6242	36			24	1.0896	2.5180	36		
25	1.0742	2.7386	35			25	1.0817	2.6223	35			25	1.0898	2.5163	35		
26	1.0743	2.7366	34			26	1.0819	2.6205	34			26	1.0899	2.5146	34		
27	1.0744	2.7346	33			27	1.0820	2.6188	33			27	1.0900	2.5129	33		
28	1.0745	2.7325	32			28	1.0821	2.6168	32			28	1.0902	2.5112	32		
29	1.0747	2.7305	31			29	1.0823	2.6150	31			29	1.0903	2.5095	31		
30	1.0748	2.7285	30			30	1.0824	2.6131	30			30	1.0904	2.5078	30		
31	1.0749	2.7265	29			31	1.0825	2.6113	29			31	1.0906	2.5062	29		
32	1.0750	2.7245	28			32	1.0827	2.6095	28			32	1.0907	2.5045	28		
33	1.0752	2.7225	27			33	1.0828	2.6076	27			33	1.0909	2.5028	27		
34	1.0753	2.7205	26			34	1.0829	2.6058	26			34	1.0910	2.5012	26		
35	1.0754	2.7185	25			35	1.0830	2.6040	25			35	1.0911	2.4995	25		
36	1.0755	2.7165	24			36	1.0832	2.6022	24			36	1.0913	2.4978	24		
37	1.0757	2.7145	23			37	1.0833	2.6003	23			37	1.0914	2.4962	23		
38	1.0758	2.7125	22			38	1.0834	2.5985	22			38	1.0916	2.4945	22		
39	1.0759	2.7105	21			39	1.0836	2.5967	21			39	1.0917	2.4928	21		
40	1.0760	2.7085	20			40	1.0837	2.5949	20			40	1.0918	2.4912	20		
41	1.0761	2.7065	19			41	1.0838	2.5931	19			41	1.0920	2.4895	19		
42	1.0763	2.7046	18			42	1.0840	2.5913	18			42	1.0921	2.4879	18		
43	1.0764	2.7026	17			43	1.0841	2.5895	17			43	1.0922	2.4862	17		
44	1.0765	2.7006	16			44	1.0842	2.5877	16			44	1.0924	2.4846	16		
45	1.0766	2.6986	15			45	1.0844	2.5859	15			45	1.0925	2.4830	15		
46	1.0768	2.6967	14			46	1.0845	2.5841	14			46	1.0927	2.4813	14		
47	1.0769	2.6947	13			47	1.0846	2.5823	13			47	1.0928	2.4797	13		
48	1.0770	2.6927	12			48	1.0848	2.5805	12			48	1.0929	2.4780	12		
49	1.0771	2.6908	11			49	1.0849	2.5788	11			49	1.0931	2.4764	11		
50	1.0773	2.6888	10			50	1.0850	2.5770	10			50	1.0932	2.4748	10		
51	1.0774	2.6869	9			51	1.0852	2.5752	9			51	1.0934	2.4731	9		
52	1.0775	2.6849	8			52	1.0853	2.5734	8			52	1.0935	2.4715	8		
53	1.0777	2.6830	7			53	1.0854	2.5716	7			53	1.0936	2.4699	7		
54	1.0778	2.6811	6			54	1.0856	2.5698	6			54	1.0938	2.4683	6		
55	1.0779	2.6791	5			55	1.0857	2.5681	5			55	1.0939	2.4667	5		
56	1.0780	2.6772	4			56	1.0858	2.5663	4			56	1.0941	2.4650	4		
57	1.0782	2.6752	3			57	1.0860	2.5646	3			57	1.0942	2.4634	3		
58	1.0783	2.6733	2			58	1.0861	2.5628	2			58	1.0944	2.4618	2		
59	1.0784	2.6714	1			59	1.0862	2.5611	1			59	1.0946	2.4602	1		
60	1.0785	2.6695	0			60	1.0864	2.5593	0			60	1.0946	2.4586	0		
'	Csc	Sec	'			'	Csc	Sec	'			'	Csc	Sec	'		

111° (291°) (248°) 68° 112° (292°) (247°) 67° 113° (293°) (246°) 66°

Table 8 NATURAL SECANTS AND COSECANTS (continued)

24° (204°)			(335°) 155°			25° (205°)			(334°) 154°			26° (206°)			(333°) 153°		
'	Sec	Csc	'	Sec	Csc	'	Sec	Csc	'	Sec	Csc	'	Sec	Csc	'	Sec	Csc
0	1.0946	2.4586	60			0	1.1034	2.3682	60			0	1.1126	2.2812	60		
1	1.0948	2.4570	59			1	1.1035	2.3647	59			1	1.1128	2.2796	59		
2	1.0949	2.4554	58			2	1.1037	2.3633	58			2	1.1129	2.2785	58		
3	1.0951	2.4538	57			3	1.1038	2.3618	57			3	1.1131	2.2771	57		
4	1.0952	2.4522	56			4	1.1040	2.3603	56			4	1.1132	2.2757	56		
5	1.0953	2.4506	55			5	1.1041	2.3588	55			5	1.1134	2.2744	55		
6	1.0955	2.4490	54			6	1.1042	2.3574	54			6	1.1136	2.2730	54		
7	1.0956	2.4474	53			7	1.1044	2.3559	53			7	1.1137	2.2717	53		
8	1.0958	2.4458	52			8	1.1046	2.3545	52			8	1.1139	2.2703	52		
9	1.0959	2.4442	51			9	1.1047	2.3530	51			9	1.1140	2.2690	51		
10	1.0961	2.4426	50			10	1.1049	2.3515	50			10	1.1142	2.2677	50		
11	1.0962	2.4411	49			11	1.1050	2.3501	49			11	1.1143	2.2663	49		
12	1.0963	2.4395	48			12	1.1052	2.3486	48			12	1.1145	2.2650	48		
13	1.0965	2.4379	47			13	1.1053	2.3472	47			13	1.1147	2.2636	47		
14	1.0966	2.4363	46			14	1.1055	2.3457	46			14	1.1148	2.2623	46		
15	1.0968	2.4348	45			15	1.1056	2.3443	45			15	1.1150	2.2610	45		
16	1.0969	2.4332	44			16	1.1058	2.3428	44			16	1.1151	2.2596	44		
17	1.0971	2.4316	43			17	1.1059	2.3414	43			17	1.1153	2.2583	43		
18	1.0972	2.4300	42			18	1.1061	2.3400	42			18	1.1155	2.2570	42		
19	1.0974	2.4285	41			19	1.1062	2.3385	41			19	1.1156	2.2556	41		
20	1.0975	2.4269	40			20	1.1064	2.3371	40			20	1.1158	2.2543	40		
21	1.0976	2.4254	39			21	1.1065	2.3356	39			21	1.1159	2.2530	39		
22	1.0978	2.4238	38			22	1.1067	2.3342	38			22	1.1161	2.2517	38		
23	1.0979	2.4222	37			23	1.1069	2.3328	37			23	1.1163	2.2504	37		
24	1.0981	2.4207	36			24	1.1070	2.3314	36			24	1.1164	2.2490	36		
25	1.0982	2.4191	35			25	1.1072	2.3299	35			25	1.1166	2.2477	35		
26	1.0984	2.4176	34			26	1.1073	2.3285	34			26	1.1168	2.2464	34		
27	1.0985	2.4160	33			27	1.1075	2.3271	33			27	1.1169	2.2451	33		
28	1.0987	2.4145	32			28	1.1076	2.3257	32			28	1.1171	2.2438	32		
29	1.0988	2.4130	31			29	1.1078	2.3242	31			29	1.1172	2.2425	31		
30	1.0989	2.4114	30			30	1.1079	2.3228	30			30	1.1174	2.2412	30		
31	1.0991	2.4099	29			31	1.1081	2.3214	29			31	1.1176	2.2399	29		
32	1.0992	2.4083	28			32	1.1082	2.3200	28			32	1.1177	2.2385	28		
33	1.0994	2.4068	27			33	1.1084	2.3186	27			33	1.1179	2.2372	27		
34	1.0995	2.4053	26			34	1.1085	2.3172	26			34	1.1180	2.2359	26		
35	1.0997	2.4038	25			35	1.1087	2.3158	25			35	1.1182	2.2346	25		
36	1.0998	2.4022	24			36	1.1089	2.3144	24			36	1.1184	2.2333	24		
37	1.1000	2.4007	23			37	1.1090	2.3130	23			37	1.1185	2.2320	23		
38	1.1001	2.3992	22			38	1.1092	2.3115	22			38	1.1187	2.2308	22		
39	1.1003	2.3977	21			39	1.1093	2.3101	21			39	1.1189	2.2295	21		
40	1.1004	2.3961	20			40	1.1095	2.3088	20			40	1.1190	2.2282	20		
41	1.1005	2.3945	19			41	1.1096	2.3074	19			41	1.1192	2.2269	19		
42	1.1007	2.3931	18			42	1.1098	2.3060	18			42	1.1194	2.2256	18		
43	1.1009	2.3915	17			43	1.1099	2.3046	17			43	1.1195	2.2243	17		
44	1.1010	2.3901	16			44	1.1101	2.3032	16			44	1.1197	2.2230	16		
45	1.1011	2.3886	15			45	1.1102	2.3018	15			45	1.1198	2.2217	15		
46	1.1013	2.3871	14			46	1.1104	2.3004	14			46	1.1200	2.2205	14		
47	1.1014	2.3855	13			47	1.1106	2.2990	13			47	1.1202	2.2192	13		
48	1.1016	2.3841	12			48	1.1107	2.2976	12			48	1.1203	2.2179	12		
49	1.1017	2.3826	11			49	1.1109	2.2962	11			49	1.1205	2.2166	11		
50	1.1019	2.3811	10			50	1.1110	2.2949	10			50	1.1207	2.2153	10		
51	1.1020	2.3796	9			51	1.1112	2.2935	9			51	1.1208	2.2141	9		
52	1.1022	2.3781	8			52	1.1113	2.2921	8			52	1.1210	2.2128	8		
53	1.1023	2.3766	7			53	1.1115	2.2907	7			53	1.1212	2.2115	7		
54	1.1025	2.3751	6			54	1.1117	2.2894	6			54	1.1213	2.2103	6		
55	1.1025	2.3736	5			55	1.1118	2.2880	5			55	1.1215	2.2090	5		
56	1.1028	2.3721	4			56	1.1120	2.2866	4			56	1.1217	2.2077	4		
57	1.1029	2.3706	3			57	1.1121	2.2853	3			57	1.1218	2.2065	3		
58	1.1031	2.3692	2			58	1.1123	2.2839	2			58	1.1220	2.2052	2		
59	1.1032	2.3677	1			59	1.1124	2.2825	1			59	1.1222	2.2039	1		
60	1.1034	2.3662	0			60	1.1126	2.2812	0			60	1.1223	2.2027	0		
	Csc	Sec					Csc	Sec					Csc	Sec			
114° (294°)			(245°) 65°			115° (295°)			(244°) 64°			116° (296°)			(243°) 63°		

Table 8 NATURAL SECANTS AND COSECANTS (continued)

27° (207°)			(332°) 152°			28° (208°)			(331°) 151°			29° (209°)			(330°) 150°		
'	Sec	Csc	'	Sec	Csc	'	Sec	Csc	'	Sec	Csc	'	Sec	Csc	'	Sec	Csc
0	1.1223	2.2027	60			0	1.1326	2.1301	60	0	1.1434	2.0527	60			0	
1	1.1225	2.2014	59			1	1.1327	2.1289	59	1	1.1435	2.0516	59			1	
2	1.1227	2.2002	58			2	1.1329	2.1277	58	2	1.1437	2.0505	58			2	
3	1.1228	2.1989	57			3	1.1331	2.1266	57	3	1.1439	2.0494	57			3	
4	1.1230	2.1977	56			4	1.1333	2.1254	56	4	1.1441	2.0483	56			4	
5	1.1232	2.1964	55			5	1.1334	2.1242	55	5	1.1443	2.0473	55			5	
6	1.1233	2.1952	54			6	1.1336	2.1231	54	6	1.1445	2.0462	54			6	
7	1.1235	2.1939	53			7	1.1338	2.1219	53	7	1.1446	2.0451	53			7	
8	1.1237	2.1927	52			8	1.1340	2.1208	52	8	1.1448	2.0440	52			8	
9	1.1238	2.1914	51			9	1.1342	2.1196	51	9	1.1450	2.0430	51			9	
10	1.1240	2.1902	50			10	1.1343	2.1185	50	10	1.1452	2.0419	50			10	
11	1.1242	2.1890	49			11	1.1345	2.1173	49	11	1.1454	2.0408	49			11	
12	1.1243	2.1877	48			12	1.1347	2.1162	48	12	1.1456	2.0398	48			12	
13	1.1245	2.1865	47			13	1.1349	2.1150	47	13	1.1458	2.0387	47			13	
14	1.1247	2.1852	46			14	1.1350	2.1139	46	14	1.1460	2.0376	46			14	
15	1.1248	2.1840	45			15	1.1352	2.1127	45	15	1.1461	2.0366	45			15	
16	1.1250	2.1828	44			16	1.1354	2.1116	44	16	1.1463	2.0355	44			16	
17	1.1252	2.1815	43			17	1.1356	2.1105	43	17	1.1465	2.0345	43			17	
18	1.1253	2.1803	42			18	1.1357	2.1093	42	18	1.1467	2.0334	42			18	
19	1.1255	2.1791	41			19	1.1359	2.1082	41	19	1.1469	2.0323	41			19	
20	1.1257	2.1779	40			20	1.1361	2.1070	40	20	1.1471	2.0313	40			20	
21	1.1259	2.1766	39			21	1.1363	2.1059	39	21	1.1473	2.0302	39			21	
22	1.1260	2.1754	38			22	1.1365	2.1048	38	22	1.1474	2.0292	38			22	
23	1.1262	2.1742	37			23	1.1366	2.1036	37	23	1.1476	2.0281	37			23	
24	1.1264	2.1730	36			24	1.1368	2.1025	36	24	1.1478	2.0271	36			24	
25	1.1265	2.1718	35			25	1.1370	2.1014	35	25	1.1480	2.0260	35			25	
26	1.1267	2.1706	34			26	1.1372	2.1002	34	26	1.1482	2.0250	34			26	
27	1.1269	2.1693	33			27	1.1374	2.0991	33	27	1.1484	2.0239	33			27	
28	1.1270	2.1681	32			28	1.1375	2.0980	32	28	1.1486	2.0229	32			28	
29	1.1272	2.1669	31			29	1.1377	2.0969	31	29	1.1488	2.0218	31			29	
30	1.1274	2.1657	30			30	1.1379	2.0957	30	30	1.1490	2.0208	30			30	
31	1.1276	2.1645	29			31	1.1381	2.0946	29	31	1.1491	2.0197	29			31	
32	1.1277	2.1633	28			32	1.1383	2.0935	28	32	1.1493	2.0187	28			32	
33	1.1279	2.1621	27			33	1.1384	2.0924	27	33	1.1495	2.0176	27			33	
34	1.1281	2.1609	26			34	1.1386	2.0913	26	34	1.1497	2.0166	26			34	
35	1.1282	2.1596	25			35	1.1388	2.0901	25	35	1.1499	2.0155	25			35	
36	1.1284	2.1584	24			36	1.1390	2.0890	24	36	1.1501	2.0145	24			36	
37	1.1286	2.1572	23			37	1.1392	2.0879	23	37	1.1503	2.0134	23			37	
38	1.1288	2.1560	22			38	1.1393	2.0868	22	38	1.1505	2.0124	22			38	
39	1.1289	2.1549	21			39	1.1395	2.0857	21	39	1.1507	2.0114	21			39	
40	1.1291	2.1537	20			40	1.1397	2.0846	20	40	1.1509	2.0104	20			40	
41	1.1293	2.1525	19			41	1.1399	2.0835	19	41	1.1510	2.0094	19			41	
42	1.1294	2.1513	18			42	1.1401	2.0824	18	42	1.1512	2.0083	18			42	
43	1.1296	2.1501	17			43	1.1402	2.0813	17	43	1.1514	2.0073	17			43	
44	1.1298	2.1489	16			44	1.1404	2.0802	16	44	1.1516	2.0063	16			44	
45	1.1300	2.1477	15			45	1.1406	2.0791	15	45	1.1518	2.0052	15			45	
46	1.1301	2.1465	14			46	1.1408	2.0779	14	46	1.1520	2.0042	14			46	
47	1.1303	2.1453	13			47	1.1410	2.0768	13	47	1.1522	2.0032	13			47	
48	1.1305	2.1441	12			48	1.1412	2.0757	12	48	1.1524	2.0022	12			48	
49	1.1307	2.1430	11			49	1.1413	2.0747	11	49	1.1526	2.0012	11			49	
50	1.1308	2.1418	10			50	1.1415	2.0736	10	50	1.1528	2.0001	10			50	
51	1.1310	2.1406	9			51	1.1417	2.0725	9	51	1.1530	1.9991	9			51	
52	1.1312	2.1394	8			52	1.1419	2.0714	8	52	1.1532	1.9981	8			52	
53	1.1313	2.1382	7			53	1.1421	2.0703	7	53	1.1533	1.9971	7			53	
54	1.1315	2.1371	6			54	1.1423	2.0692	6	54	1.1535	1.9961	6			54	
55	1.1317	2.1360	5			55	1.1424	2.0681	5	55	1.1537	1.9951	5			55	
56	1.1319	2.1347	4			56	1.1426	2.0670	4	56	1.1539	1.9940	4			56	
57	1.1320	2.1336	3			57	1.1428	2.0659	3	57	1.1541	1.9930	3			57	
58	1.1322	2.1324	2			58	1.1430	2.0648	2	58	1.1543	1.9920	2			58	
59	1.1324	2.1312	1			59	1.1432	2.0637	1	59	1.1545	1.9910	1			59	
60	1.1326	2.1301	0			60	1.1434	2.0627	0	60	1.1547	1.9900	0			60	
'	Csc	Sec	'			'	Csc	Sec	'	'	Csc	Sec	'			'	Csc

117° (297°) (242°) 62° 118° (298°) (241°) 61° 119° (299°) (240°) 60°

Table B NATURAL SECANTS AND COSECANTS (continued)

30° (210°)			(329°) 149°			31° (211°)			(328°) 148°			32° (212°)			(327°) 147°		
	Sec	Csc		Sec	Csc		Sec	Csc		Sec	Csc		Sec	Csc		Sec	Csc
0	1.1547	2.0300	60	1.1666	1.9416	60	1.1792	1.8871	60	1.1792	1.8871	60	1.1792	1.8871	60	1.1792	1.8871
1	1.1549	1.9990	59	1.1658	1.9407	59	1.1794	1.8862	59	1.1794	1.8862	59	1.1794	1.8862	59	1.1794	1.8862
2	1.1551	1.9980	58	1.1670	1.9397	58	1.1796	1.8853	58	1.1796	1.8853	58	1.1796	1.8853	58	1.1796	1.8853
3	1.1553	1.9970	57	1.1672	1.9388	57	1.1798	1.8844	57	1.1798	1.8844	57	1.1798	1.8844	57	1.1798	1.8844
4	1.1555	1.9960	56	1.1675	1.9379	56	1.1800	1.8836	56	1.1800	1.8836	56	1.1800	1.8836	56	1.1800	1.8836
5	1.1557	1.9950	55	1.1677	1.9369	55	1.1803	1.8827	55	1.1803	1.8827	55	1.1803	1.8827	55	1.1803	1.8827
6	1.1559	1.9940	54	1.1679	1.9360	54	1.1805	1.8818	54	1.1805	1.8818	54	1.1805	1.8818	54	1.1805	1.8818
7	1.1561	1.9930	53	1.1681	1.9351	53	1.1807	1.8810	53	1.1807	1.8810	53	1.1807	1.8810	53	1.1807	1.8810
8	1.1563	1.9920	52	1.1683	1.9341	52	1.1809	1.8801	52	1.1809	1.8801	52	1.1809	1.8801	52	1.1809	1.8801
9	1.1565	1.9910	51	1.1685	1.9332	51	1.1811	1.8792	51	1.1811	1.8792	51	1.1811	1.8792	51	1.1811	1.8792
10	1.1566	1.9900	50	1.1687	1.9323	50	1.1813	1.8783	50	1.1813	1.8783	50	1.1813	1.8783	50	1.1813	1.8783
11	1.1568	1.9890	49	1.1689	1.9313	49	1.1815	1.8775	49	1.1815	1.8775	49	1.1815	1.8775	49	1.1815	1.8775
12	1.1570	1.9880	48	1.1691	1.9304	48	1.1818	1.8766	48	1.1818	1.8766	48	1.1818	1.8766	48	1.1818	1.8766
13	1.1572	1.9870	47	1.1693	1.9295	47	1.1820	1.8757	47	1.1820	1.8757	47	1.1820	1.8757	47	1.1820	1.8757
14	1.1574	1.9860	46	1.1695	1.9285	46	1.1822	1.8749	46	1.1822	1.8749	46	1.1822	1.8749	46	1.1822	1.8749
15	1.1576	1.9850	45	1.1697	1.9276	45	1.1824	1.8740	45	1.1824	1.8740	45	1.1824	1.8740	45	1.1824	1.8740
16	1.1578	1.9840	44	1.1699	1.9267	44	1.1826	1.8731	44	1.1826	1.8731	44	1.1826	1.8731	44	1.1826	1.8731
17	1.1580	1.9830	43	1.1701	1.9258	43	1.1828	1.8723	43	1.1828	1.8723	43	1.1828	1.8723	43	1.1828	1.8723
18	1.1582	1.9821	42	1.1703	1.9249	42	1.1831	1.8714	42	1.1831	1.8714	42	1.1831	1.8714	42	1.1831	1.8714
19	1.1584	1.9811	41	1.1705	1.9239	41	1.1833	1.8706	41	1.1833	1.8706	41	1.1833	1.8706	41	1.1833	1.8706
20	1.1586	1.9801	40	1.1707	1.9230	40	1.1835	1.8697	40	1.1835	1.8697	40	1.1835	1.8697	40	1.1835	1.8697
21	1.1588	1.9791	39	1.1710	1.9221	39	1.1837	1.8688	39	1.1837	1.8688	39	1.1837	1.8688	39	1.1837	1.8688
22	1.1590	1.9781	38	1.1712	1.9212	38	1.1839	1.8680	38	1.1839	1.8680	38	1.1839	1.8680	38	1.1839	1.8680
23	1.1592	1.9771	37	1.1714	1.9203	37	1.1842	1.8671	37	1.1842	1.8671	37	1.1842	1.8671	37	1.1842	1.8671
24	1.1594	1.9762	36	1.1716	1.9194	36	1.1844	1.8663	36	1.1844	1.8663	36	1.1844	1.8663	36	1.1844	1.8663
25	1.1596	1.9752	35	1.1718	1.9184	35	1.1846	1.8654	35	1.1846	1.8654	35	1.1846	1.8654	35	1.1846	1.8654
26	1.1598	1.9742	34	1.1720	1.9176	34	1.1848	1.8646	34	1.1848	1.8646	34	1.1848	1.8646	34	1.1848	1.8646
27	1.1600	1.9732	33	1.1722	1.9166	33	1.1850	1.8637	33	1.1850	1.8637	33	1.1850	1.8637	33	1.1850	1.8637
28	1.1602	1.9722	32	1.1724	1.9157	32	1.1852	1.8629	32	1.1852	1.8629	32	1.1852	1.8629	32	1.1852	1.8629
29	1.1604	1.9713	31	1.1726	1.9148	31	1.1855	1.8620	31	1.1855	1.8620	31	1.1855	1.8620	31	1.1855	1.8620
30	1.1606	1.9703	30	1.1728	1.9139	30	1.1857	1.8612	30	1.1857	1.8612	30	1.1857	1.8612	30	1.1857	1.8612
31	1.1608	1.9693	29	1.1730	1.9130	29	1.1859	1.8603	29	1.1859	1.8603	29	1.1859	1.8603	29	1.1859	1.8603
32	1.1610	1.9684	28	1.1732	1.9121	28	1.1861	1.8596	28	1.1861	1.8596	28	1.1861	1.8596	28	1.1861	1.8596
33	1.1612	1.9674	27	1.1735	1.9112	27	1.1863	1.8586	27	1.1863	1.8586	27	1.1863	1.8586	27	1.1863	1.8586
34	1.1614	1.9664	26	1.1737	1.9103	26	1.1866	1.8578	26	1.1866	1.8578	26	1.1866	1.8578	26	1.1866	1.8578
35	1.1616	1.9654	25	1.1739	1.9094	25	1.1868	1.8569	25	1.1868	1.8569	25	1.1868	1.8569	25	1.1868	1.8569
36	1.1618	1.9645	24	1.1741	1.9084	24	1.1870	1.8561	24	1.1870	1.8561	24	1.1870	1.8561	24	1.1870	1.8561
37	1.1620	1.9635	23	1.1743	1.9075	23	1.1872	1.8552	23	1.1872	1.8552	23	1.1872	1.8552	23	1.1872	1.8552
38	1.1622	1.9625	22	1.1745	1.9066	22	1.1875	1.8544	22	1.1875	1.8544	22	1.1875	1.8544	22	1.1875	1.8544
39	1.1624	1.9616	21	1.1747	1.9057	21	1.1877	1.8535	21	1.1877	1.8535	21	1.1877	1.8535	21	1.1877	1.8535
40	1.1626	1.9606	20	1.1749	1.9048	20	1.1879	1.8527	20	1.1879	1.8527	20	1.1879	1.8527	20	1.1879	1.8527
41	1.1628	1.9597	19	1.1751	1.9039	19	1.1881	1.8519	19	1.1881	1.8519	19	1.1881	1.8519	19	1.1881	1.8519
42	1.1630	1.9587	18	1.1753	1.9031	18	1.1883	1.8510	18	1.1883	1.8510	18	1.1883	1.8510	18	1.1883	1.8510
43	1.1632	1.9577	17	1.1755	1.9022	17	1.1886	1.8502	17	1.1886	1.8502	17	1.1886	1.8502	17	1.1886	1.8502
44	1.1634	1.9568	16	1.1758	1.9013	16	1.1888	1.8494	16	1.1888	1.8494	16	1.1888	1.8494	16	1.1888	1.8494
45	1.1636	1.9558	15	1.1760	1.9004	15	1.1890	1.8485	15	1.1890	1.8485	15	1.1890	1.8485	15	1.1890	1.8485
46	1.1638	1.9549	14	1.1762	1.8995	14	1.1892	1.8477	14	1.1892	1.8477	14	1.1892	1.8477	14	1.1892	1.8477
47	1.1640	1.9539	13	1.1764	1.8986	13	1.1895	1.8468	13	1.1895	1.8468	13	1.1895	1.8468	13	1.1895	1.8468
48	1.1642	1.9530	12	1.1766	1.8977	12	1.1897	1.8460	12	1.1897	1.8460	12	1.1897	1.8460	12	1.1897	1.8460
49	1.1644	1.9520	11	1.1768	1.8968	11	1.1899	1.8452	11	1.1899	1.8452	11	1.1899	1.8452	11	1.1899	1.8452
50	1.1646	1.9511	10	1.1770	1.8959	10	1.1901	1.8443	10	1.1901	1.8443	10	1.1901	1.8443	10	1.1901	1.8443
51	1.1648	1.9501	9	1.1773	1.8950	9	1.1903	1.8435	9	1.1903	1.8435	9	1.1903	1.8435	9	1.1903	1.8435
52	1.1650	1.9492	8	1.1775	1.8941	8	1.1906	1.8427	8	1.1906	1.8427	8	1.1906	1.8427	8	1.1906	1.8427
53	1.1652	1.9482	7	1.1777	1.8933	7	1.1908	1.8419	7	1.1908	1.8419	7	1.1908	1.8419	7	1.1908	1.8419
54	1.1654	1.9473	6	1.1779	1.8924	6	1.1910	1.8410	6	1.1910	1.8410	6	1.1910	1.8410	6	1.1910	1.8410
55	1.1656	1.9463	5	1.1781	1.8915	5	1.1912	1.8402	5	1.1912	1.8402	5	1.1912	1.8402	5	1.1912	1.8402
56	1.1658	1.9454	4	1.1783	1.8906	4	1.1915	1.8394	4	1.1915	1.8394	4	1.1915	1.8394	4	1.1915	1.8394
57	1.1660	1.9444	3	1.1785	1.8897	3	1.1917	1.8385	3	1.1917	1.8385	3	1.1917	1.8385	3	1.1917	1.8385
58	1.1662	1.9435	2	1.1788	1.8888	2	1.1919	1.8377	2	1.1919	1.8377	2	1.1919	1.8377	2	1.1919	1.8377
59	1.1664	1.9425	1	1.1790	1.8880	1	1.1921	1.8369	1	1.1921	1.8369	1	1.1921	1.8369	1	1.1921	1.8369
60	1.1666	1.9416	0	1.1792	1.8871	0	1.1924	1.8361	0	1.1924	1.8361	0	1.1924	1.8361	0	1.1924	1.8361
120° (300°)			(329°) 59°			121° (301°)			(328°) 58°			122° (302°)			(327°) 57°		

Table 8 NATURAL SECANTS AND COSECANTS (continued)

33° (213°)			34° (214°)			35° (215°)			
	Sec	Csc		Sec	Csc		Sec	Csc	
0	1.1924	1.8361	60	1.2062	1.7882	60	1.2208	1.7434	60
1	1.1926	1.8353	59	1.2065	1.7875	59	1.2210	1.7427	59
2	1.1928	1.8344	58	1.2067	1.7868	58	1.2213	1.7420	58
3	1.1930	1.8336	57	1.2069	1.7860	57	1.2215	1.7413	57
4	1.1933	1.8328	56	1.2072	1.7852	56	1.2218	1.7406	56
5	1.1935	1.8320	55	1.2074	1.7844	55	1.2220	1.7398	55
6	1.1927	1.8312	54	1.2076	1.7837	54	1.2223	1.7391	54
7	1.1939	1.8303	53	1.2079	1.7829	53	1.2225	1.7384	53
8	1.1942	1.8295	52	1.2081	1.7821	52	1.2228	1.7377	52
9	1.1944	1.8287	51	1.2084	1.7814	51	1.2230	1.7370	51
10	1.1946	1.8279	50	1.2086	1.7806	50	1.2233	1.7362	50
11	1.1949	1.8271	49	1.2088	1.7799	49	1.2235	1.7355	49
12	1.1951	1.8263	48	1.2091	1.7791	48	1.2238	1.7348	48
13	1.1953	1.8255	47	1.2093	1.7783	47	1.2240	1.7341	47
14	1.1955	1.8247	46	1.2096	1.7776	46	1.2243	1.7334	46
15	1.1958	1.8238	45	1.2098	1.7768	45	1.2245	1.7327	45
16	1.1960	1.8230	44	1.2100	1.7761	44	1.2248	1.7320	44
17	1.1962	1.8222	43	1.2102	1.7753	43	1.2250	1.7313	43
18	1.1964	1.8214	42	1.2105	1.7745	42	1.2253	1.7306	42
19	1.1967	1.8206	41	1.2108	1.7738	41	1.2255	1.7298	41
20	1.1969	1.8198	40	1.2110	1.7730	40	1.2258	1.7291	40
21	1.1971	1.8190	39	1.2112	1.7723	39	1.2260	1.7284	39
22	1.1974	1.8182	38	1.2115	1.7715	38	1.2263	1.7277	38
23	1.1976	1.8174	37	1.2117	1.7708	37	1.2265	1.7270	37
24	1.1978	1.8166	36	1.2120	1.7700	36	1.2268	1.7263	36
25	1.1981	1.8158	35	1.2122	1.7693	35	1.2271	1.7256	35
26	1.1983	1.8150	34	1.2124	1.7685	34	1.2273	1.7249	34
27	1.1985	1.8142	33	1.2127	1.7678	33	1.2276	1.7242	33
28	1.1987	1.8134	32	1.2129	1.7670	32	1.2278	1.7235	32
29	1.1990	1.8126	31	1.2132	1.7663	31	1.2281	1.7228	31
30	1.1992	1.8118	30	1.2134	1.7655	30	1.2283	1.7221	30
31	1.1994	1.8110	29	1.2136	1.7648	29	1.2286	1.7213	29
32	1.1997	1.8102	28	1.2139	1.7640	28	1.2288	1.7206	28
33	1.1999	1.8094	27	1.2141	1.7633	27	1.2291	1.7199	27
34	1.2001	1.8086	26	1.2144	1.7625	26	1.2293	1.7192	26
35	1.2004	1.8078	25	1.2146	1.7618	25	1.2296	1.7185	25
36	1.2006	1.8070	24	1.2149	1.7610	24	1.2299	1.7178	24
37	1.2008	1.8062	23	1.2151	1.7603	23	1.2301	1.7172	23
38	1.2011	1.8055	22	1.2154	1.7596	22	1.2304	1.7165	22
39	1.2013	1.8047	21	1.2156	1.7588	21	1.2306	1.7158	21
40	1.2015	1.8039	20	1.2158	1.7581	20	1.2309	1.7151	20
41	1.2018	1.8031	19	1.2161	1.7573	19	1.2311	1.7144	19
42	1.2020	1.8023	18	1.2163	1.7566	18	1.2314	1.7137	18
43	1.2022	1.8015	17	1.2166	1.7559	17	1.2317	1.7130	17
44	1.2025	1.8007	16	1.2168	1.7551	16	1.2319	1.7123	16
45	1.2027	1.8000	15	1.2171	1.7544	15	1.2322	1.7116	15
46	1.2029	1.7992	14	1.2173	1.7537	14	1.2324	1.7109	14
47	1.2032	1.7984	13	1.2176	1.7529	13	1.2327	1.7102	13
48	1.2034	1.7976	12	1.2178	1.7522	12	1.2328	1.7095	12
49	1.2036	1.7968	11	1.2181	1.7515	11	1.2332	1.7088	11
50	1.2039	1.7960	10	1.2183	1.7507	10	1.2335	1.7081	10
51	1.2041	1.7953	9	1.2185	1.7500	9	1.2337	1.7075	9
52	1.2043	1.7945	8	1.2188	1.7493	8	1.2340	1.7068	8
53	1.2046	1.7937	7	1.2190	1.7485	7	1.2342	1.7061	7
54	1.2048	1.7929	6	1.2193	1.7478	6	1.2345	1.7054	6
55	1.2050	1.7922	5	1.2195	1.7471	5	1.2348	1.7047	5
56	1.2053	1.7914	4	1.2198	1.7463	4	1.2350	1.7040	4
57	1.2055	1.7906	3	1.2200	1.7456	3	1.2353	1.7033	3
58	1.2057	1.7898	2	1.2203	1.7449	2	1.2355	1.7027	2
59	1.2060	1.7891	1	1.2205	1.7442	1	1.2358	1.7020	1
60	1.2062	1.7883	0	1.2208	1.7434	0	1.2361	1.7013	0

123° (303°) (236°) 56° 124° (304°) (235°) 55° 125° (305°) (234°) 54°

Table 8 NATURAL SECANTS AND COSECANTS (continued)

36° (216°)			(323°) 141°			37° (217°)			(322°) 142°			38° (218°)			(321°) 141°		
<i>r</i>	Sec	Csc	<i>r</i>	Sec	Csc	<i>r</i>	Sec	Csc	<i>r</i>	Sec	Csc	<i>r</i>	Sec	Csc	<i>r</i>	Sec	Csc
0	1.2291	1.7013	60	1.2521	1.6516	60	1.2590	1.6243	60	1.2590	1.6243	60	1.2590	1.6243	60	1.2590	1.6243
1	1.2283	1.7006	59	1.2524	1.6510	59	1.2593	1.6237	59	1.2593	1.6237	59	1.2593	1.6237	59	1.2593	1.6237
2	1.2266	1.6999	58	1.2527	1.6504	58	1.2596	1.6231	58	1.2596	1.6231	58	1.2596	1.6231	58	1.2596	1.6231
3	1.2259	1.6993	57	1.2530	1.6497	57	1.2599	1.6225	57	1.2599	1.6225	57	1.2599	1.6225	57	1.2599	1.6225
4	1.2251	1.6986	56	1.2533	1.6491	56	1.2602	1.6219	56	1.2602	1.6219	56	1.2602	1.6219	56	1.2602	1.6219
5	1.2237	1.6979	55	1.2535	1.6484	55	1.2605	1.6213	55	1.2605	1.6213	55	1.2605	1.6213	55	1.2605	1.6213
6	1.2236	1.6972	54	1.2538	1.6478	54	1.2608	1.6207	54	1.2608	1.6207	54	1.2608	1.6207	54	1.2608	1.6207
7	1.2229	1.6966	53	1.2541	1.6472	53	1.2611	1.6201	53	1.2611	1.6201	53	1.2611	1.6201	53	1.2611	1.6201
8	1.2222	1.6959	52	1.2543	1.6465	52	1.2613	1.6195	52	1.2613	1.6195	52	1.2613	1.6195	52	1.2613	1.6195
9	1.2214	1.6952	51	1.2546	1.6459	51	1.2616	1.6189	51	1.2616	1.6189	51	1.2616	1.6189	51	1.2616	1.6189
10	1.2207	1.6945	50	1.2549	1.6453	50	1.2619	1.6183	50	1.2619	1.6183	50	1.2619	1.6183	50	1.2619	1.6183
11	1.2200	1.6939	49	1.2552	1.6446	49	1.2622	1.6177	49	1.2622	1.6177	49	1.2622	1.6177	49	1.2622	1.6177
12	1.2192	1.6932	48	1.2554	1.6440	48	1.2625	1.6171	48	1.2625	1.6171	48	1.2625	1.6171	48	1.2625	1.6171
13	1.2185	1.6925	47	1.2557	1.6434	47	1.2628	1.6165	47	1.2628	1.6165	47	1.2628	1.6165	47	1.2628	1.6165
14	1.2177	1.6918	46	1.2559	1.6427	46	1.2631	1.6159	46	1.2631	1.6159	46	1.2631	1.6159	46	1.2631	1.6159
15	1.2170	1.6912	45	1.2562	1.6421	45	1.2634	1.6153	45	1.2634	1.6153	45	1.2634	1.6153	45	1.2634	1.6153
16	1.2163	1.6905	44	1.2565	1.6415	44	1.2637	1.6147	44	1.2637	1.6147	44	1.2637	1.6147	44	1.2637	1.6147
17	1.2155	1.6898	43	1.2568	1.6408	43	1.2640	1.6141	43	1.2640	1.6141	43	1.2640	1.6141	43	1.2640	1.6141
18	1.2148	1.6892	42	1.2571	1.6402	42	1.2643	1.6135	42	1.2643	1.6135	42	1.2643	1.6135	42	1.2643	1.6135
19	1.2141	1.6885	41	1.2574	1.6396	41	1.2646	1.6129	41	1.2646	1.6129	41	1.2646	1.6129	41	1.2646	1.6129
20	1.2134	1.6878	40	1.2577	1.6389	40	1.2649	1.6123	40	1.2649	1.6123	40	1.2649	1.6123	40	1.2649	1.6123
21	1.2126	1.6871	39	1.2579	1.6383	39	1.2651	1.6117	39	1.2651	1.6117	39	1.2651	1.6117	39	1.2651	1.6117
22	1.2119	1.6865	38	1.2582	1.6377	38	1.2654	1.6111	38	1.2654	1.6111	38	1.2654	1.6111	38	1.2654	1.6111
23	1.2111	1.6858	37	1.2585	1.6371	37	1.2657	1.6105	37	1.2657	1.6105	37	1.2657	1.6105	37	1.2657	1.6105
24	1.2104	1.6852	36	1.2588	1.6364	36	1.2660	1.6099	36	1.2660	1.6099	36	1.2660	1.6099	36	1.2660	1.6099
25	1.2097	1.6845	35	1.2591	1.6358	35	1.2663	1.6093	35	1.2663	1.6093	35	1.2663	1.6093	35	1.2663	1.6093
26	1.2090	1.6838	34	1.2593	1.6352	34	1.2666	1.6087	34	1.2666	1.6087	34	1.2666	1.6087	34	1.2666	1.6087
27	1.2082	1.6832	33	1.2596	1.6346	33	1.2669	1.6082	33	1.2669	1.6082	33	1.2669	1.6082	33	1.2669	1.6082
28	1.2075	1.6825	32	1.2599	1.6340	32	1.2672	1.6076	32	1.2672	1.6076	32	1.2672	1.6076	32	1.2672	1.6076
29	1.2067	1.6818	31	1.2602	1.6333	31	1.2675	1.6070	31	1.2675	1.6070	31	1.2675	1.6070	31	1.2675	1.6070
30	1.2060	1.6812	30	1.2605	1.6327	30	1.2678	1.6064	30	1.2678	1.6064	30	1.2678	1.6064	30	1.2678	1.6064
31	1.2052	1.6805	29	1.2608	1.6321	29	1.2681	1.6058	29	1.2681	1.6058	29	1.2681	1.6058	29	1.2681	1.6058
32	1.2045	1.6799	28	1.2611	1.6314	28	1.2684	1.6052	28	1.2684	1.6052	28	1.2684	1.6052	28	1.2684	1.6052
33	1.2038	1.6792	27	1.2613	1.6308	27	1.2687	1.6046	27	1.2687	1.6046	27	1.2687	1.6046	27	1.2687	1.6046
34	1.2031	1.6785	26	1.2615	1.6302	26	1.2690	1.6040	26	1.2690	1.6040	26	1.2690	1.6040	26	1.2690	1.6040
35	1.2024	1.6779	25	1.2619	1.6296	25	1.2693	1.6035	25	1.2693	1.6035	25	1.2693	1.6035	25	1.2693	1.6035
36	1.2016	1.6772	24	1.2622	1.6290	24	1.2696	1.6029	24	1.2696	1.6029	24	1.2696	1.6029	24	1.2696	1.6029
37	1.2009	1.6766	23	1.2624	1.6283	23	1.2699	1.6023	23	1.2699	1.6023	23	1.2699	1.6023	23	1.2699	1.6023
38	1.2002	1.6759	22	1.2627	1.6277	22	1.2702	1.6017	22	1.2702	1.6017	22	1.2702	1.6017	22	1.2702	1.6017
39	1.2004	1.6753	21	1.2630	1.6271	21	1.2704	1.6011	21	1.2704	1.6011	21	1.2704	1.6011	21	1.2704	1.6011
40	1.2017	1.6746	20	1.2633	1.6265	20	1.2707	1.6005	20	1.2707	1.6005	20	1.2707	1.6005	20	1.2707	1.6005
41	1.2030	1.6739	19	1.2635	1.6259	19	1.2710	1.6000	19	1.2710	1.6000	19	1.2710	1.6000	19	1.2710	1.6000
42	1.2043	1.6733	18	1.2639	1.6253	18	1.2713	1.5994	18	1.2713	1.5994	18	1.2713	1.5994	18	1.2713	1.5994
43	1.2056	1.6726	17	1.2641	1.6246	17	1.2716	1.5988	17	1.2716	1.5988	17	1.2716	1.5988	17	1.2716	1.5988
44	1.2069	1.6720	16	1.2644	1.6240	16	1.2719	1.5982	16	1.2719	1.5982	16	1.2719	1.5982	16	1.2719	1.5982
45	1.2082	1.6713	15	1.2647	1.6234	15	1.2722	1.5976	15	1.2722	1.5976	15	1.2722	1.5976	15	1.2722	1.5976
46	1.2095	1.6707	14	1.2650	1.6228	14	1.2725	1.5971	14	1.2725	1.5971	14	1.2725	1.5971	14	1.2725	1.5971
47	1.2108	1.6700	13	1.2653	1.6222	13	1.2728	1.5965	13	1.2728	1.5965	13	1.2728	1.5965	13	1.2728	1.5965
48	1.2121	1.6694	12	1.2656	1.6216	12	1.2731	1.5959	12	1.2731	1.5959	12	1.2731	1.5959	12	1.2731	1.5959
49	1.2134	1.6687	11	1.2659	1.6210	11	1.2734	1.5953	11	1.2734	1.5953	11	1.2734	1.5953	11	1.2734	1.5953
50	1.2147	1.6681	10	1.2661	1.6203	10	1.2737	1.5948	10	1.2737	1.5948	10	1.2737	1.5948	10	1.2737	1.5948
51	1.2160	1.6674	9	1.2664	1.6207	9	1.2740	1.5942	9	1.2740	1.5942	9	1.2740	1.5942	9	1.2740	1.5942
52	1.2173	1.6668	8	1.2667	1.6201	8	1.2743	1.5936	8	1.2743	1.5936	8	1.2743	1.5936	8	1.2743	1.5936
53	1.2186	1.6661	7	1.2670	1.6205	7	1.2746	1.5930	7	1.2746	1.5930	7	1.2746	1.5930	7	1.2746	1.5930
54	1.2199	1.6655	6	1.2673	1.6209	6	1.2749	1.5925	6	1.2749	1.5925	6	1.2749	1.5925	6	1.2749	1.5925
55	1.2212	1.6649	5	1.2676	1.6213	5	1.2752	1.5919	5	1.2752	1.5919	5	1.2752	1.5919	5	1.2752	1.5919
56	1.2225	1.6642	4	1.2679	1.6217	4	1.2755	1.5913	4	1.2755	1.5913	4	1.2755	1.5913	4	1.2755	1.5913
57	1.2238	1.6635	3	1.2682	1.6221	3	1.2758	1.5907	3	1.2758	1.5907	3	1.2758	1.5907	3	1.2758	1.5907
58	1.2251	1.6629	2	1.2684	1.6225	2	1.2761	1.5902	2	1.2761	1.5902	2	1.2761	1.5902	2	1.2761	1.5902
59	1.2264	1.6623	1	1.2687	1.6229	1	1.2764	1.5896	1	1.2764	1.5896	1	1.2764	1.5896	1	1.2764	1.5896
60	1.2277	1.6617	0	1.2690	1.6233	0	1.2767	1.5890	0	1.2767	1.5890	0	1.2767	1.5890	0	1.2767	1.5890

126° (306°) (233°) 53° 127° (307°) (232°) 52° 128° (308°) (231°) 51°

Table 8 NATURAL SECANTS AND COSECANTS (continued)

39° (219°)			(320°) 140°			40° (220°)			(319°) 139°			41° (221°)			(318°) 138°		
°	Sec	Csc	°	Sec	Csc	°	Sec	Csc	°	Sec	Csc	°	Sec	Csc	°	Sec	Csc
0	1.2868	1.5890	60	1.3084	1.5557	60	1.3250	1.5243	0	1.3250	1.5243	60	1.3250	1.5243	60	1.3455	1.4945
1	1.2871	1.5884	59	1.3057	1.5552	59	1.3253	1.5237	1	1.3253	1.5237	59	1.3452	1.4948	59	1.3442	1.4954
2	1.2874	1.5879	58	1.3060	1.5546	58	1.3257	1.5232	2	1.3257	1.5232	58	1.3449	1.4959	4	1.3446	1.4959
3	1.2877	1.5873	57	1.3064	1.5541	57	1.3260	1.5227	3	1.3260	1.5227	57	1.3443	1.4979	7	1.3443	1.4979
4	1.2880	1.5867	56	1.3067	1.5536	56	1.3264	1.5222	4	1.3264	1.5222	56	1.3435	1.4974	6	1.3435	1.4974
5	1.2883	1.5862	55	1.3070	1.5530	55	1.3267	1.5217	5	1.3267	1.5217	55	1.3439	1.4969	5	1.3439	1.4969
6	1.2886	1.5856	54	1.3073	1.5525	54	1.3270	1.5212	6	1.3270	1.5212	54	1.3442	1.4964	4	1.3442	1.4964
7	1.2889	1.5850	53	1.3076	1.5520	53	1.3274	1.5207	7	1.3274	1.5207	53	1.3449	1.4959	3	1.3449	1.4959
8	1.2892	1.5845	52	1.3080	1.5514	52	1.3277	1.5202	8	1.3277	1.5202	52	1.3459	1.4950	2	1.3459	1.4950
9	1.2895	1.5839	51	1.3083	1.5509	51	1.3280	1.5197	9	1.3280	1.5197	51	1.3453	1.4950	1	1.3453	1.4950
10	1.2898	1.5833	50	1.3086	1.5504	50	1.3284	1.5192	10	1.3284	1.5192	50	1.3455	1.4945	0	1.3455	1.4945
11	1.2901	1.5828	49	1.3089	1.5498	49	1.3287	1.5187	11	1.3287	1.5187	49					
12	1.2904	1.5822	48	1.3093	1.5493	48	1.3291	1.5182	12	1.3291	1.5182	48					
13	1.2907	1.5816	47	1.3096	1.5488	47	1.3294	1.5177	13	1.3294	1.5177	47					
14	1.2910	1.5811	46	1.3099	1.5482	46	1.3297	1.5172	14	1.3297	1.5172	46					
15	1.2913	1.5805	45	1.3102	1.5477	45	1.3301	1.5167	15	1.3301	1.5167	45					
16	1.2916	1.5800	44	1.3105	1.5472	44	1.3304	1.5162	16	1.3304	1.5162	44					
17	1.2919	1.5794	43	1.3109	1.5466	43	1.3307	1.5156	17	1.3307	1.5156	43					
18	1.2923	1.5788	42	1.3112	1.5461	42	1.3311	1.5151	18	1.3311	1.5151	42					
19	1.2926	1.5783	41	1.3115	1.5456	41	1.3314	1.5146	19	1.3314	1.5146	41					
20	1.2929	1.5777	40	1.3118	1.5450	40	1.3318	1.5141	20	1.3318	1.5141	40					
21	1.2932	1.5771	39	1.3122	1.5445	39	1.3321	1.5136	21	1.3321	1.5136	39					
22	1.2935	1.5766	38	1.3125	1.5440	38	1.3325	1.5131	22	1.3325	1.5131	38					
23	1.2938	1.5760	37	1.3128	1.5435	37	1.3328	1.5126	23	1.3328	1.5126	37					
24	1.2941	1.5755	36	1.3131	1.5429	36	1.3331	1.5121	24	1.3331	1.5121	36					
25	1.2944	1.5749	35	1.3135	1.5424	35	1.3335	1.5116	25	1.3335	1.5116	35					
26	1.2947	1.5744	34	1.3138	1.5419	34	1.3338	1.5111	26	1.3338	1.5111	34					
27	1.2950	1.5738	33	1.3141	1.5413	33	1.3342	1.5107	27	1.3342	1.5107	33					
28	1.2953	1.5732	32	1.3144	1.5408	32	1.3345	1.5102	28	1.3345	1.5102	32					
29	1.2957	1.5727	31	1.3148	1.5403	31	1.3348	1.5097	29	1.3348	1.5097	31					
30	1.2960	1.5721	30	1.3151	1.5398	30	1.3352	1.5092	30	1.3352	1.5092	30					
31	1.2963	1.5716	29	1.3154	1.5392	29	1.3355	1.5087	31	1.3355	1.5087	29					
32	1.2966	1.5710	28	1.3157	1.5387	28	1.3359	1.5082	32	1.3359	1.5082	28					
33	1.2969	1.5705	27	1.3161	1.5382	27	1.3362	1.5077	33	1.3362	1.5077	27					
34	1.2972	1.5699	26	1.3164	1.5377	26	1.3366	1.5072	34	1.3366	1.5072	26					
35	1.2975	1.5694	25	1.3167	1.5372	25	1.3369	1.5067	35	1.3369	1.5067	25					
36	1.2978	1.5688	24	1.3171	1.5366	24	1.3373	1.5062	36	1.3373	1.5062	24					
37	1.2981	1.5683	23	1.3174	1.5361	23	1.3376	1.5057	37	1.3376	1.5057	23					
38	1.2985	1.5677	22	1.3177	1.5356	22	1.3380	1.5052	38	1.3380	1.5052	22					
39	1.2988	1.5672	21	1.3180	1.5351	21	1.3383	1.5047	39	1.3383	1.5047	21					
40	1.2991	1.5666	20	1.3184	1.5345	20	1.3386	1.5042	40	1.3386	1.5042	20					
41	1.2994	1.5661	19	1.3187	1.5340	19	1.3390	1.5037	41	1.3390	1.5037	19					
42	1.2997	1.5655	18	1.3190	1.5335	18	1.3393	1.5032	42	1.3393	1.5032	18					
43	1.3000	1.5650	17	1.3194	1.5330	17	1.3397	1.5027	43	1.3397	1.5027	17					
44	1.3003	1.5644	16	1.3197	1.5325	16	1.3400	1.5023	44	1.3400	1.5023	16					
45	1.3007	1.5639	15	1.3200	1.5320	15	1.3404	1.5018	45	1.3404	1.5018	15					
46	1.3010	1.5633	14	1.3203	1.5314	14	1.3407	1.5013	46	1.3407	1.5013	14					
47	1.3013	1.5628	13	1.3207	1.5309	13	1.3411	1.5008	47	1.3411	1.5008	13					
48	1.3016	1.5622	12	1.3210	1.5304	12	1.3414	1.5003	48	1.3414	1.5003	12					
49	1.3019	1.5617	11	1.3213	1.5299	11	1.3418	1.4998	49	1.3418	1.4998	11					
50	1.3022	1.5611	10	1.3217	1.5294	10	1.3421	1.4993	50	1.3421	1.4993	10					
51	1.3026	1.5606	9	1.3220	1.5289	9	1.3425	1.4988	51	1.3425	1.4988	9					
52	1.3029	1.5601	8	1.3223	1.5283	8	1.3428	1.4984	52	1.3428	1.4984	8					
53	1.3032	1.5595	7	1.3227	1.5278	7	1.3432	1.4979	53	1.3432	1.4979	7					
54	1.3035	1.5590	6	1.3230	1.5273	6	1.3435	1.4974	54	1.3435	1.4974	6					
55	1.3038	1.5584	5	1.3233	1.5268	5	1.3439	1.4969	55	1.3439	1.4969	5					
56	1.3041	1.5579	4	1.3237	1.5263	4	1.3442	1.4964	56	1.3442	1.4964	4					
57	1.3045	1.5573	3	1.3240	1.5258	3	1.3446	1.4959	57	1.3446	1.4959	3					
58	1.3048	1.5568	2	1.3243	1.5253	2	1.3449	1.4954	58	1.3449	1.4954	2					
59	1.3051	1.5563	1	1.3247	1.5248	1	1.3453	1.4950	59	1.3453	1.4950	1					
60	1.3054	1.5557	0	1.3250	1.5243	0	1.3455	1.4945	60	1.3455	1.4945	0					

129° (309°)

(230°) 50°

130° (310°)

(229°) 49°

131° (311°)

(228°) 48°

Table 8 NATURAL SECANTS AND COSECANTS (continued)

42° (222°)			(317°) 137°	43° (223°)			(316°) 136°	44° (224°)			(315°) 135°	
'	Sec	Csc	'	Sec	Csc	'	Sec	Csc	'	Sec	Csc	
0	1.3456	1.4945	60			0	1.3673	1.4663	60	1.3902	1.4388	60
1	1.3460	1.4940	59			1	1.3677	1.4658	59	1.3906	1.4381	59
2	1.3463	1.4935	58			2	1.3681	1.4654	58	1.3909	1.4377	58
3	1.3467	1.4930	57			3	1.3684	1.4649	57	1.3913	1.4373	57
4	1.3470	1.4925	56			4	1.3688	1.4645	56	1.3917	1.4378	56
5	1.3474	1.4921	55			5	1.3692	1.4640	55	1.3921	1.4374	55
6	1.3478	1.4916	54			6	1.3696	1.4635	54	1.3925	1.4370	54
7	1.3481	1.4911	53			7	1.3699	1.4631	53	1.3929	1.4365	53
8	1.3485	1.4905	52			8	1.3703	1.4626	52	1.3933	1.4361	52
9	1.3488	1.4901	51			9	1.3707	1.4622	51	1.3937	1.4357	51
10	1.3492	1.4897	50			10	1.3711	1.4617	50	1.3941	1.4352	50
11	1.3495	1.4892	49			11	1.3714	1.4613	49	1.3945	1.4348	49
12	1.3499	1.4887	48			12	1.3718	1.4608	48	1.3949	1.4344	48
13	1.3502	1.4882	47			13	1.3722	1.4604	47	1.3953	1.4340	47
14	1.3506	1.4878	46			14	1.3726	1.4599	46	1.3957	1.4335	46
15	1.3510	1.4873	45			15	1.3729	1.4595	45	1.3961	1.4331	45
16	1.3513	1.4868	44			16	1.3733	1.4590	44	1.3965	1.4327	44
17	1.3517	1.4863	43			17	1.3737	1.4586	43	1.3969	1.4322	43
18	1.3520	1.4859	42			18	1.3741	1.4581	42	1.3972	1.4318	42
19	1.3524	1.4854	41			19	1.3744	1.4577	41	1.3976	1.4314	41
20	1.3527	1.4849	40			20	1.3748	1.4572	40	1.3980	1.4310	40
21	1.3531	1.4844	39			21	1.3752	1.4568	39	1.3984	1.4305	39
22	1.3535	1.4840	38			22	1.3756	1.4563	38	1.3988	1.4301	38
23	1.3538	1.4835	37			23	1.3759	1.4559	37	1.3992	1.4297	37
24	1.3542	1.4830	36			24	1.3763	1.4554	36	1.3996	1.4293	36
25	1.3545	1.4825	35			25	1.3767	1.4550	35	1.4000	1.4289	35
26	1.3549	1.4821	34			26	1.3771	1.4545	34	1.4004	1.4284	34
27	1.3553	1.4816	33			27	1.3775	1.4541	33	1.4008	1.4280	33
28	1.3556	1.4811	32			28	1.3778	1.4536	32	1.4012	1.4275	32
29	1.3560	1.4807	31			29	1.3782	1.4532	31	1.4016	1.4271	31
30	1.3563	1.4802	30			30	1.3786	1.4527	30	1.4020	1.4267	30
31	1.3567	1.4797	29			31	1.3790	1.4523	29	1.4024	1.4262	29
32	1.3571	1.4792	28			32	1.3794	1.4518	28	1.4028	1.4258	28
33	1.3574	1.4788	27			33	1.3797	1.4514	27	1.4032	1.4253	27
34	1.3578	1.4783	26			34	1.3801	1.4510	26	1.4036	1.4249	26
35	1.3582	1.4778	25			35	1.3805	1.4505	25	1.4040	1.4244	25
36	1.3585	1.4774	24			36	1.3809	1.4501	24	1.4044	1.4242	24
37	1.3589	1.4769	23			37	1.3813	1.4496	23	1.4048	1.4238	23
38	1.3592	1.4764	22			38	1.3817	1.4492	22	1.4052	1.4234	22
39	1.3596	1.4760	21			39	1.3820	1.4487	21	1.4057	1.4229	21
40	1.3600	1.4755	20			40	1.3824	1.4483	20	1.4061	1.4225	20
41	1.3603	1.4750	19			41	1.3828	1.4479	19	1.4065	1.4221	19
42	1.3607	1.4746	18			42	1.3832	1.4474	18	1.4069	1.4217	18
43	1.3611	1.4741	17			43	1.3836	1.4470	17	1.4073	1.4213	17
44	1.3614	1.4737	16			44	1.3840	1.4465	16	1.4077	1.4208	16
45	1.3618	1.4732	15			45	1.3843	1.4461	15	1.4081	1.4204	15
46	1.3622	1.4727	14			46	1.3847	1.4457	14	1.4085	1.4200	14
47	1.3625	1.4723	13			47	1.3851	1.4452	13	1.4089	1.4195	13
48	1.3629	1.4718	12			48	1.3855	1.4448	12	1.4093	1.4191	12
49	1.3633	1.4713	11			49	1.3858	1.4443	11	1.4097	1.4186	11
50	1.3636	1.4709	10			50	1.3863	1.4439	10	1.4101	1.4182	10
51	1.3640	1.4704	9			51	1.3867	1.4435	9	1.4105	1.4178	9
52	1.3644	1.4700	8			52	1.3871	1.4430	8	1.4109	1.4175	8
53	1.3647	1.4695	7			53	1.3874	1.4426	7	1.4113	1.4171	7
54	1.3651	1.4690	6			54	1.3878	1.4422	6	1.4118	1.4167	6
55	1.3655	1.4686	5			55	1.3882	1.4417	5	1.4122	1.4163	5
56	1.3658	1.4681	4			56	1.3886	1.4413	4	1.4126	1.4159	4
57	1.3662	1.4677	3			57	1.3890	1.4409	3	1.4130	1.4154	3
58	1.3666	1.4672	2			58	1.3894	1.4404	2	1.4134	1.4150	2
59	1.3670	1.4667	1			59	1.3898	1.4400	1	1.4138	1.4146	1
60	1.3673	1.4663	0			60	1.3902	1.4396	0	1.4142	1.4142	0
'	Csc	Sec	'	'	Csc	'	Csc	Sec	'	Csc	Sec	'

132° (312°)

(227°) 47°

133° (313°)

(226°) 46°

134° (314°)

(225°) 45°

Table 9 MINUTES AND SECONDS TO DECIMAL PARTS OF A DEGREE

MINUTES AND SECONDS TO DECIMAL PARTS OF A DEGREE				DECIMAL PARTS OF A DEGREE TO MINUTES AND SECONDS			
Min.	Degrees	Sec.	Degrees	Deg.	"	Deg.	"
0	0.00000	0	0.00000	0.60	0 00	0.60	36 00
1	.01667	1	.00028	.61	0 36	.61	36 36
2	.03333	2	.00055	.62	1 12	.62	37 12
3	.05000	3	.00083	.63	1 48	.63	37 48
4	.06667	4	.00111	.64	2 24	.64	38 24
5	.08333	5	.00139	.65	3 00	.65	39 00
6	.10000	6	.00167	.66	3 36	.66	39 36
7	.11667	7	.00194	.67	4 12	.67	40 12
8	.13333	8	.00222	.68	4 48	.68	40 48
9	.15000	9	.00250	.69	5 24	.69	41 24
10	0.16667	10	0.00278	0.70	6 00	0.70	42 00
11	.18333	11	.00305	.71	6 36	.71	42 36
12	.20000	12	.00333	.72	7 12	.72	43 12
13	.21667	13	.00361	.73	7 48	.73	43 48
14	.23333	14	.00389	.74	8 24	.74	44 24
15	.25000	15	.00417	.75	9 00	.75	45 00
16	.26667	16	.00444	.76	9 36	.76	45 36
17	.28333	17	.00472	.77	10 12	.77	46 12
18	.30000	18	.00500	.78	10 48	.78	46 48
19	.31667	19	.00527	.79	11 24	.79	47 24
20	0.33333	20	0.00556	0.80	12 00	0.80	48 00
21	.35000	21	.00583	.81	12 36	.81	48 36
22	.36667	22	.00611	.82	13 12	.82	49 12
23	.38333	23	.00639	.83	13 48	.83	49 48
24	.40000	24	.00667	.84	14 24	.84	50 24
25	.41667	25	.00694	.85	15 00	.85	51 00
26	.43333	26	.00722	.86	15 36	.86	51 36
27	.45000	27	.00750	.87	16 12	.87	52 12
28	.46667	28	.00778	.88	16 48	.88	52 48
29	.48333	29	.00805	.89	17 24	.89	53 24
30	0.50000	30	0.00833	0.90	18 00	0.90	54 00
31	.51667	31	.00861	.91	18 36	.91	54 36
32	.53333	32	.00889	.92	19 12	.92	55 12
33	.55000	33	.00916	.93	19 48	.93	55 48
34	.56667	34	.00944	.94	20 24	.94	56 24
35	.58333	35	.00972	.95	21 00	.95	57 00
36	.60000	36	.01000	.96	21 36	.96	57 36
37	.61667	37	.01028	.97	22 12	.97	58 12
38	.63333	38	.01055	.98	22 48	.98	58 48
39	.65000	39	.01083	.99	23 24	.99	59 24
40	0.66667	40	0.01111	1.00	24 00	1.00	60 00
41	.68333	41	.01139	.41	24 36		
42	.70000	42	.01167	.42	25 12		
43	.71667	43	.01194	.43	25 48		
44	.73333	44	.01222	.44	26 24		
45	.75000	45	.01250	.45	27 00		
46	.76667	46	.01278	.46	27 36	0.036	0.0
47	.78333	47	.01305	.47	28 12	.001	3.6
48	.80000	48	.01333	.48	28 48	.032	7.2
49	.81667	49	.01361	.49	29 24	.002	10.8
50	0.83333	50	0.01389	.50	30 00	.004	14.4
51	.85000	51	.01416	.51	30 36	.035	18.0
52	.86667	52	.01444	.52	31 12	.036	21.6
53	.88333	53	.01472	.53	31 48	.007	25.2
54	.90000	54	.01500	.54	32 24	.008	28.8
55	.91667	55	.01527	.55	33 00	.009	32.4
56	.93333	56	.01555	.56	33 36		
57	.95000	57	.01583	.57	34 12	0.010	36.0
58	.96667	58	.01611	.58	34 48		
59	.98333	59	.01639	.59	35 24		
60	1.00000	60	0.01667	0.60	36		

Table 10 NATURAL TRIGONOMETRIC FUNCTIONS FOR DECIMAL FRACTIONS OF A DEGREE (continued)

Table with 5 columns (Deg, Sin, Tan, Ctn, Cos) and 20 rows of data, followed by a second table with 5 columns (Deg, Sin, Tan, Ctn, Cos) and 20 rows of data. Includes a legend at the bottom for Cos, Ctn, Tan, Sin, and Deg.

Table 10 NATURAL TRIGONOMETRIC FUNCTIONS FOR DECIMAL FRACTIONS OF A DEGREE (continued)

Deg.	Sin	Tan	Ctn	Cos	Deg.	Sin	Tan	Ctn	Cos	
24.0	.40674	.44523	2.2460	.91355	56.0	.86000	.57735	1.7321	.86603	
24.1	.40833	.44732	2.2355	.91283	56.9	.86151	.57968	1.7251	.86515	
24.2	.40992	.44942	2.2251	.91212	57.8	.86302	.58201	1.7182	.86427	
24.3	.41151	.45152	2.2148	.91140	58.7	.86453	.58435	1.7113	.86340	
24.4	.41310	.45362	2.2045	.91068	58.6	.86603	.58670	1.7045	.86251	
24.5	.41469	.45573	2.1943	.90996	58.5	.86754	.58905	1.6977	.86163	
24.6	.41628	.45784	2.1842	.90924	58.4	.86904	.59140	1.6909	.86074	
24.7	.41787	.45995	2.1742	.90851	58.3	.87054	.59375	1.6842	.85985	
24.8	.41945	.46205	2.1642	.90778	58.2	.87204	.59612	1.6775	.85896	
24.9	.42104	.46418	2.1543	.90704	58.1	.87354	.59849	1.6709	.85806	
25.0	.42262	.46631	2.1445	.90631	58.0	.87504	.60086	1.6643	.85717	
25.1	.42420	.46843	2.1348	.90557	57.9	.87653	.60324	1.6577	.85627	
25.2	.42578	.47056	2.1251	.90483	57.8	.87803	.60562	1.6512	.85536	
25.3	.42735	.47270	2.1155	.90408	57.7	.87952	.60801	1.6447	.85445	
25.4	.42894	.47483	2.1060	.90334	57.6	.88101	.61040	1.6383	.85355	
25.5	.43051	.47698	2.0965	.90259	57.5	.88250	.61280	1.6319	.85264	
25.6	.43209	.47912	2.0872	.90183	57.4	.88399	.61520	1.6255	.85173	
25.7	.43365	.48127	2.0778	.90108	57.3	.88547	.61761	1.6191	.85081	
25.8	.43523	.48342	2.0686	.90032	57.2	.88696	.62003	1.6128	.84989	
25.9	.43680	.48557	2.0594	.89956	57.1	.88844	.62245	1.6066	.84897	
26.0	.43837	.48773	2.0503	.89879	57.0	.88992	.62487	1.6003	.84805	
26.1	.43994	.48989	2.0413	.89802	56.9	.89140	.62730	1.5941	.84712	
26.2	.44151	.49205	2.0323	.89726	56.8	.89288	.62973	1.5880	.84619	
26.3	.44307	.49423	2.0233	.89649	56.7	.89435	.63217	1.5818	.84526	
26.4	.44464	.49640	2.0145	.89571	56.6	.89583	.63462	1.5757	.84433	
26.5	.44620	.49858	2.0057	.89493	56.5	.89730	.63707	1.5697	.84339	
26.6	.44776	.50076	1.9970	.89415	56.4	.89877	.63953	1.5637	.84245	
26.7	.44932	.50295	1.9883	.89337	56.3	.89924	.64199	1.5577	.84151	
26.8	.45088	.50514	1.9797	.89259	56.2	.90071	.64446	1.5517	.84057	
26.9	.45243	.50733	1.9711	.89180	56.1	.90217	.64693	1.5458	.83962	
27.0	.45399	.50953	1.9626	.89101	56.0	.90364	.64941	1.5399	.83867	
27.1	.45554	.51173	1.9542	.89021	55.9	.90510	.65189	1.5340	.83772	
27.2	.45710	.51393	1.9458	.88942	55.8	.90656	.65438	1.5282	.83676	
27.3	.45865	.51614	1.9375	.88862	55.7	.90802	.65688	1.5224	.83581	
27.4	.46020	.51835	1.9292	.88782	55.6	.90948	.65938	1.5166	.83485	
27.5	.46175	.52057	1.9210	.88701	55.5	.91094	.66189	1.5108	.83389	
27.6	.46330	.52279	1.9128	.88620	55.4	.91239	.66440	1.5051	.83292	
27.7	.46484	.52501	1.9047	.88539	55.3	.91384	.66692	1.4994	.83195	
27.8	.46639	.52724	1.8967	.88458	55.2	.91529	.66944	1.4938	.83098	
27.9	.46793	.52947	1.8887	.88377	55.1	.91674	.67197	1.4882	.83001	
28.0	.46947	.53171	1.8807	.88295	55.0	.91819	.67451	1.4826	.82904	
28.1	.47101	.53395	1.8728	.88213	54.9	.91964	.67705	1.4770	.82806	
28.2	.47255	.53620	1.8650	.88130	54.8	.92109	.67960	1.4715	.82708	
28.3	.47409	.53844	1.8572	.88048	54.7	.92253	.68215	1.4660	.82610	
28.4	.47562	.54070	1.8495	.87965	54.6	.92398	.68471	1.4605	.82511	
28.5	.47716	.54296	1.8418	.87882	54.5	.92542	.68728	1.4550	.82413	
28.6	.47869	.54522	1.8341	.87799	54.4	.92687	.68985	1.4496	.82314	
28.7	.48022	.54748	1.8265	.87715	54.3	.92831	.69243	1.4442	.82214	
28.8	.48175	.54975	1.8190	.87631	54.2	.92975	.69502	1.4388	.82115	
28.9	.48328	.55203	1.8115	.87546	54.1	.93119	.69761	1.4335	.82015	
29.0	.48481	.55431	1.8040	.87462	54.0	.93263	.70021	1.4281	.81915	
29.1	.48634	.55659	1.7966	.87377	53.9	.93407	.70281	1.4229	.81815	
29.2	.48786	.55888	1.7893	.87292	53.8	.93551	.70542	1.4178	.81714	
29.3	.48938	.56117	1.7820	.87207	53.7	.93695	.70804	1.4128	.81614	
29.4	.49090	.56347	1.7747	.87121	53.6	.93839	.71066	1.4077	.81513	
29.5	.49242	.56577	1.7675	.87036	53.5	.93983	.71329	1.4029	.81412	
29.6	.49394	.56806	1.7603	.86949	53.4	.94127	.71593	1.3982	.81310	
29.7	.49546	.57035	1.7532	.86863	53.3	.94271	.71857	1.3936	.81208	
29.8	.49697	.57264	1.7461	.86777	53.2	.94415	.72122	1.3891	.81106	
29.9	.49849	.57503	1.7391	.86690	53.1	.94559	.72388	1.3846	.81004	
30.0	.50000	.57735	1.7321	.86603	53.0	.94703	.72654	1.3802	.80902	
	Cos	Ctn	Tan	Sin	Deg.	Cos	Ctn	Tan	Sin	Deg.

Table 10 NATURAL TRIGONOMETRIC FUNCTIONS FOR DECIMAL FRACTIONS
OF A DEGREE (continued)

Deg.	Sin	Tan	Ctn	Cos	Deg.	Sin	Tan	Ctn	Cos	
36.0	.58779	.72654	1.3764	.80902	54.0	.80902	1.3764	.72654	.58779	
36.1	.58920	.72921	1.3713	.80799	53.9	.80799	1.3713	.72921	.58920	
36.2	.59061	.73189	1.3663	.80696	53.8	.80696	1.3663	.73189	.59061	
36.3	.59201	.73457	1.3613	.80593	53.7	.80593	1.3613	.73457	.59201	
36.4	.59342	.73726	1.3564	.80489	53.6	.80489	1.3564	.73726	.59342	
36.5	.59482	.73996	1.3514	.80386	53.5	.80386	1.3514	.73996	.59482	
36.6	.59622	.74267	1.3465	.80282	53.4	.80282	1.3465	.74267	.59622	
36.7	.59763	.74538	1.3416	.80178	53.3	.80178	1.3416	.74538	.59763	
36.8	.59902	.74810	1.3367	.80073	53.2	.80073	1.3367	.74810	.59902	
36.9	.60042	.75082	1.3319	.79968	53.1	.79968	1.3319	.75082	.60042	
37.0	.60182	.75355	1.3270	.79864	53.0	.79864	1.3270	.75355	.60182	
37.1	.60321	.75629	1.3222	.79758	52.9	.79758	1.3222	.75629	.60321	
37.2	.60460	.75904	1.3175	.79653	52.8	.79653	1.3175	.75904	.60460	
37.3	.60599	.76180	1.3127	.79547	52.7	.79547	1.3127	.76180	.60599	
37.4	.60738	.76456	1.3079	.79441	52.6	.79441	1.3079	.76456	.60738	
37.5	.60878	.76733	1.3032	.79335	52.5	.79335	1.3032	.76733	.60878	
37.6	.61018	.77010	1.2985	.79229	52.4	.79229	1.2985	.77010	.61018	
37.7	.61153	.77289	1.2938	.79122	52.3	.79122	1.2938	.77289	.61153	
37.8	.61291	.77568	1.2892	.79016	52.2	.79016	1.2892	.77568	.61291	
37.9	.61429	.77848	1.2845	.78909	52.1	.78909	1.2845	.77848	.61429	
38.0	.61566	.78129	1.2799	.78801	52.0	.78801	1.2799	.78129	.61566	
38.1	.61704	.78410	1.2753	.78694	51.9	.78694	1.2753	.78410	.61704	
38.2	.61841	.78692	1.2708	.78586	51.8	.78586	1.2708	.78692	.61841	
38.3	.61978	.78975	1.2662	.78478	51.7	.78478	1.2662	.78975	.61978	
38.4	.62115	.79259	1.2617	.78369	51.6	.78369	1.2617	.79259	.62115	
38.5	.62251	.79544	1.2572	.78261	51.5	.78261	1.2572	.79544	.62251	
38.6	.62386	.79829	1.2527	.78152	51.4	.78152	1.2527	.79829	.62386	
38.7	.62524	.80115	1.2482	.78043	51.3	.78043	1.2482	.80115	.62524	
38.8	.62660	.80402	1.2437	.77934	51.2	.77934	1.2437	.80402	.62660	
38.9	.62796	.80690	1.2393	.77824	51.1	.77824	1.2393	.80690	.62796	
39.0	.62932	.80978	1.2349	.77715	51.0	.77715	1.2349	.80978	.62932	
39.1	.63068	.81268	1.2305	.77606	50.9	.77606	1.2305	.81268	.63068	
39.2	.63203	.81558	1.2261	.77494	50.8	.77494	1.2261	.81558	.63203	
39.3	.63338	.81849	1.2218	.77384	50.7	.77384	1.2218	.81849	.63338	
39.4	.63473	.82141	1.2174	.77273	50.6	.77273	1.2174	.82141	.63473	
39.5	.63608	.82434	1.2131	.77162	50.5	.77162	1.2131	.82434	.63608	
39.6	.63742	.82727	1.2088	.77051	50.4	.77051	1.2088	.82727	.63742	
39.7	.63877	.83022	1.2045	.76940	50.3	.76940	1.2045	.83022	.63877	
39.8	.64011	.83317	1.2002	.76828	50.2	.76828	1.2002	.83317	.64011	
39.9	.64145	.83613	1.1960	.76717	50.1	.76717	1.1960	.83613	.64145	
40.0	.64279	.83910	1.1918	.76604	50.0	.76604	1.1918	.83910	.64279	
40.1	.64412	.84208	1.1875	.76492	49.9	.76492	1.1875	.84208	.64412	
40.2	.64546	.84507	1.1833	.76380	49.8	.76380	1.1833	.84507	.64546	
40.3	.64679	.84805	1.1792	.76267	49.7	.76267	1.1792	.84805	.64679	
40.4	.64812	.85107	1.1750	.76154	49.6	.76154	1.1750	.85107	.64812	
40.5	.64945	.85403	1.1708	.76041	49.5	.76041	1.1708	.85403	.64945	
40.6	.65078	.85701	1.1667	.75927	49.4	.75927	1.1667	.85701	.65078	
40.7	.65210	.86001	1.1626	.75813	49.3	.75813	1.1626	.86001	.65210	
40.8	.65342	.86303	1.1585	.75700	49.2	.75700	1.1585	.86303	.65342	
40.9	.65474	.86608	1.1544	.75585	49.1	.75585	1.1544	.86608	.65474	
41.0	.65606	.86919	1.1504	.75471	49.0	.75471	1.1504	.86919	.65606	
41.1	.65738	.87236	1.1463	.75356	48.9	.75356	1.1463	.87236	.65738	
41.2	.65869	.87553	1.1423	.75241	48.8	.75241	1.1423	.87553	.65869	
41.3	.66000	.87873	1.1382	.75126	48.7	.75126	1.1382	.87873	.66000	
41.4	.66131	.88197	1.1342	.75011	48.6	.75011	1.1342	.88197	.66131	
41.5	.66262	.88523	1.1303	.74896	48.5	.74896	1.1303	.88523	.66262	
41.6	.66393	.88854	1.1263	.74780	48.4	.74780	1.1263	.88854	.66393	
41.7	.66523	.89191	1.1224	.74664	48.3	.74664	1.1224	.89191	.66523	
41.8	.66653	.89534	1.1184	.74548	48.2	.74548	1.1184	.89534	.66653	
41.9	.66783	.89884	1.1145	.74431	48.1	.74431	1.1145	.89884	.66783	
42.0	.66913	.90240	1.1106	.74314	48.0	.74314	1.1106	.90240	.66913	
42.1	.67043	.90602	1.1067	.74198	47.9	.74198	1.1067	.90602	.67043	
42.2	.67172	.90971	1.1028	.74080	47.8	.74080	1.1028	.90971	.67172	
42.3	.67301	.91348	1.0989	.73963	47.7	.73963	1.0989	.91348	.67301	
42.4	.67430	.91733	1.0951	.73846	47.6	.73846	1.0951	.91733	.67430	
42.5	.67559	.92123	1.0913	.73728	47.5	.73728	1.0913	.92123	.67559	
42.6	.67688	.92520	1.0875	.73610	47.4	.73610	1.0875	.92520	.67688	
42.7	.67816	.92924	1.0837	.73491	47.3	.73491	1.0837	.92924	.67816	
42.8	.67944	.93336	1.0799	.73372	47.2	.73372	1.0799	.93336	.67944	
42.9	.68072	.93755	1.0761	.73254	47.1	.73254	1.0761	.93755	.68072	
43.0	.68200	.94182	1.0724	.73135	47.0	.73135	1.0724	.94182	.68200	
43.1	.68327	.94617	1.0686	.73016	46.9	.73016	1.0686	.94617	.68327	
43.2	.68455	.95061	1.0649	.72897	46.8	.72897	1.0649	.95061	.68455	
43.3	.68582	.95514	1.0612	.72777	46.7	.72777	1.0612	.95514	.68582	
43.4	.68709	.95977	1.0575	.72657	46.6	.72657	1.0575	.95977	.68709	
43.5	.68836	.96450	1.0538	.72537	46.5	.72537	1.0538	.96450	.68836	
43.6	.68962	.96932	1.0501	.72417	46.4	.72417	1.0501	.96932	.68962	
43.7	.69088	.97425	1.0464	.72297	46.3	.72297	1.0464	.97425	.69088	
43.8	.69214	.97929	1.0428	.72176	46.2	.72176	1.0428	.97929	.69214	
43.9	.69340	.98445	1.0392	.72055	46.1	.72055	1.0392	.98445	.69340	
44.0	.69466	.98972	1.0355	.71934	46.0	.71934	1.0355	.98972	.69466	
44.1	.69591	.99511	1.0319	.71813	45.9	.71813	1.0319	.99511	.69591	
44.2	.69717	.97246	1.0283	.71691	45.8	.71691	1.0283	.99772	.69717	
44.3	.69842	.97565	1.0247	.71569	45.7	.71569	1.0247	.99888	.69842	
44.4	.69966	.97927	1.0212	.71447	45.6	.71447	1.0212	.99999	.69966	
44.5	.70091	.98270	1.0176	.71325	45.5	.71325	1.0176	.99999	.70091	
44.6	.70215	.98613	1.0141	.71203	45.4	.71203	1.0141	.99999	.70215	
44.7	.70339	.98965	1.0105	.71080	45.3	.71080	1.0105	.99999	.70339	
44.8	.70463	.99324	1.0070	.70957	45.2	.70957	1.0070	.99999	.70463	
44.9	.70587	.99692	1.0035	.70834	45.1	.70834	1.0035	.99999	.70587	
45.0	.70711	1.00000	1.0000	.70711	45.0	.70711	1.00000	1.00000	.70711	
	Cos	Ctn	Tan	Sin	Deg.	Cos	Ctn	Tan	Sin	Deg.

Table 11 COMMON LOGARITHMS OF TRIGONOMETRIC FUNCTIONS IN RADIAN MEASURE

Rad	L Sin	L Tan	L Ctn	L Cos	Rad	L Sin	L Tan	L Ctn	L Cos
.00				10.0000	.50	9.6897	9.7374	10.2528	9.9433
.01	8.6900	8.0000	12.0000	.0000	.51	.6886	.7477	.2523	.9409
.02	.3010	.3011	11.6989	9.9999	.52	.6953	.7578	.2422	.9384
.03	.4771	.4773	.6227	.9998	.53	.7037	.7678	.2322	.9359
.04	.6019	.6023	.3977	.9997	.54	.7111	.7777	.2223	.9333
.05	8.6988	8.6993	11.3007	9.9995	.55	9.7182	9.7875	10.2125	9.9307
.06	.7779	.7787	.2213	.9992	.56	.7262	.7972	.2028	.9280
.07	.8447	.8455	.1542	.9989	.57	.7321	.8068	.1932	.9253
.08	.9026	.9040	.0960	.9986	.58	.7388	.8164	.1836	.9224
.09	.9537	.9554	.0446	.9982	.59	.7454	.8258	.1742	.9196
.10	8.9993	9.0015	10.9985	9.9978	.60	9.7518	9.8351	10.1649	9.9166
.11	9.0405	.0431	.9369	.9974	.61	.7581	.8444	.1556	.9136
.12	.0781	.0813	.9187	.9969	.62	.7642	.8536	.1464	.9109
.13	.1127	.1164	.8836	.9963	.63	.7702	.8628	.1372	.9074
.14	.1447	.1480	.8510	.9957	.64	.7761	.8719	.1281	.9042
.15	9.1745	9.1794	10.8206	9.9951	.65	9.7819	9.8809	10.1191	9.9010
.16	.2023	.2078	.7922	.9944	.66	.7875	.8899	.1101	.9975
.17	.2284	.2347	.7553	.9937	.67	.7931	.8989	.1011	.9942
.18	.2529	.2600	.7100	.9929	.68	.7985	.9078	.0922	.9907
.19	.2761	.2840	.6660	.9921	.69	.8038	.9166	.0834	.9872
.20	9.2981	9.3069	10.6931	9.9913	.70	9.8090	9.9255	10.0745	9.8836
.21	.3190	.3287	.6713	.9904	.71	.8141	.9343	.0657	.8799
.22	.3389	.3495	.6305	.9894	.72	.8191	.9430	.0570	.8767
.23	.3579	.3695	.5856	.9884	.73	.8240	.9518	.0482	.8733
.24	.3760	.3887	.5413	.9874	.74	.8288	.9605	.0395	.8698
.25	9.3934	9.4071	10.5929	9.9863	.75	9.8335	9.9692	10.0308	9.8643
.26	.4101	.4249	.5751	.9852	.76	.8382	.9779	.0221	.8602
.27	.4261	.4421	.5379	.9840	.77	.8427	.9866	.0134	.8561
.28	.4415	.4587	.4953	.9827	.78	.8471	9.9953	10.0047	.8518
.29	.4563	.4748	.4522	.9815	.79	.8515	10.0040	9.9960	.8475
.30	9.4708	9.4904	10.5096	9.9802	.80	9.8557	10.0127	9.9873	9.8421
.31	.4844	.5058	.4944	.9788	.81	.8599	.0214	.9786	.8385
.32	.4977	.5203	.4797	.9774	.82	.8640	.0301	.9699	.8339
.33	.5106	.5347	.4563	.9759	.83	.8680	.0388	.9612	.8292
.34	.5231	.5487	.4253	.9744	.84	.8719	.0475	.9525	.8244
.35	9.5352	9.5623	10.4377	9.9728	.85	9.8758	10.0563	9.9437	9.8195
.36	.5469	.5757	.4243	.9712	.86	.8796	.0650	.9350	.8146
.37	.5582	.5887	.4113	.9695	.87	.8833	.0738	.9262	.8094
.38	.5693	.6014	.3986	.9679	.88	.8869	.0827	.9173	.8042
.39	.5800	.6139	.3861	.9661	.89	.8905	.0915	.9085	.7989
.40	9.5904	9.6261	10.3739	9.9643	.90	9.8939	10.1004	9.8996	9.7935
.41	.6005	.6381	.3619	.9624	.91	.8974	.1094	.8906	.7880
.42	.6104	.6499	.3501	.9605	.92	.9007	.1184	.8816	.7823
.43	.6200	.6616	.3385	.9585	.93	.9040	.1274	.8726	.7766
.44	.6293	.6728	.3272	.9565	.94	.9072	.1365	.8635	.7707
.45	9.6385	9.6840	10.3160	9.9545	.95	9.9103	10.1456	9.8544	9.7647
.46	.6473	.6950	.3050	.9523	.96	.9134	.1548	.8452	.7585
.47	.6550	.7058	.2942	.9502	.97	.9164	.1641	.8359	.7523
.48	.6644	.7165	.2835	.9479	.98	.9193	.1735	.8265	.7459
.49	.6727	.7270	.2730	.9456	.99	.9222	.1829	.8171	.7393
.50	9.6807	9.7374	10.2628	9.9433	1.00	9.9250	10.1924	9.8076	9.7325

The -10 portion of the characteristic of the logarithm is not printed in Table 11 but must be written down whenever such a logarithm is used.

Table 11 COMMON LOGARITHMS OF TRIGONOMETRIC FUNCTIONS IN RADIAN MEASURE (continued) 306

Rad	L Sin	L Tan	L Ctn	L Cos	Rad	L Sin	L Tan	L Ctn	L Cos
1.00	9.9250	10.1924	9.8076	9.7326	1.30	9.9639	10.5666	9.4434	9.4273
1.01	.9278	.2020	.7980	.7258	1.31	.9651	.5737	.4263	.4114
1.02	.9306	.2117	.7883	.7188	1.32	.9662	.5814	.4386	.3948
1.03	.9331	.2216	.7785	.7117	1.33	.9673	.5898	.4502	.3774
1.04	.9357	.2314	.7685	.7048	1.34	.9683	.5990	.4610	.3594
1.05	9.9382	10.2414	9.7585	9.6969	1.35	9.9693	10.6489	9.3611	9.3405
1.06	.9407	.2515	.7485	.6892	1.36	.9703	.6696	.3304	.3206
1.07	.9431	.2617	.7383	.6814	1.37	.9712	.6814	.3386	.2998
1.08	.9454	.2721	.7279	.6733	1.38	.9720	.7141	.2859	.2779
1.09	.9477	.2826	.7174	.6651	1.39	.9729	.7380	.2620	.2548
1.10	9.9500	10.2933	9.7067	9.6567	1.40	9.9736	10.7633	9.2367	9.2304
1.11	.9522	.3041	.6959	.6480	1.41	.9744	.7900	.2160	.2044
1.12	.9543	.3151	.6849	.6392	1.42	.9750	.8183	.1817	.1767
1.13	.9564	.3263	.6737	.6301	1.43	.9757	.8485	.1515	.1472
1.14	.9584	.3376	.6624	.6208	1.44	.9763	.8809	.1191	.1154
1.15	9.9604	10.3492	9.6508	9.6112	1.45	9.9768	10.9156	9.0842	9.0810
1.16	.9623	.3609	.6391	.6013	1.46	.9773	.9537	.6482	.6436
1.17	.9641	.3729	.6271	.5912	1.47	.9778	.9951	.6040	.6027
1.18	.9659	.3851	.6149	.5808	1.48	.9782	11.0407	5.9593	5.9575
1.19	.9677	.3976	.6024	.5701	1.49	.9786	.0917	.9083	.9069
1.20	9.9694	10.4103	9.5997	9.5591	1.50	9.9789	11.1493	5.8507	5.8498
1.21	.9711	.4233	.5767	.5478	1.51	.9792	.2168	.7844	.7838
1.22	.9727	.4366	.5634	.5361	1.52	.9794	.2938	.7062	.7055
1.23	.9743	.4502	.5498	.5241	1.53	.9796	.3891	.6109	.6105
1.24	.9758	.4642	.5358	.5116	1.54	.9798	.5114	.4886	.4884
1.25	9.9773	10.4785	9.5215	9.4888	1.55	9.9799	11.6820	5.3180	5.3160
1.26	.9787	.4932	.5068	.4856	1.56	10.0000	11.9667	5.0333	5.0333
1.27	.9800	.5083	.4917	.4717	1.57	10.0000	13.0980	5.0011	5.0011
1.28	.9814	.5239	.4761	.4575	1.58	10.0000	12.0360*	7.9640*	7.9640*
1.29	.9826	.5400	.4600	.4427	1.59	9.9999	11.7166*	8.2834*	8.2834*
1.30	9.9839	10.5565	9.4434	9.4273	1.60	9.9998	11.5344*	8.4656*	8.4654*

* The tangent, cotangent, and cosine of these angles are negative.

$$\pi \text{ radians} = 180^\circ \quad \pi = 3.14159\ 26536$$

$$1 \text{ radian} = 57^\circ 17' 44'' .80625 = 57^\circ .29577\ 95131$$

$$1^\circ = 0.01745\ 32925\ 19943 \text{ radian} = 60' = 3600''$$

Table 12 DEGREES, MINUTES, AND SECONDS TO RADIAN

Deg	Rad	Min	Rad	Sec	Rad
1	0.01745 33	1	0.00029 09	1	0.00000 48
2	0.03490 66	2	0.00058 18	2	0.00000 97
3	0.05235 99	3	0.00087 27	3	0.00001 46
4	0.06981 32	4	0.00116 36	4	0.00001 94
5	0.08728 65	5	0.00145 44	5	0.00002 42
6	0.10471 98	6	0.00174 53	6	0.00002 91
7	0.12217 30	7	0.00203 62	7	0.00003 39
8	0.13962 63	8	0.00232 71	8	0.00003 88
9	0.15707 96	9	0.00261 80	9	0.00004 36
10	0.17453 29	10	0.00290 89	10	0.00004 85
20	0.34906 59	20	0.00581 78	20	0.00009 70
30	0.52359 88	30	0.00872 66	30	0.00014 54
40	0.69813 17	40	0.01163 55	40	0.00019 39
50	0.87266 46	50	0.01454 44	50	0.00024 24
60	1.04719 75	60	0.01745 33	60	0.00029 09
70	1.22173 05				
80	1.39626 34				
90	1.57079 63				

Table 13 TRIGONOMETRIC FUNCTIONS IN

RADIAN MEASURE

Rad	Sin	Tan	Ctn	Cos	Rad	Sin	Tan	Ctn	Cos
.00	.0000	.0000	1.0000	.50	.4794	.5493	1.330	.8776
.01	.0100	.0100	99.997	1.0000	.51	.4882	.5594	1.788	.8727
.02	.0200	.0200	49.893	.9999	.52	.4969	.5726	1.747	.8678
.03	.0300	.0300	33.323	.9996	.53	.5055	.5859	1.707	.8628
.04	.0400	.0400	24.987	.9992	.54	.5141	.5994	1.668	.8577
.05	.0500	.0500	19.983	.9988	.55	.5227	.6131	1.631	.8525
.06	.0600	.0601	16.647	.9982	.56	.5312	.6269	1.595	.8473
.07	.0699	.0701	14.262	.9976	.57	.5398	.6410	1.560	.8419
.08	.0799	.0802	12.473	.9968	.58	.5480	.6552	1.526	.8365
.09	.0899	.0902	11.081	.9960	.59	.5564	.6696	1.494	.8309
.10	.0999	.1003	9.967	.9950	.60	.5646	.6841	1.462	.8253
.11	.1098	.1104	9.054	.9940	.61	.5729	.6989	1.431	.8196
.12	.1197	.1206	8.293	.9928	.62	.5810	.7139	1.401	.8139
.13	.1296	.1307	7.649	.9916	.63	.5891	.7291	1.372	.8080
.14	.1395	.1409	7.098	.9902	.64	.5972	.7445	1.343	.8021
.15	.1494	.1511	6.617	.9888	.65	.6052	.7602	1.315	.7961
.16	.1593	.1614	6.197	.9872	.66	.6131	.7761	1.289	.7900
.17	.1692	.1717	5.826	.9856	.67	.6210	.7923	1.262	.7839
.18	.1790	.1820	5.495	.9836	.68	.6288	.8087	1.237	.7776
.19	.1889	.1923	5.200	.9820	.69	.6365	.8253	1.212	.7712
.20	.1987	.2027	4.933	.9801	.70	.6442	.8423	1.187	.7648
.21	.2085	.2131	4.692	.9780	.71	.6518	.8595	1.163	.7584
.22	.2182	.2236	4.472	.9759	.72	.6594	.8771	1.140	.7519
.23	.2279	.2341	4.271	.9737	.73	.6669	.8949	1.117	.7455
.24	.2377	.2447	4.086	.9713	.74	.6743	.9131	1.095	.7389
.25	.2474	.2553	3.916	.9689	.75	.6816	.9316	1.073	.7317
.26	.2571	.2660	3.759	.9664	.76	.6889	.9505	1.052	.7248
.27	.2667	.2763	3.613	.9638	.77	.6961	.9697	1.031	.7179
.28	.2764	.2874	3.478	.9611	.78	.7033	.9893	1.011	.7109
.29	.2860	.2984	3.351	.9582	.79	.7104	1.009	.9908	.7038
.30	.2956	.3093	3.233	.9553	.80	.7174	1.030	.9712	.6967
.31	.3051	.3203	3.122	.9523	.81	.7243	1.050	.9520	.6895
.32	.3146	.3314	3.018	.9492	.82	.7311	1.072	.9331	.6822
.33	.3240	.3425	2.920	.9460	.83	.7379	1.093	.9146	.6749
.34	.3335	.3537	2.827	.9428	.84	.7446	1.116	.8964	.6675
.35	.3429	.3650	2.740	.9394	.85	.7513	1.138	.8785	.6600
.36	.3523	.3764	2.657	.9359	.86	.7578	1.162	.8609	.6524
.37	.3616	.3879	2.578	.9323	.87	.7643	1.185	.8437	.6448
.38	.3709	.3994	2.504	.9287	.88	.7707	1.210	.8267	.6372
.39	.3802	.4111	2.433	.9249	.89	.7771	1.235	.8100	.6294
.40	.3894	.4228	2.365	.9211	.90	.7833	1.260	.7936	.6216
.41	.3986	.4345	2.301	.9171	.91	.7895	1.286	.7774	.6137
.42	.4078	.4465	2.239	.9131	.92	.7956	1.313	.7615	.6058
.43	.4169	.4586	2.180	.9090	.93	.8016	1.341	.7458	.5978
.44	.4259	.4708	2.124	.9048	.94	.8076	1.369	.7303	.5898
.45	.4350	.4831	2.070	.9004	.95	.8134	1.398	.7151	.5817
.46	.4439	.4954	2.018	.8961	.96	.8192	1.428	.7001	.5735
.47	.4529	.5080	1.969	.8916	.97	.8249	1.459	.6853	.5653
.48	.4616	.5205	1.921	.8870	.98	.8305	1.491	.6707	.5570
.49	.4706	.5334	1.875	.8823	.99	.8360	1.524	.6563	.5487
.50	.4794	.5463	1.830	.8776	1.00	.8415	1.557	.6421	.5403

π radians = 180° $\pi = 3.14159\ 26536$
 1 radian = 57°17'44" .80625 = 57° .29577 95131
 1° = 0.01745 32925 19943 radian = 60' = 3600"

Table 13 TRIGONOMETRIC FUNCTIONS IN RADIAN MEASURE (continued) 308

Rad	Sin	Tan	Ctn	Cos	Rad	Sin	Tan	Ctn	Cos
1.00	.8415	1.557	.6421	.5403	1.30	.9636	3.602	.2776	.2675
1.01	.8468	1.592	.6281	.5319	1.31	.9662	3.747	.2669	.2579
1.02	.8521	1.628	.6142	.5234	1.32	.9687	3.893	.2562	.2482
1.03	.8573	1.666	.6005	.5148	1.33	.9711	4.042	.2456	.2385
1.04	.8624	1.704	.5870	.5062	1.34	.9735	4.206	.2350	.2288
1.05	.8674	1.743	.5736	.4976	1.35	.9757	4.455	.2245	.2190
1.06	.8724	1.784	.5604	.4889	1.36	.9779	4.673	.2140	.2092
1.07	.8772	1.827	.5473	.4801	1.37	.9799	4.913	.2035	.1994
1.08	.8820	1.871	.5344	.4713	1.38	.9819	5.177	.1931	.1896
1.09	.8866	1.917	.5216	.4625	1.39	.9837	5.471	.1828	.1798
1.10	.8912	1.965	.5090	.4536	1.40	.9854	5.798	.1725	.1700
1.11	.8957	2.014	.4964	.4447	1.41	.9871	6.165	.1622	.1601
1.12	.9001	2.066	.4840	.4357	1.42	.9887	6.531	.1519	.1502
1.13	.9044	2.120	.4718	.4267	1.43	.9901	7.055	.1417	.1403
1.14	.9085	2.178	.4596	.4176	1.44	.9915	7.602	.1315	.1304
1.15	.9126	2.234	.4475	.4085	1.45	.9927	8.238	.1214	.1205
1.16	.9166	2.296	.4356	.3993	1.46	.9939	8.989	.1113	.1106
1.17	.9206	2.360	.4237	.3902	1.47	.9949	9.887	.1011	.1006
1.18	.9246	2.427	.4120	.3809	1.48	.9959	10.933	.0910	.0907
1.19	.9284	2.498	.4003	.3717	1.49	.9967	12.350	.0810	.0807
1.20	.9320	2.572	.3888	.3624	1.50	.9975	14.101	.0709	.0707
1.21	.9355	2.650	.3773	.3530	1.51	.9982	16.428	.0608	.0608
1.22	.9391	2.733	.3658	.3436	1.52	.9987	19.670	.0508	.0508
1.23	.9425	2.820	.3546	.3342	1.53	.9992	24.498	.0408	.0408
1.24	.9458	2.912	.3434	.3248	1.54	.9995	32.461	.0308	.0308
1.25	.9490	3.010	.3323	.3153	1.55	.9998	48.078	.0208	.0208
1.26	.9521	3.113	.3212	.3058	1.56	.9999	92.620	.0108	.0108
1.27	.9551	3.224	.3102	.2963	1.57	1.0000	1256.8	.0008	.0008
1.28	.9580	3.341	.2993	.2867	1.58	1.0000	-108.65	-.0032	-.0382
1.29	.9608	3.467	.2884	.2771	1.59	.9998	-52.067	-.0192	-.0192
1.30	.9636	3.602	.2776	.2675	1.60	.9996	-34.233	-.0292	-.0292

Table 14 RADIANS TO DEGREES, MINUTES, AND SECONDS

Rad	Rad	Rad	Rad	Rad	Rad
1	57°17'44".8	.1	5°43'46".5	.01	0°34'22".6
2	114°35'29".6	.2	11°27'33".0	.02	1°8'45".3
3	171°53'14".4	.3	17°11'18".4	.03	1°43'07".9
4	229°10'59".2	.4	22°55'05".9	.04	2°17'30".6
5	286°28'44".0	.5	28°38'52".4	.05	2°51'53".2
6	343°46'28".8	.6	34°22'38".9	.06	3°26'15".9
7	401°4'13".6	.7	40°6'25".4	.07	4°0'38".5
8	458°21'58".4	.8	45°50'11".8	.08	4°35'01".2
9	515°39'43".3	.9	51°33'58".3	.09	5°9'23".8
				.001	0°3'26".3
				.002	0°6'52".5
				.003	0°10'18".8
				.004	0°13'45".1
				.005	0°17'11".3
				.006	0°20'37".6
				.007	0°24'03".9
				.008	0°27'30".1
				.009	0°30'56".4
				.0001	0°0'20".6
				.0002	0°0'41".3
				.0003	0°1'01".9
				.0004	0°1'22".5
				.0005	0°1'43".1
				.0006	0°2'03".8
				.0007	0°2'24".4
				.0008	0°2'45".0
				.0009	0°3'05".6

RADIANS TO DEGREES

Rad	Degrees	Rad	Degrees	Rad	Degrees
1	57.2958	4	229.1831	7	401.0705
2	114.5916	5	286.4789	8	458.3662
3	171.8873	6	343.7747	9	515.6620

**EXPONENTIAL AND
HYPERBOLIC FUNCTIONS**

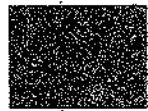


TABLE 15. *Natural Logarithms of Numbers, 311*

TABLE 16. *Values and Common Logarithms of Exponential and Hyperbolic Functions, 315*

TABLE 17. *Multiples of M and of $1/M$, 322*

Table 15 NATURAL LOGARITHMS OF NUMBERS 311

0.00-5.99†

N	0	1	2	3	4	5	6	7	8	9
0.0		5.395	6.088	6.493	6.781	7.004	7.187	7.341	7.474	7.592
0.1	7.697	7.793	7.880	7.960	8.034	8.103	8.167	8.228	8.285	8.339
0.2	8.391	8.439	8.486	8.530	8.573	8.614	8.653	8.691	8.727	8.762
0.3	8.796	8.829	8.861	8.891	8.921	8.950	8.978	9.006	9.032	9.058
0.4	9.084	9.108	9.132	9.156	9.179	9.201	9.223	9.245	9.266	9.287
0.5	9.307	9.327	9.346	9.365	9.384	9.402	9.420	9.438	9.455	9.472
0.6	9.489	9.506	9.522	9.538	9.554	9.569	9.584	9.600	9.614	9.629
0.7	9.643	9.658	9.671	9.685	9.699	9.712	9.726	9.739	9.752	9.764
0.8	9.777	9.789	9.802	9.814	9.826	9.837	9.849	9.861	9.872	9.883
0.9	9.895	9.906	9.917	9.927	9.938	9.949	9.959	9.970	9.980	9.990
1.0	0.0	0000	0995	1980	2955	3922	4879	5827	6765	7695
1.1		9531	*0436	*1333	*2222	*3103	*3976	*4842	*5700	*6551
1.2	0.1	8232	9082	9885	*0701	*1511	*2314	*3111	*3902	*4686
1.3	0.2	6255	7003	7763	8518	9267	*0010	*0748	*1481	*2208
1.4	0.3	3847	4359	5065	5767	6464	7155	7844	8525	9204
1.5	0.4	0547	1211	1871	2527	3178	3825	4469	5108	5742
1.6		7000	7623	8243	8858	9470	*0078	*0682	*1282	*1879
1.7	0.5	3003	3649	4282	4912	5539	5962	6531	7098	7661
1.8		6779	9353	9884	*0432	*0977	*1519	*2058	*2594	*3127
1.9	0.6	4185	4710	5233	5752	6269	6783	7294	7803	8310
2.0		9315	9813	*0319	*0804	*1285	*1784	*2271	*2755	*3237
2.1	0.7	4194	4669	5142	5612	6081	6547	7011	7473	7932
2.2		8846	9299	9751	*0209	*0648	*1093	*1536	*1978	*2418
2.3	0.8	3291	3725	4157	4587	5015	5442	5866	6289	6710
2.4		7547	7963	8377	8789	9200	9509	*0015	*0422	*0825
2.5	0.9	1629	2028	2425	2822	3216	3609	4001	4391	4779
2.6		5551	5935	6317	6698	7076	7455	7833	8208	8582
2.7		9325	9695	*0063	*0430	*0786	*1160	*1523	*1885	*2245
2.8	1.0	2962	3318	3674	4028	4380	4732	5082	5431	5779
2.9		6471	6815	7158	7500	7841	8181	8519	8855	9192
3.0		9861	*0164	*0526	*0885	*1166	*1514	*1841	*2168	*2493
3.1	1.1	3140	3462	3783	4103	4422	4740	5057	5373	5688
3.2		6315	6627	6938	7248	7557	7865	8173	8479	8784
3.3		9392	9695	9996	*0297	*0597	*0896	*1194	*1491	*1785
3.4	1.2	2373	2671	2964	3256	3547	3837	4127	4415	4703
3.5		5275	5562	5846	6130	6413	6695	6976	7257	7535
3.6		8002	8271	8547	8823	9108	9373	9648	*0019	*0291
3.7	1.3	0833	1103	1372	1641	1909	2176	2442	2708	2972
3.8		3500	3763	4025	4286	4547	4807	5067	5325	5584
3.9		6098	6354	6608	6864	7118	7372	7624	7877	8128
4.0		8829	8879	9128	9377	9624	9872	*0118	*0364	*0610
4.1	1.4	1099	1342	1585	1828	2070	2311	2552	2792	3031
4.2		3508	3746	3984	4220	4456	4692	4927	5161	5395
4.3		5962	6194	6426	6657	6887	7016	7247	7476	7705
4.4		9160	9387	9614	9840	9965	9290	9513	9739	9962
4.5	1.5	0408	0630	0851	1072	1293	1513	1732	1951	2170
4.6		2806	2823	3039	3256	3471	3687	3902	4116	4330
4.7		4756	4969	5181	5393	5604	5814	6025	6235	6444
4.8		6862	7070	7277	7485	7691	7898	8104	8309	8515
4.9		8924	9127	9331	9534	9737	9939	*0141	*0342	*0543
5.0	1.6	0944	1144	1343	1542	1741	1939	2137	2334	2531
5.1		2924	3120	3315	3511	3705	3900	4094	4287	4481
5.2		4888	5083	5276	5471	5662	5853	6043	6233	6423
5.3		6771	6959	7147	7335	7523	7710	7899	8083	8269
5.4		8540	8925	9010	9194	9378	9562	9745	9928	*0111
5.5	1.7	0473	0555	0638	1010	1199	1380	1560	1740	1919
5.6		2277	2455	2633	2811	2988	3166	3342	3519	3695
5.7		4047	4222	4397	4572	4745	4920	5094	5267	5440
5.8		5786	5958	6130	6302	6473	6644	6815	6985	7155
5.9		7495	7665	7834	8002	8171	8339	8507	8675	8842

Base e = 2.718 ...

log_e 0.10 = 7.69741 49070 - 10

† Entries in Table 15 are values of log_e N for the indicated values of N.

Table 15 NATURAL LOGARITHMS OF NUMBERS (continued)

6.00-10.09

N	0	1	2	3	4	5	6	7	8	9
6.0	1.7 8176	9342	9509	9675	9840	*0006	*0171	*0336	*0500	*0665
6.1	1.8 0829	0993	1156	1319	1482	1645	1808	1970	2132	2294
6.2		2455	2616	2777	2938	3098	3258	3418	3578	3737
6.3		4055	4214	4372	4530	4688	4845	5003	5160	5317
6.4		5630	5786	5942	6097	6253	6408	6563	6719	6872
6.5		7180	7334	7487	7641	7794	7947	8099	8251	8403
6.6		8707	8858	9010	9160	9311	9462	9612	9762	9912
6.7	1.9 0211	0360	0509	0658	0806	0954	1102	1250	1398	1545
6.8		1692	1839	1986	2132	2279	2425	2571	2716	2862
6.9		3152	3297	3442	3586	3730	3874	4018	4162	4305
7.0		4591	4734	4876	5019	5161	5303	5445	5586	5727
7.1		6009	6150	6291	6431	6571	6711	6851	6991	7130
7.2		7408	7547	7685	7824	7962	8100	8238	8376	8513
7.3		8787	8924	9061	9198	9334	9470	9606	9742	*0013
7.4	2.0 0148	0283	0418	0553	0687	0821	0956	1089	1223	1357
7.5		1490	1624	1757	1890	2022	2155	2287	2419	2551
7.6		2615	2946	3078	3209	3340	3471	3601	3732	3862
7.7		4122	4252	4381	4511	4640	4769	4898	5027	5156
7.8		5412	5540	5668	5796	5924	6051	6179	6306	6433
7.9		6660	6785	6910	7035	7160	7284	7408	7532	7655
8.0		7944	8069	8194	8318	8443	8567	8691	8815	8939
8.1		9186	9310	9433	9556	9679	9802	9924	*0047	*0169
8.2	2.1 0413	0535	0657	0779	0900	1021	1142	1263	1384	1505
8.3		1626	1746	1866	1986	2106	2226	2346	2466	2585
8.4		2823	2942	3061	3180	3298	3417	3535	3653	3771
8.5		4007	4124	4242	4359	4476	4593	4710	4827	4943
8.6		5176	5292	5409	5524	5640	5756	5871	5987	6102
8.7		6332	6447	6562	6677	6791	6905	7020	7134	7248
8.8		7475	7589	7702	7816	7929	8042	8155	8267	8380
8.9		8605	8717	8830	8942	9054	9165	9277	9388	9500
9.0		9722	9834	9944	*0055	*0166	*0276	*0387	*0497	*0607
9.1	2.2 0827	0937	1047	1157	1266	1375	1485	1594	1703	1812
9.2		1920	2029	2138	2246	2354	2462	2570	2678	2786
9.3		3001	3109	3216	3324	3431	3538	3645	3751	3858
9.4		4071	4177	4284	4390	4496	4601	4707	4813	4918
9.5		5129	5234	5339	5444	5549	5654	5759	5863	5968
9.6		6176	6280	6384	6488	6592	6696	6799	6903	7006
9.7		7213	7316	7419	7521	7624	7727	7829	7932	8034
9.8		8238	8340	8442	8544	8646	8747	8849	8950	9051
9.9		9253	9354	9455	9556	9657	9757	9858	9958	*0058
10.0	2.3 0259	0358	0458	0558	0658	0757	0857	0956	1055	1154
N	0	1	2	3	4	5	6	7	8	9

Table 15 NATURAL LOGARITHMS OF NUMBERS (continued)

10-99

N	0	1	2	3	4	5	6	7	8	9
1	2.30259	39790	48491	56495	63906	70805	77259	83321	89037	94444
2	99573	*04462	*09104	*13549	*17805	*21888	*25810	*29534	*33220	*36730
3	3.40120	43369	46574	49581	52336	55555	58352	61092	63739	66355
4	68868	71357	73767	76129	78419	80666	82864	85015	87120	89182
5	91202	93189	95124	97029	98898	*00733	*02535	*04305	*06044	*07754
6	4.09434	11087	12713	14313	15888	17438	18965	20469	21951	23411
7	24850	26268	27667	29049	30407	31749	33073	34381	35671	36948
8	38203	39445	40672	41884	43082	44265	45435	46591	47734	48864
9	49981	51086	52179	53260	54329	55388	56436	57471	58497	59512

log_e 10 = 2.30258 50930

Table 15 NATURAL LOGARITHMS OF NUMBERS (continued)

100-609

N	0	1	2	3	4	5	6	7	8	9
10	4.6 0517	1512	2497	3473	4439	5396	6344	7283	8213	9135
11	4.7 3048	0953	1850	2739	3620	4493	5359	6217	7068	7912
12		8749	9679	*0402	*1218	*2028	*2831	*3628	*4419	*5203
13	4.8 6753	7620	8280	8935	9784	*0527	*1265	*1998	*2725	*3447
14	4.9 4164	4875	5583	6284	6981	7673	8361	9043	9721	*0389
15	5.0 1054	1728	2388	3044	3695	4343	4986	5625	6260	6890
16		7517	8149	8760	9375	*0595	*1199	*1799	*2396	*2990
17	5.1 3560	4165	4749	5329	5906	6479	7048	7615	8178	8739
18		9298	9850	*0401	*0849	*1494	*2035	*2575	*3111	*3644
19	5.2 4702	5227	5750	6269	6786	7300	7811	8320	8827	9330
20		9832	*0330	*0827	*1321	*1812	*2301	*2788	*3272	*3754
21	5.3 4711	5185	5659	6129	6590	7064	7528	7990	8450	8907
22		9363	9816	*0268	*0717	*1165	*1610	*2053	*2495	*2935
23	5.4 3808	4242	4674	5104	5532	5959	6383	6806	7227	7646
24		8064	8480	8894	9306	9717	*0126	*0533	*0939	*1343
25	5.5 2146	2545	2943	3339	3733	4128	4518	4908	5296	5683
26		6068	6452	6834	7215	7595	7973	8350	8725	9099
27		9842	*0212	*0580	*0947	*1313	*1677	*2040	*2402	*2762
28	5.6 3479	3835	4191	4545	4897	5249	5599	5948	6296	6643
29		6868	7322	7675	8017	8358	8698	9036	9373	*0044
30	5.7 0378	0711	1043	1373	1703	2031	2359	2685	3010	3334
31		2667	3079	3490	3899	4307	4715	5122	5528	5934
32		6832	7144	7455	7765	8074	8382	8690	8998	9306
33		9909	*0212	*0513	*0814	*1114	*1413	*1711	*2008	*2305
34	5.8 2895	3188	3481	3773	4064	4354	4644	4932	5220	5507
35		5798	6079	6363	6647	6930	7212	7493	7774	8053
36		8810	8888	9164	9440	9715	9990	*0263	*0536	*0808
37	5.9 1350	1520	1889	2153	2426	2698	2969	3225	3489	3754
38		4017	4280	4542	4803	5064	5324	5584	5842	6101
39		6815	6971	7125	7381	7635	7889	8141	8394	8645
40		9146	9396	9645	9894	*0141	*0389	*0635	*0881	*1127
41	6.0 1816	1859	2102	2345	2587	2828	3069	3309	3548	3787
42		4925	4263	4501	4737	4973	5209	5444	5678	5912
43		6379	6611	6843	7074	7304	7535	7764	7993	8222
44		8577	8904	9131	9357	9582	9807	*0032	*0258	*0479
45	6.1 0925	1147	1368	1589	1810	2030	2249	2468	2687	2905
46		3123	3340	3559	3773	3988	4204	4419	4633	4847
47		5273	5486	5698	5910	6121	6331	6542	6752	6961
48		7379	7587	7794	8002	8208	8415	8621	8826	9032
49		9441	9544	9848	*0051	*0254	*0458	*0658	*0859	*1060
50	6.2 1461	1661	1860	2059	2258	2456	2654	2851	3048	3245
51		3441	3637	3832	4028	4222	4417	4611	4804	4998
52		5363	5575	5767	5958	6149	6340	6530	6720	6910
53		7288	7476	7664	7852	8040	8227	8413	8600	8786
54		9157	9342	9527	9711	9895	*0079	*0262	*0445	*0628
55	6.3 0992	1173	1355	1536	1716	1897	2077	2257	2436	2615
56		2754	2972	3150	3328	3505	3683	3859	4036	4212
57		4564	4739	4914	5089	5263	5437	5611	5784	5957
58		6303	6475	6647	6819	6990	7161	7332	7502	7673
59		8012	8182	8351	8519	8688	8856	9024	9192	9359
60		9593	9859	*0026	*0192	*0357	*0523	*0688	*0853	*1017
N	0	1	2	3	4	5	6	7	8	9

log_e 100 = 4.60517 01860

Table 15 NATURAL LOGARITHMS OF NUMBERS (continued)

600-1109

N	0	1	2	3	4	5	6	7	8	9
60	6.3	9693	9859	*0026	*0192	*0357	*0523	*0688	*0853	*1017
61	6.4	1346	1510	1673	1836	1999	2162	2325	2487	2649
62		2872	3133	3294	3456	3615	3775	3935	4095	4254
63		4572	4731	4889	5047	5205	5362	5520	5677	5834
64		6147	6303	6459	6614	6770	6925	7080	7235	7389
65		7697	7851	8004	8158	8311	8464	8616	8768	8920
66		9224	9375	9527	9677	9828	9979	*0129	*0279	*0429
67	6.5	0728	0877	1025	1175	1323	1471	1619	1767	1915
68		2208	2356	2503	2649	2795	2942	3088	3233	3378
69		3669	3814	3959	4103	4247	4391	4535	4679	4822
70		5108	5251	5393	5536	5678	5820	5962	6103	6244
71		6526	6667	6808	6948	7088	7228	7368	7508	7647
72		7925	8064	8203	8341	8479	8617	8755	8893	9030
73		9304	9441	9578	9715	9851	9988	*0123	*0259	*0394
74	6.6	0695	0800	0935	1070	1204	1338	1473	1607	1740
75		2007	2141	2274	2407	2539	2672	2804	2936	3068
76		3332	3463	3596	3726	3857	3988	4118	4249	4379
77		4639	4769	4898	5028	5157	5286	5415	5544	5673
78		5929	6056	6185	6313	6441	6569	6696	6823	6950
79		7203	7330	7456	7582	7708	7834	7960	8085	8211
80		8461	8586	8711	8835	8960	9084	9208	9332	9455
81		9703	9827	9950	*0073	*0196	*0319	*0441	*0564	*0686
82	6.7	0930	1052	1174	1296	1417	1538	1659	1780	1901
83		2143	2263	2383	2503	2623	2743	2863	2982	3102
84		3340	3459	3578	3697	3815	3934	4052	4170	4288
85		4524	4641	4759	4876	4993	5110	5227	5344	5460
86		5693	5809	5926	6041	6157	6273	6388	6504	6619
87		6849	6964	7079	7194	7308	7422	7537	7651	7765
88		7992	8106	8219	8333	8446	8559	8672	8784	8897
89		9122	9234	9347	9459	9571	9682	9794	9906	*0017
90	6.8	0239	0351	0461	0572	0683	0793	0904	1014	1124
91		1344	1454	1564	1674	1783	1892	2002	2111	2220
92		2437	2546	2655	2763	2871	2979	3087	3195	3303
93		3518	3626	3733	3841	3948	4055	4162	4268	4375
94		4588	4694	4801	4907	5013	5118	5224	5330	5435
95		5546	5751	5857	5961	6066	6171	6276	6380	6485
96		6693	6797	6901	7005	7109	7213	7316	7420	7523
97		7730	7833	7936	8038	8141	8244	8346	8449	8551
98		8755	8857	8959	9061	9163	9264	9366	9467	9568
99		9770	9871	9972	*0073	*0174	*0274	*0375	*0475	*0575
100	6.9	0776	0875	0975	1075	1175	1274	1374	1473	1572
101		1771	1870	1968	2067	2166	2264	2363	2461	2560
102		2756	2854	2952	3049	3147	3245	3342	3440	3537
103		3731	3828	3925	4022	4119	4216	4312	4409	4505
104		4698	4794	4890	4986	5081	5177	5273	5368	5464
105		5550	5750	5845	5940	6036	6130	6224	6319	6414
106		6602	6697	6791	6886	6979	7073	7167	7261	7354
107		7541	7635	7728	7821	7915	8008	8101	8193	8286
108		8472	8564	8657	8749	8841	8934	9026	9118	9210
109		9393	9485	9577	9668	9760	9851	9942	*0033	*0125
110	7.0	0307	0397	0488	0579	0670	0760	0851	0941	1031
111										1121
N	0	1	2	3	4	5	6	7	8	9

log_e 1000 = 6.90775 52790

To find the logarithm of a number which is 10 (or 1/10) times a number whose logarithm is given, add to (or subtract from) the given logarithm the logarithm of 10.

Table 16 VALUES AND COMMON LOGARITHMS OF
EXPONENTIAL AND HYPERBOLIC FUNCTIONS 315

x	e^x		e^{-x}	Sinh x		Cosh x		Tanh x
	Value	Log ₁₀		Value	Value	Log ₁₀	Value	
0.00	1.0000	.00000	1.00000	0.0000	—∞	1.0000	.00000	.00000
0.01	1.0101	.00434	0.99005	0.0100	2.00001	1.0001	.00002	.01000
0.02	1.0202	.00869	.98020	0.0200	2.30106	1.0002	.00009	.02000
0.03	1.0303	.01303	.97045	0.0300	2.47719	1.0005	.00020	.02999
0.04	1.0408	.01737	.96079	0.0400	2.60218	1.0008	.00035	.03998
0.05	1.0513	.02171	.95123	0.0500	2.69915	1.0013	.00054	.04996
0.06	1.0618	.02605	.94176	0.0600	2.77841	1.0018	.00078	.05993
0.07	1.0725	.03040	.93239	0.0701	2.84665	1.0025	.00106	.06989
0.08	1.0833	.03474	.92312	0.0801	2.90355	1.0032	.00139	.07983
0.09	1.0942	.03909	.91393	0.0901	2.95483	1.0041	.00176	.08976
0.10	1.1052	.04343	.90484	0.1002	3.00072	1.0050	.00217	.09967
0.11	1.1163	.04777	.89583	0.1102	3.04227	1.0061	.00262	.10956
0.12	1.1275	.05212	.88692	0.1203	3.08022	1.0072	.00312	.11943
0.13	1.1388	.05646	.87809	0.1304	3.11517	1.0085	.00366	.12927
0.14	1.1503	.06080	.86936	0.1405	3.14755	1.0098	.00424	.13909
0.15	1.1618	.06514	.86071	0.1506	3.17772	1.0113	.00487	.14889
0.16	1.1735	.06949	.85214	0.1607	3.20697	1.0128	.00554	.15865
0.17	1.1853	.07383	.84366	0.1708	3.23554	1.0145	.00625	.16838
0.18	1.1972	.07817	.83527	0.1810	3.26262	1.0162	.00700	.17808
0.19	1.2092	.08252	.82696	0.1911	3.28836	1.0181	.00779	.18775
0.20	1.2214	.08686	.81873	0.2013	3.30392	1.0201	.00863	.19738
0.21	1.2337	.09120	.81058	0.2115	3.32541	1.0221	.00951	.20697
0.22	1.2461	.09554	.80252	0.2218	3.34592	1.0243	.01043	.21652
0.23	1.2586	.09989	.79453	0.2320	3.36555	1.0266	.01139	.22603
0.24	1.2712	.10423	.78663	0.2423	3.38437	1.0289	.01239	.23550
0.25	1.2840	.10857	.77880	0.2526	3.40245	1.0314	.01343	.24492
0.26	1.2968	.11292	.77105	0.2629	3.41986	1.0340	.01452	.25430
0.27	1.3100	.11726	.76338	0.2733	3.43663	1.0367	.01564	.26362
0.28	1.3231	.12160	.75578	0.2837	3.45282	1.0395	.01681	.27291
0.29	1.3364	.12595	.74826	0.2941	3.46847	1.0425	.01801	.28213
0.30	1.3499	.13029	.74082	0.3045	3.48362	1.0456	.01926	.29131
0.31	1.3634	.13463	.73345	0.3150	3.49830	1.0488	.02054	.30044
0.32	1.3771	.13897	.72615	0.3255	3.51254	1.0521	.02187	.30951
0.33	1.3910	.14332	.71892	0.3360	3.52637	1.0554	.02323	.31852
0.34	1.4049	.14766	.71177	0.3465	3.53981	1.0588	.02463	.32748
0.35	1.4191	.15200	.70469	0.3572	3.55290	1.0621	.02507	.33638
0.36	1.4333	.15635	.69768	0.3678	3.56564	1.0655	.02555	.34521
0.37	1.4477	.16069	.69073	0.3785	3.57807	1.0692	.02607	.35399
0.38	1.4623	.16503	.68386	0.3892	3.59019	1.0731	.02663	.36271
0.39	1.4770	.16937	.67706	0.4000	3.60202	1.0770	.02722	.37136
0.40	1.4918	.17372	.67032	0.4108	3.61358	1.0811	.02785	.37995
0.41	1.5068	.17806	.66365	0.4216	3.62488	1.0852	.02852	.38847
0.42	1.5220	.18240	.65705	0.4325	3.63594	1.0895	.02923	.39693
0.43	1.5373	.18675	.65051	0.4434	3.64677	1.0939	.02997	.40532
0.44	1.5527	.19109	.64404	0.4543	3.65738	1.0984	.03073	.41364
0.45	1.5683	.19543	.63763	0.4653	3.66777	1.1030	.03153	.42190
0.46	1.5841	.19977	.63128	0.4764	3.67797	1.1077	.03236	.43008
0.47	1.6000	.20412	.62500	0.4875	3.68797	1.1125	.03322	.43820
0.48	1.6161	.20846	.61876	0.4986	3.69779	1.1174	.03411	.44624
0.49	1.6323	.21280	.61263	0.5098	3.70744	1.1225	.03501	.45422
0.50	1.6487	.21715	.60663	0.5211	3.71692	1.1276	.03591	.46212

Table 16 VALUES AND COMMON LOGARITHMS OF EXPONENTIAL AND HYPERBOLIC FUNCTIONS (continued)

x	e ^x		e ^{-x}	Sinh x		Cosh x		Tanh x
	Value	Log ₁₀		Value	Log ₁₀	Value	Log ₁₀	
0.50	1.6487	.21715	.60653	0.5211	1.71692	1.1276	.05217	.46212
0.51	1.6652	.22149	.60059	0.5324	1.72624	1.1329	.05419	.46395
0.52	1.6820	.22593	.59452	0.5438	1.73540	1.1383	.05625	.46770
0.53	1.6989	.23018	.58860	0.5552	1.74442	1.1436	.05834	.46938
0.54	1.7160	.23452	.58275	0.5666	1.75330	1.1489	.06046	.47299
0.55	1.7333	.23886	.57695	0.5782	1.76204	1.1551	.06262	.47652
0.56	1.7507	.24320	.57121	0.5897	1.77065	1.1609	.06481	.47998
0.57	1.7683	.24755	.56553	0.6014	1.77914	1.1669	.06703	.48336
0.58	1.7860	.25189	.55990	0.6131	1.78751	1.1730	.06929	.48667
0.59	1.8040	.25623	.55433	0.6248	1.79576	1.1792	.07157	.48990
0.60	1.8221	.26058	.54881	0.6367	1.80390	1.1855	.07389	.49305
0.61	1.8404	.26492	.54335	0.6485	1.81194	1.1919	.07624	.49613
0.62	1.8589	.26926	.53794	0.6605	1.81987	1.1984	.07861	.49913
0.63	1.8776	.27361	.53259	0.6725	1.82770	1.2051	.08102	.50205
0.64	1.8965	.27795	.52729	0.6846	1.83543	1.2119	.08346	.50490
0.65	1.9155	.28229	.52205	0.6967	1.84308	1.2188	.08593	.50767
0.66	1.9346	.28663	.51686	0.7088	1.85063	1.2258	.08842	.51038
0.67	1.9538	.29096	.51171	0.7213	1.85809	1.2330	.09095	.51303
0.68	1.9730	.29532	.50662	0.7336	1.86548	1.2402	.09351	.51562
0.69	1.9923	.29966	.50158	0.7461	1.87278	1.2476	.09609	.51816
0.70	2.0118	.30401	.49659	0.7586	1.88000	1.2552	.09870	.52063
0.71	2.0314	.30835	.49164	0.7712	1.88715	1.2628	.10134	.52305
0.72	2.0511	.31269	.48675	0.7838	1.89423	1.2705	.10401	.52541
0.73	2.0709	.31703	.48191	0.7966	1.90123	1.2783	.10670	.52772
0.74	2.0909	.32138	.47711	0.8094	1.90817	1.2863	.10942	.52998
0.75	2.1110	.32572	.47237	0.8223	1.91504	1.2944	.11216	.53219
0.76	2.1313	.33006	.46767	0.8353	1.92185	1.3026	.11493	.53436
0.77	2.1518	.33441	.46301	0.8484	1.92859	1.3110	.11773	.53649
0.78	2.1724	.33875	.45841	0.8615	1.93527	1.3195	.12055	.53857
0.79	2.1932	.34309	.45384	0.8748	1.94189	1.3282	.12340	.54061
0.80	2.2142	.34744	.44933	0.8881	1.94846	1.3370	.12627	.54260
0.81	2.2353	.35178	.44486	0.9015	1.95498	1.3460	.12917	.54455
0.82	2.2565	.35612	.44043	0.9150	1.96144	1.3551	.13209	.54646
0.83	2.2778	.36046	.43605	0.9286	1.96784	1.3644	.13503	.54833
0.84	2.2993	.36481	.43171	0.9423	1.97420	1.3740	.13800	.55016
0.85	2.3209	.36915	.42741	0.9561	1.98051	1.3835	.14099	.55197
0.86	2.3426	.37349	.42315	0.9700	1.98677	1.3932	.14400	.55375
0.87	2.3644	.37784	.41895	0.9840	1.99299	1.4029	.14704	.55549
0.88	2.3863	.38218	.41478	0.9981	1.99916	1.4128	.15009	.55720
0.89	2.4083	.38652	.41066	1.0122	0.00528	1.4229	.15317	.55888
0.90	2.4304	.39087	.40657	1.0265	0.01137	1.4331	.15627	.56053
0.91	2.4526	.39521	.40252	1.0409	.01741	1.4434	.15939	.56215
0.92	2.4749	.39955	.39852	1.0554	.02341	1.4539	.16254	.56374
0.93	2.4973	.40389	.39455	1.0700	.02937	1.4644	.16570	.56530
0.94	2.5200	.40824	.39063	1.0847	.03530	1.4751	.16888	.56682
0.95	2.5427	.41258	.38674	1.0995	.04119	1.4862	.17208	.56831
0.96	2.5656	.41692	.38288	1.1144	.04704	1.4973	.17531	.56978
0.97	2.5886	.42127	.37906	1.1294	.05286	1.5085	.17856	.57122
0.98	2.6117	.42561	.37531	1.1445	.05864	1.5199	.18181	.57263
0.99	2.6349	.42995	.37158	1.1598	.06439	1.5314	.18509	.57401
1.00	2.7183	.43429	.36788	1.1752	.07011	1.5431	.18839	.57536

Table 16 VALUES AND COMMON LOGARITHMS OF EXPONENTIAL AND HYPERBOLIC FUNCTIONS (continued)

x	e^x		e^{-x}	Sinh x		Cosh x		Tanh x
	Value	Log ₁₀		Value	Value	Log ₁₀	Value	
1.00	2.7183	.43429	.36788	1.1752	.07011	1.6431	.21839	.76159
1.01	2.7456	.43864	.36422	1.1907	.07580	1.6549	.21971	.76576
1.02	2.7732	.44288	.36050	1.2033	.08146	1.6669	.22104	.76987
1.03	2.8011	.44732	.35701	1.2220	.08703	1.6790	.22238	.77391
1.04	2.8292	.45167	.35345	1.2379	.09265	1.6913	.22376	.77789
1.05	2.8577	.45601	.34994	1.2539	.09825	1.6038	.22515	.78181
1.06	2.8864	.46035	.34646	1.2700	.10379	1.6164	.22655	.78566
1.07	2.9154	.46470	.34301	1.2862	.10930	1.6292	.22797	.78946
1.08	2.9447	.46904	.33959	1.3025	.11479	1.6421	.22941	.79320
1.09	2.9743	.47338	.33622	1.3190	.12025	1.6552	.23086	.79688
1.10	3.0042	.47772	.33287	1.3356	.12569	1.6685	.22233	.80050
1.11	3.0344	.48207	.32956	1.3524	.13111	1.6820	.22382	.80406
1.12	3.0649	.48641	.32628	1.3693	.13651	1.6956	.22533	.80757
1.13	3.0957	.49075	.32303	1.3863	.14189	1.7093	.22683	.81102
1.14	3.1268	.49510	.31982	1.4035	.14726	1.7233	.22836	.81441
1.15	3.1582	.49944	.31664	1.4208	.15263	1.7374	.22990	.81775
1.16	3.1899	.50378	.31349	1.4382	.15793	1.7517	.23146	.82104
1.17	3.2220	.50812	.31037	1.4558	.16311	1.7662	.23303	.82427
1.18	3.2544	.51247	.30728	1.4735	.16835	1.7808	.23462	.82745
1.19	3.2871	.51681	.30422	1.4914	.17360	1.7957	.23622	.83058
1.20	3.3201	.52115	.30110	1.5095	.17882	1.8107	.23784	.83365
1.21	3.3535	.52550	.29802	1.5276	.18402	1.8259	.23948	.83668
1.22	3.3872	.52984	.29500	1.5460	.18920	1.8412	.24113	.83965
1.23	3.4212	.53418	.29203	1.5645	.19437	1.8568	.24280	.84258
1.24	3.4556	.53853	.28910	1.5831	.19951	1.8725	.24448	.84546
1.25	3.4903	.54287	.28620	1.6019	.20464	1.8884	.24617	.84828
1.26	3.5254	.54721	.28335	1.6209	.20975	1.9045	.24787	.85106
1.27	3.5609	.55155	.28053	1.6400	.21485	1.9208	.24959	.85380
1.28	3.5966	.55590	.27774	1.6593	.21993	1.9373	.25132	.85648
1.29	3.6328	.56024	.27500	1.6788	.22499	1.9540	.25307	.85913
1.30	3.6693	.56458	.27230	1.6984	.23004	1.9709	.25483	.86172
1.31	3.7062	.56893	.26962	1.7182	.23507	1.9880	.25660	.86428
1.32	3.7434	.57327	.26704	1.7381	.24009	2.0053	.25837	.86678
1.33	3.7810	.57761	.26448	1.7583	.24509	2.0228	.26014	.86925
1.34	3.8190	.58195	.26195	1.7786	.25008	2.0404	.26192	.87167
1.35	3.8574	.58630	.25944	1.7991	.25505	2.0582	.26371	.87405
1.36	3.8962	.59064	.25696	1.8198	.26002	2.0762	.26551	.87639
1.37	3.9354	.59498	.25451	1.8408	.26496	2.0944	.26732	.87869
1.38	3.9749	.59933	.25208	1.8617	.26990	2.1128	.26914	.88093
1.39	4.0149	.60367	.24968	1.8829	.27482	2.1320	.27097	.88317
1.40	4.0552	.60801	.24730	1.9043	.27974	2.1509	.27282	.88535
1.41	4.0960	.61236	.24494	1.9259	.28464	2.1700	.27468	.88749
1.42	4.1371	.61670	.24261	1.9477	.28952	2.1894	.27656	.88960
1.43	4.1787	.62104	.24031	1.9697	.29440	2.2090	.27844	.89167
1.44	4.2207	.62538	.23803	1.9919	.29926	2.2288	.28033	.89370
1.45	4.2631	.62973	.23577	2.0143	.30412	2.2488	.28223	.89569
1.46	4.3060	.63407	.23354	2.0369	.30896	2.2689	.28414	.89765
1.47	4.3492	.63841	.23133	2.0597	.31379	2.2892	.28606	.89958
1.48	4.3929	.64276	.22914	2.0827	.31862	2.3103	.28800	.90147
1.49	4.4371	.64710	.22697	2.1059	.32343	2.3312	.29000	.90332
1.50	4.4817	.65144	.22483	2.1293	.32823	2.3524	.29201	.90515

Table 16 VALUES AND COMMON LOGARITHMS OF EXPONENTIAL AND HYPERBOLIC FUNCTIONS (continued)

x	e ^x		e ^{-x} Value	Sinh x		Cosh x		Tanh x Value
	Value	Log ₁₀		Value	Log ₁₀	Value	Log ₁₀	
1.50	4.4817	.65144	.22313	2.1263	.32823	2.3524	.37151	.90515
1.51	4.5267	.65578	.22091	2.1529	.33303	2.3738	.37545	.90594
1.52	4.5722	.66013	.21871	2.1768	.33781	2.3955	.37939	.90670
1.53	4.6182	.66447	.21654	2.2008	.34258	2.4174	.38334	.91042
1.54	4.6646	.66881	.21438	2.2251	.34735	2.4395	.38730	.91212
1.55	4.7115	.67316	.21226	2.2496	.35211	2.4619	.39126	.91379
1.56	4.7588	.67750	.21014	2.2743	.35688	2.4845	.39524	.91542
1.57	4.8066	.68184	.20805	2.2993	.36160	2.5073	.39921	.91703
1.58	4.8550	.68619	.20598	2.3245	.36633	2.5305	.40320	.91860
1.59	4.9037	.69053	.20393	2.3499	.37105	2.5538	.40719	.92015
1.60	4.9530	.69487	.20190	2.3756	.37577	2.5775	.41119	.92167
1.61	5.0028	.69921	.19989	2.4015	.38048	2.6013	.41520	.92316
1.62	5.0531	.70356	.19790	2.4276	.38518	2.6255	.41921	.92462
1.63	5.1039	.70790	.19593	2.4540	.38987	2.6499	.42323	.92606
1.64	5.1552	.71224	.19398	2.4806	.39456	2.6746	.42725	.92747
1.65	5.2070	.71659	.19205	2.5073	.39923	2.6995	.43129	.92886
1.66	5.2593	.72093	.19014	2.5346	.40391	2.7247	.43532	.93022
1.67	5.3122	.72527	.18825	2.5620	.40857	2.7502	.43937	.93155
1.68	5.3656	.72961	.18637	2.5896	.41323	2.7760	.44341	.93286
1.69	5.4195	.73396	.18452	2.6175	.41788	2.8020	.44747	.93415
1.70	5.4739	.73830	.18268	2.6456	.42253	2.8283	.45153	.93541
1.71	5.5290	.74264	.18087	2.6740	.42717	2.8549	.45559	.93665
1.72	5.5845	.74699	.17907	2.7027	.43180	2.8818	.45965	.93788
1.73	5.6407	.75133	.17728	2.7317	.43643	2.9090	.46374	.93906
1.74	5.6973	.75567	.17552	2.7609	.44105	2.9364	.46782	.94023
1.75	5.7546	.76002	.17377	2.7904	.44567	2.9642	.47191	.94138
1.76	5.8124	.76436	.17204	2.8202	.45028	2.9922	.47600	.94250
1.77	5.8709	.76870	.17033	2.8503	.45488	3.0206	.48009	.94361
1.78	5.9299	.77304	.16864	2.8806	.45948	3.0492	.48419	.94470
1.79	5.9895	.77739	.16698	2.9112	.46408	3.0782	.48830	.94576
1.80	6.0496	.78173	.16530	2.9422	.46867	3.1075	.49241	.94681
1.81	6.1104	.78607	.16365	2.9734	.47325	3.1371	.49652	.94783
1.82	6.1719	.79042	.16203	3.0049	.47783	3.1669	.50064	.94884
1.83	6.2339	.79476	.16041	3.0367	.48241	3.1972	.50476	.94983
1.84	6.2965	.79910	.15882	3.0689	.48698	3.2277	.50889	.95080
1.85	6.3598	.80344	.15724	3.1013	.49154	3.2585	.51302	.95175
1.86	6.4237	.80779	.15567	3.1340	.49610	3.2897	.51716	.95268
1.87	6.4883	.81213	.15412	3.1671	.50066	3.3212	.52130	.95359
1.88	6.5535	.81647	.15259	3.2005	.50521	3.3530	.52544	.95449
1.89	6.6194	.82082	.15107	3.2341	.50978	3.3852	.52959	.95537
1.90	6.6859	.82516	.14957	3.2682	.51430	3.4177	.53374	.95624
1.91	6.7531	.82950	.14808	3.3025	.51884	3.4506	.53789	.95709
1.92	6.8210	.83385	.14661	3.3372	.52338	3.4839	.54205	.95792
1.93	6.8895	.83819	.14515	3.3722	.52791	3.5173	.54621	.95873
1.94	6.9588	.84253	.14370	3.4075	.53244	3.5512	.55038	.95955
1.95	7.0287	.84687	.14227	3.4432	.53698	3.5855	.55455	.96032
1.96	7.0993	.85122	.14086	3.4792	.54148	3.6201	.55872	.96109
1.97	7.1707	.85556	.13946	3.5156	.54600	3.6551	.56290	.96185
1.98	7.2427	.85990	.13807	3.5523	.55051	3.6904	.56707	.96259
1.99	7.3155	.86425	.13670	3.5894	.55502	3.7261	.57126	.96331
2.00	7.3891	.86859	.13534	3.6259	.55953	3.7622	.57544	.96403

Table 16 VALUES AND COMMON LOGARITHMS OF EXPONENTIAL AND HYPERBOLIC FUNCTIONS (continued)

x	e ^x		e ^{-x}	Sinh x		Cosh x		Tanh x
	Value	Log ₁₀		Value	Log ₁₀	Value	Log ₁₀	
2.00	7.3891	.86859	.13534	3.6269	.55953	3.7622	.57544	.96403
2.01	7.4633	.87293	.13399	3.6647	.56403	3.7987	.57963	.96473
2.02	7.5363	.87727	.13266	3.7028	.56853	3.8355	.58382	.96541
2.03	7.6141	.88162	.13134	3.7414	.57303	3.8727	.58802	.96609
2.04	7.6903	.88596	.13003	3.7803	.57753	3.9103	.59221	.96675
2.05	7.7679	.89030	.12873	3.8195	.58202	3.9483	.59641	.96740
2.06	7.8460	.89465	.12745	3.8592	.58650	3.9867	.60061	.96803
2.07	7.9248	.89900	.12619	3.8993	.59099	4.0255	.60482	.96865
2.08	8.0045	.90333	.12493	3.9398	.59547	4.0647	.60903	.96928
2.09	8.0849	.90768	.12369	3.9806	.59995	4.1043	.61324	.96988
2.10	8.1662	.91202	.12246	4.0219	.60443	4.1443	.61745	.97048
2.11	8.2482	.91635	.12124	4.0635	.60890	4.1847	.62167	.97103
2.12	8.3311	.92070	.12003	4.1056	.61337	4.2256	.62589	.97159
2.13	8.4149	.92505	.11884	4.1480	.61784	4.2669	.63011	.97215
2.14	8.4994	.92939	.11765	4.1909	.62231	4.3085	.63433	.97269
2.15	8.5849	.93373	.11648	4.2342	.62677	4.3507	.63856	.97323
2.16	8.6711	.93808	.11533	4.2779	.63123	4.3932	.64278	.97378
2.17	8.7583	.94242	.11418	4.3221	.63569	4.4362	.64701	.97432
2.18	8.8463	.94676	.11304	4.3668	.64015	4.4797	.65125	.97487
2.19	8.9352	.95110	.11192	4.4116	.64460	4.5236	.65548	.97542
2.20	9.0250	.95545	.11080	4.4571	.64905	4.5679	.65972	.97597
2.21	9.1157	.95979	.10970	4.5030	.65350	4.6127	.66396	.97652
2.22	9.2073	.96413	.10861	4.5494	.65795	4.6580	.66820	.97707
2.23	9.2999	.96848	.10753	4.5962	.66240	4.7037	.67244	.97761
2.24	9.3933	.97282	.10646	4.6434	.66684	4.7499	.67668	.97815
2.25	9.4877	.97716	.10540	4.6912	.67128	4.7966	.68093	.97869
2.26	9.5831	.98151	.10435	4.7394	.67572	4.8437	.68518	.97923
2.27	9.6794	.98585	.10331	4.7880	.68016	4.8914	.68943	.97977
2.28	9.7767	.99019	.10228	4.8372	.68459	4.9395	.69368	.98031
2.29	9.8749	.99453	.10127	4.8868	.68903	4.9881	.69794	.98084
2.30	9.9742	.99888	.10026	4.9370	.69346	5.0372	.70219	.98137
2.31	10.074	1.00322	.09926	4.9876	.69789	5.0868	.70645	.98190
2.32	10.176	1.00755	.09827	5.0387	.70232	5.1370	.71071	.98242
2.33	10.278	1.01191	.09730	5.0903	.70675	5.1876	.71497	.98294
2.34	10.381	1.01625	.09633	5.1425	.71117	5.2388	.71923	.98346
2.35	10.486	1.02059	.09537	5.1951	.71559	5.2905	.72349	.98397
2.36	10.591	1.02493	.09442	5.2483	.72002	5.3427	.72776	.98448
2.37	10.697	1.02928	.09348	5.3020	.72444	5.3954	.73203	.98499
2.38	10.805	1.03362	.09255	5.3562	.72885	5.4487	.73630	.98550
2.39	10.913	1.03796	.09163	5.4109	.73327	5.5026	.74056	.98601
2.40	11.023	1.04231	.09072	5.4662	.73769	5.5569	.74484	.98652
2.41	11.134	1.04665	.08982	5.5221	.74210	5.6119	.74911	.98703
2.42	11.246	1.05099	.08892	5.5785	.74652	5.6674	.75338	.98754
2.43	11.359	1.05534	.08804	5.6354	.75093	5.7235	.75766	.98805
2.44	11.473	1.05968	.08716	5.6928	.75534	5.7801	.76194	.98856
2.45	11.588	1.06402	.08629	5.7510	.75975	5.8373	.76621	.98907
2.46	11.705	1.06836	.08543	5.8097	.76415	5.8951	.77049	.98958
2.47	11.822	1.07271	.08458	5.8689	.76856	5.9535	.77477	.99009
2.48	11.941	1.07705	.08374	5.9288	.77296	6.0125	.77906	.99060
2.49	12.061	1.08139	.08291	5.9892	.77737	6.0721	.78334	.99111
2.50	12.182	1.08574	.08208	6.0502	.78177	6.1323	.78762	.99162

Table 16 VALUES AND COMMON LOGARITHMS OF EXPONENTIAL AND HYPERBOLIC FUNCTIONS (continued)

x	e^x		e^{-x} Value	Sinh x		Cosh x		Tanh x Value
	Value	Log ₁₀		Value	Log ₁₀	Value	Log ₁₀	
2.50	12.182	1.08574	.08208	6.0502	.78177	6.1323	.78762	.98661
2.51	12.305	1.09008	.08127	6.1118	.78817	6.1931	.79191	.98688
2.52	12.429	1.09442	.08046	6.1741	.79057	6.2545	.79519	.98714
2.53	12.554	1.09877	.07966	6.2369	.79497	6.3166	.80048	.98739
2.54	12.680	1.10311	.07887	6.3004	.79937	6.3793	.80477	.98764
2.55	12.807	1.10745	.07808	6.3645	.80377	6.4426	.80906	.98788
2.56	12.936	1.11179	.07730	6.4293	.80816	6.5066	.81335	.98812
2.57	13.066	1.11614	.07654	6.4948	.81256	6.5712	.81764	.98836
2.58	13.197	1.12048	.07577	6.5607	.81695	6.6365	.82194	.98858
2.59	13.330	1.12482	.07502	6.6274	.82134	6.7024	.82623	.98881
2.60	13.464	1.12917	.07427	6.6947	.82573	6.7690	.83052	.98903
2.61	13.599	1.13351	.07353	6.7628	.83012	6.8363	.83482	.98924
2.62	13.736	1.13785	.07280	6.8315	.83451	6.9043	.83912	.98946
2.63	13.874	1.14219	.07208	6.9008	.83890	6.9729	.84341	.98968
2.64	14.013	1.14654	.07136	6.9706	.84329	7.0423	.84771	.98987
2.65	14.154	1.15088	.07065	7.0417	.84768	7.1123	.85201	.99007
2.66	14.296	1.15522	.06995	7.1132	.85208	7.1831	.85631	.99026
2.67	14.440	1.15957	.06925	7.1854	.85645	7.2545	.86061	.99045
2.68	14.585	1.16391	.06856	7.2583	.86083	7.3265	.86492	.99063
2.69	14.732	1.16825	.06788	7.3319	.86522	7.3998	.86922	.99082
2.70	14.880	1.17260	.06721	7.4063	.86960	7.4735	.87352	.99101
2.71	15.029	1.17694	.06654	7.4814	.87398	7.5479	.87783	.99119
2.72	15.180	1.18128	.06587	7.5572	.87836	7.6231	.88213	.99136
2.73	15.333	1.18562	.06522	7.6338	.88274	7.6991	.88644	.99153
2.74	15.487	1.18997	.06457	7.7112	.88712	7.7758	.89074	.99170
2.75	15.643	1.19431	.06393	7.7894	.89150	7.8533	.89505	.99186
2.76	15.800	1.19865	.06329	7.8683	.89586	7.9316	.89936	.99202
2.77	15.959	1.20300	.06266	7.9480	.90026	8.0106	.90367	.99219
2.78	16.119	1.20734	.06204	8.0285	.90463	8.0905	.90798	.99233
2.79	16.281	1.21168	.06142	8.1098	.90901	8.1712	.91229	.99248
2.80	16.445	1.21602	.06081	8.1919	.91339	8.2527	.91660	.99263
2.81	16.610	1.22037	.06020	8.2749	.91776	8.3351	.92091	.99278
2.82	16.777	1.22471	.05961	8.3586	.92213	8.4182	.92522	.99292
2.83	16.945	1.22905	.05901	8.4432	.92651	8.5022	.92953	.99306
2.84	17.115	1.23340	.05843	8.5287	.93088	8.5871	.93385	.99320
2.85	17.288	1.23774	.05784	8.6150	.93525	8.6728	.93816	.99333
2.86	17.462	1.24208	.05727	8.7021	.93963	8.7594	.94247	.99346
2.87	17.637	1.24643	.05670	8.7902	.94400	8.8468	.94679	.99359
2.88	17.814	1.25077	.05613	8.8791	.94837	8.9352	.95110	.99372
2.89	17.993	1.25511	.05558	8.9686	.95274	9.0244	.95542	.99384
2.90	18.174	1.25945	.05502	9.0596	.95711	9.1146	.95974	.99396
2.91	18.357	1.26380	.05448	9.1512	.96149	9.2056	.96405	.99408
2.92	18.541	1.26814	.05393	9.2437	.96584	9.2976	.96837	.99420
2.93	18.728	1.27248	.05340	9.3371	.97021	9.3905	.97269	.99431
2.94	18.916	1.27683	.05287	9.4315	.97458	9.4844	.97701	.99443
2.95	19.106	1.28117	.05234	9.5268	.97895	9.5791	.98133	.99454
2.96	19.298	1.28551	.05182	9.6231	.98331	9.6749	.98565	.99464
2.97	19.492	1.28985	.05130	9.7203	.98768	9.7716	.98997	.99475
2.98	19.688	1.29420	.05079	9.8185	.99205	9.8693	.99429	.99485
2.99	19.886	1.29854	.05029	9.9177	0.99641	9.9680	0.99861	.99495
3.00	20.086	1.30288	.04979	10.018	1.00078	10.068	1.00293	.99505

Table 16 VALUES AND COMMON LOGARITHMS OF EXPONENTIAL AND HYPERBOLIC FUNCTIONS (continued)

x	e^x		e^{-x}	Sinh x		Cosh x		Tanh x Value
	Value	Log ₁₀		Value	Log ₁₀	Value	Log ₁₀	
3.00	20.086	1.30288	.04979	10.018	1.00078	10.068	1.00293	.99505
3.05	21.116	1.32480	.04735	10.534	1.02259	10.581	1.02454	.99582
3.10	22.198	1.34631	.04506	11.078	1.04440	11.122	1.04616	.99659
3.15	23.335	1.35803	.04285	11.647	1.06620	11.690	1.06779	.99733
3.20	24.533	1.38974	.04076	12.246	1.08799	12.287	1.08943	.99798
3.25	25.790	1.41146	.03877	12.876	1.10977	12.915	1.11108	.99700
3.30	27.113	1.43317	.03688	13.538	1.13156	13.575	1.13273	.99728
3.35	28.503	1.45489	.03508	14.234	1.15332	14.269	1.15439	.99754
3.40	29.964	1.47660	.03337	14.965	1.17509	14.989	1.17695	.99777
3.45	31.500	1.49832	.03175	15.734	1.19686	15.768	1.19772	.99799
3.50	33.115	1.52003	.03020	16.543	1.21860	16.673	1.21940	.99818
3.55	34.813	1.54175	.02872	17.392	1.24036	17.421	1.24107	.99835
3.60	36.598	1.56348	.02732	18.286	1.26211	18.313	1.26275	.99851
3.65	38.475	1.58517	.02599	19.224	1.28385	19.250	1.28444	.99865
3.70	40.447	1.60689	.02472	20.211	1.30559	20.235	1.30612	.99878
3.75	42.521	1.62860	.02352	21.249	1.32733	21.272	1.32781	.99889
3.80	44.701	1.65032	.02237	22.339	1.34907	22.362	1.34951	.99900
3.85	46.993	1.67205	.02128	23.486	1.37081	23.507	1.37120	.99909
3.90	49.402	1.69378	.02024	24.691	1.39254	24.711	1.39290	.99918
3.95	51.935	1.71546	.01925	25.958	1.41427	25.977	1.41458	.99926
4.00	54.598	1.73718	.01832	27.290	1.43600	27.308	1.43629	.99933
4.10	60.340	1.78061	.01657	30.162	1.47946	30.179	1.47970	.99945
4.20	66.266	1.82404	.01500	33.338	1.52291	33.351	1.52310	.99955
4.30	72.490	1.86747	.01357	36.843	1.56636	36.857	1.56652	.99963
4.40	79.031	1.91090	.01227	40.719	1.60980	40.732	1.60993	.99970
4.50	85.897	1.95433	.01111	45.003	1.65324	45.014	1.65336	.99976
4.60	93.104	1.99775	.01005	49.737	1.69668	49.747	1.69677	.99980
4.70	100.759	2.04118	.00910	54.969	1.74012	54.978	1.74019	.99983
4.80	108.971	2.08461	.00823	60.751	1.78355	60.759	1.78361	.99986
4.90	117.849	2.12804	.00745	67.141	1.82699	67.149	1.82704	.99989
5.00	127.426	2.17147	.00674	74.203	1.87042	74.210	1.87046	.99991
5.10	137.810	2.21490	.00610	82.008	1.91389	82.014	1.91389	.99993
5.20	149.027	2.25833	.00552	90.633	1.95729	90.639	1.95731	.99994
5.30	161.094	2.30176	.00499	100.17	2.00074	100.17	2.00074	.99995
5.40	174.031	2.34519	.00452	110.70	2.04416	110.71	2.04417	.99996
5.50	187.869	2.38862	.00409	122.34	2.08758	122.35	2.08760	.99997
5.60	202.637	2.43205	.00370	135.21	2.13101	135.22	2.13103	.99997
5.70	218.367	2.47548	.00335	149.43	2.17444	149.44	2.17445	.99998
5.80	235.091	2.51891	.00303	165.15	2.21787	165.15	2.21788	.99998
5.90	252.844	2.56234	.00274	182.52	2.26130	182.52	2.26131	.99998
6.00	271.687	2.60577	.00248	201.71	2.30473	201.72	2.30474	.99999
6.25	318.01	2.71434	.00193	259.01	2.41331	259.01	2.41331	.99999
6.50	365.14	2.82291	.00150	332.57	2.52188	332.57	2.52188	1.0000
6.75	424.06	2.93149	.00117	427.03	2.63046	427.03	2.63046	1.0000
7.00	495.0	3.04006	.00091	548.32	2.73903	548.32	2.73903	1.0000
7.50	1808.0	3.25721	.00055	904.02	2.95618	904.02	2.95618	1.0000
8.00	2981.0	3.47436	.00034	1490.5	3.17333	1490.5	3.17333	1.0000
8.50	4914.8	3.69150	.00020	2457.4	3.39047	2457.4	3.39047	1.0000
9.00	8103.1	3.90865	.00012	4051.5	3.60762	4051.5	3.60762	1.0000
9.50	13386.	4.12580	.00007	6579.9	3.82477	6579.9	3.82477	1.0000
10.00	22026.	4.34294	.00005	11013.	4.04191	11013.	4.04191	1.0000

Table 17 MULTIPLES OF M AND OF $1/M$

N	N-M	N	N-M	N	N+M	N	N+M
0	0.00000 000	50	21.71472 410	0	0.00000 000	50	115.12925 465
1	0.43429 448	51	22.14901 658	1	2.30258 509	51	117.43133 974
2	0.86858 896	52	22.58331 306	2	4.60517 019	52	119.73442 484
3	1.30288 345	53	23.01760 754	3	6.90775 528	53	122.03750 993
4	1.73717 793	54	23.45190 202	4	9.21034 037	54	124.34059 502
5	2.17147 241	55	23.88619 650	5	11.51292 546	55	126.64368 011
6	2.60576 689	56	24.32049 098	6	13.81551 056	56	128.94676 521
7	3.04006 137	57	24.75478 547	7	16.11809 565	57	131.24985 030
8	3.47435 586	58	25.18907 995	8	18.42068 074	58	133.55293 539
9	3.90865 034	59	25.62337 443	9	20.72326 584	59	135.85602 049
10	4.34294 482	60	26.05766 891	10	23.02585 093	60	138.15910 558
11	4.77723 930	61	26.49196 340	11	25.32843 602	61	140.46219 067
12	5.21153 378	62	26.92625 788	12	27.63102 112	62	142.76527 577
13	5.64582 826	63	27.36055 236	13	29.93360 621	63	145.06836 086
14	6.08012 275	64	27.79484 684	14	32.23619 130	64	147.37144 595
15	6.51441 723	65	28.22914 132	15	34.53877 639	65	149.67453 104
16	6.94871 171	66	28.66343 581	16	36.84136 149	66	151.97761 614
17	7.38300 619	67	29.09773 029	17	39.14394 658	67	154.28070 123
18	7.81730 067	68	29.53202 477	18	41.44653 167	68	156.58378 632
19	8.25159 516	69	29.96631 925	19	43.74911 677	69	158.88687 142
20	8.68588 964	70	30.40061 373	20	46.05170 186	70	161.18995 651
21	9.12018 412	71	30.83490 822	21	48.35428 695	71	163.49304 160
22	9.55447 860	72	31.26920 270	22	50.65687 205	72	165.79612 670
23	9.98877 308	73	31.70349 718	23	52.95945 714	73	168.09921 179
24	10.42306 757	74	32.13779 166	24	55.26204 223	74	170.40229 688
25	10.85736 205	75	32.57208 614	25	57.56462 732	75	172.70538 197
26	11.29165 653	76	33.00638 062	26	59.86721 242	76	175.00846 707
27	11.72595 101	77	33.44067 511	27	62.16979 751	77	177.31155 216
28	12.16024 549	78	33.87496 959	28	64.47238 260	78	179.61463 725
29	12.59453 998	79	34.30926 407	29	66.77496 770	79	181.91772 235
30	13.02883 446	80	34.74355 855	30	69.07755 279	80	184.22080 744
31	13.46312 894	81	35.17785 303	31	71.38013 788	81	186.52389 253
32	13.89742 342	82	35.61214 752	32	73.68272 298	82	188.82697 763
33	14.33171 790	83	36.04644 200	33	75.98530 807	83	191.13006 272
34	14.76601 238	84	36.48073 648	34	78.28789 316	84	193.43314 781
35	15.20030 687	85	36.91503 096	35	80.59047 825	85	195.73623 290
36	15.63460 135	86	37.34932 544	36	82.89306 335	86	198.03931 800
37	16.06889 583	87	37.78361 993	37	85.19564 844	87	200.34240 309
38	16.50319 031	88	38.21791 441	38	87.49823 353	88	202.64548 818
39	16.93748 479	89	38.65220 889	39	89.80081 863	89	204.94857 328
40	17.37177 928	90	39.08650 337	40	92.10340 372	90	207.25165 837
41	17.80607 376	91	39.52079 785	41	94.40598 881	91	209.55474 346
42	18.24036 824	92	39.95509 234	42	96.70857 391	92	211.85782 855
43	18.67466 272	93	40.38938 682	43	99.01115 900	93	214.16091 365
44	19.10895 720	94	40.82368 130	44	101.31374 409	94	216.46399 874
45	19.54325 169	95	41.25797 578	45	103.61632 918	95	218.76708 383
46	19.97754 617	96	41.69227 026	46	105.91891 428	96	221.07016 893
47	20.41184 065	97	42.12656 474	47	108.22149 937	97	223.37325 402
48	20.84613 513	98	42.56085 923	48	110.52408 446	98	225.67633 911
49	21.28042 961	99	42.99515 371	49	112.82666 956	99	227.97942 421
50	21.71472 410	100	43.42944 819	50	115.12925 465	100	230.28250 930

$$M = \log_{10} e = .43429 44819 03251 82765 \quad \frac{1}{M} = \log_e 10 = 2.30258 50929 94045 68402$$

$$\log_{10} n = \log_e n \log_{10} e = M \log_e n \quad \log_e n = \log_{10} n \log_e 10 = \frac{1}{M} \log_{10} n$$

$$\log_{10} e^x = x \log_{10} e = xM \quad \log_e (10^x) = \log_e x + n \frac{1}{M}$$

**POWERS, ROOTS, RECIPROCAL,
AREAS, AND CIRCUMFERENCES**

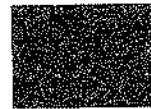


TABLE 18. *Squares, Cubes, Square Roots,
and Cube Roots, 325*

TABLE 19. *Reciprocals, Circumferences,
and Areas of Circles, 345*

**Table 18 SQUARES, CUBES, SQUARE ROOTS,
AND CUBE ROOTS**

n	n^2	n^3	\sqrt{n}	$\sqrt{10n}$	$\sqrt[3]{n}$	$\sqrt[3]{10n}$	$\sqrt[3]{100n}$
1	1	1	1.000 000	3.162 278	1.000 000	2.154 435	4.641 589
2	4	8	1.414 214	4.472 136	1.259 921	2.714 418	6.848 035
3	9	27	1.732 051	5.477 226	1.442 250	3.107 233	6.894 330
4	16	64	2.000 000	6.324 555	1.587 401	3.419 952	7.368 063
5	25	125	2.236 068	7.071 068	1.709 978	3.684 031	7.937 095
6	36	216	2.449 490	7.745 967	1.817 121	3.914 858	8.434 327
7	49	343	2.645 751	8.366 600	1.912 931	4.121 285	8.879 040
8	64	512	2.828 427	8.944 272	2.000 000	4.308 659	9.283 178
9	81	729	3.000 000	9.486 833	2.080 084	4.481 405	9.654 894
10	100	1 000	3.162 278	10.000 00	2.154 435	4.641 589	10.000 00
11	121	1 331	3.316 625	10.488 09	2.223 980	4.791 420	10.322 80
12	144	1 728	3.464 102	10.954 45	2.289 428	4.932 424	10.626 59
13	169	2 197	3.605 551	11.491 75	2.351 335	5.068 797	10.913 93
14	196	2 744	3.741 657	11.832 16	2.410 142	5.192 494	11.186 89
15	225	3 375	3.872 983	12.247 45	2.466 212	5.313 293	11.447 14
16	256	4 096	4.000 000	12.649 11	2.519 842	5.428 835	11.696 07
17	289	4 913	4.123 106	13.038 40	2.571 232	5.539 658	11.934 83
18	324	5 832	4.242 641	13.416 41	2.620 741	5.646 216	12.164 40
19	361	6 859	4.358 899	13.784 05	2.668 402	5.748 897	12.385 62
20	400	8 000	4.472 136	14.142 14	2.714 418	5.846 035	12.599 21
21	441	9 261	4.582 576	14.491 38	2.758 924	5.938 822	12.805 79
22	484	10 648	4.690 416	14.832 40	2.802 039	6.036 811	13.006 91
23	529	12 167	4.795 832	15.165 75	2.843 867	6.129 628	13.200 06
24	576	13 824	4.898 979	15.491 93	2.884 499	6.214 455	13.388 86
25	625	15 625	5.000 000	15.811 39	2.924 018	6.299 605	13.572 03
26	676	17 578	5.099 020	16.124 52	2.962 496	6.382 594	13.750 69
27	729	19 683	5.196 152	16.431 68	3.000 000	6.463 304	13.924 77
28	784	21 952	5.291 503	16.733 20	3.036 559	6.542 133	14.094 00
29	841	24 389	5.385 165	17.029 39	3.072 317	6.619 106	14.260 43
30	900	27 000	5.477 226	17.320 51	3.107 233	6.694 330	14.422 50
31	961	29 791	5.567 764	17.606 82	3.141 381	6.757 899	14.581 00
32	1 024	32 768	5.656 854	17.888 64	3.174 802	6.839 904	14.736 13
33	1 089	35 937	5.744 563	18.166 90	3.207 534	6.910 423	14.888 06
34	1 156	39 304	5.830 952	18.439 09	3.239 612	6.979 532	15.036 95
35	1 225	42 875	5.916 080	18.706 29	3.271 066	7.047 269	15.182 94
36	1 296	46 656	6.000 000	18.973 67	3.301 927	7.113 787	15.326 19
37	1 369	50 653	6.082 763	19.235 38	3.332 222	7.179 034	15.466 86
38	1 444	54 872	6.164 414	19.493 69	3.361 975	7.243 156	15.604 91
39	1 521	59 319	6.244 993	19.748 42	3.391 211	7.306 144	15.740 61
40	1 600	64 000	6.324 555	20.000 00	3.419 952	7.368 063	15.874 01
41	1 681	68 921	6.403 124	20.248 46	3.448 217	7.428 859	16.005 21
42	1 764	74 088	6.480 741	20.493 90	3.476 027	7.488 672	16.134 29
43	1 849	79 507	6.557 439	20.736 44	3.503 388	7.547 842	16.261 33
44	1 936	85 184	6.633 250	20.976 13	3.530 348	7.605 905	16.386 43
45	2 025	91 125	6.708 204	21.213 20	3.556 893	7.663 064	16.509 64
46	2 116	97 336	6.782 330	21.447 61	3.583 048	7.719 443	16.631 03
47	2 209	103 823	6.855 655	21.679 48	3.608 829	7.774 950	16.750 89
48	2 304	110 592	6.928 203	21.908 80	3.634 241	7.829 735	16.868 85
49	2 401	117 649	7.000 000	22.135 94	3.659 308	7.883 735	16.984 99

Table 18 SQUARES, CUBES, SQUARE ROOTS, AND CUBE ROOTS (continued) 326

n	n^2	n^3	\sqrt{n}	$\sqrt{10n}$	$\sqrt[3]{n}$	$\sqrt[3]{10n}$	$\sqrt[3]{100n}$
50	2 500	126 000	7.071 068	22.360 68	3.684 031	7.937 005	17.089 76
51	2 601	132 651	7.141 428	22.583 18	3.708 430	7.989 870	17.213 01
52	2 704	140 608	7.211 103	22.803 51	3.732 511	8.041 482	17.324 78
53	2 809	148 877	7.280 110	23.021 73	3.756 286	8.092 672	17.435 13
54	2 916	157 464	7.348 469	23.237 90	3.779 763	8.143 253	17.544 11
55	3 025	166 375	7.416 192	23.452 08	3.802 952	8.193 213	17.651 74
56	3 136	175 616	7.483 315	23.664 32	3.825 862	8.242 571	17.758 08
57	3 249	185 193	7.549 834	23.874 67	3.848 501	8.291 344	17.863 16
58	3 364	195 112	7.615 773	24.083 19	3.870 877	8.339 551	17.967 02
59	3 481	205 379	7.681 146	24.289 92	3.892 986	8.387 207	18.069 69
60	3 600	216 000	7.745 967	24.494 93	3.914 868	8.434 327	18.171 21
61	3 721	226 981	7.810 250	24.698 16	3.936 497	8.480 926	18.271 60
62	3 844	238 328	7.874 008	24.899 60	3.957 892	8.527 019	18.370 91
63	3 969	250 047	7.937 254	25.099 60	3.979 057	8.572 619	18.469 15
64	4 096	262 144	8.000 000	25.298 22	4.000 000	8.617 739	18.566 36
65	4 225	274 625	8.062 258	25.495 10	4.020 726	8.662 391	18.662 56
66	4 356	287 496	8.124 038	25.690 47	4.041 240	8.706 588	18.757 77
67	4 489	300 763	8.185 353	25.884 36	4.061 548	8.750 340	18.852 04
68	4 624	314 432	8.246 211	26.076 81	4.081 655	8.793 650	18.945 36
69	4 761	328 509	8.306 624	26.267 85	4.101 566	8.836 556	19.037 78
70	4 900	343 000	8.366 500	26.457 51	4.121 286	8.879 040	19.129 31
71	5 041	357 911	8.425 150	26.645 83	4.140 818	8.921 121	19.219 97
72	5 184	373 248	8.483 281	26.832 82	4.160 168	8.962 839	19.309 79
73	5 329	389 017	8.541 004	27.018 51	4.179 339	9.004 113	19.398 77
74	5 476	405 224	8.598 325	27.202 94	4.198 336	9.045 042	19.486 95
75	5 625	421 875	8.660 254	27.386 13	4.217 163	9.085 603	19.574 34
76	5 776	438 976	8.717 798	27.568 10	4.235 824	9.125 805	19.660 95
77	5 929	456 533	8.774 964	27.748 87	4.254 321	9.165 656	19.746 81
78	6 084	474 562	8.831 761	27.928 48	4.272 659	9.205 164	19.831 92
79	6 241	493 039	8.888 194	28.106 94	4.290 840	9.244 335	19.916 32
80	6 400	512 000	8.944 272	28.284 27	4.308 869	9.283 178	20.000 00
81	6 561	531 441	9.000 000	28.460 50	4.326 749	9.321 698	20.062 99
82	6 724	551 368	9.055 325	28.635 64	4.344 481	9.359 902	20.165 30
83	6 889	571 787	9.110 434	28.809 72	4.362 071	9.397 796	20.246 84
84	7 056	592 704	9.165 151	28.982 75	4.379 519	9.435 388	20.327 93
85	7 225	614 125	9.219 544	29.154 75	4.396 830	9.472 682	20.408 28
86	7 396	636 056	9.273 616	29.325 76	4.414 005	9.509 685	20.488 00
87	7 569	658 503	9.327 379	29.495 76	4.431 048	9.546 408	20.567 10
88	7 744	681 472	9.380 832	29.664 79	4.447 960	9.582 840	20.645 50
89	7 921	704 969	9.433 981	29.832 87	4.464 745	9.619 002	20.723 51
90	8 100	729 000	9.486 833	30.000 00	4.481 405	9.654 894	20.800 84
91	8 281	753 571	9.539 392	30.166 21	4.497 941	9.690 521	20.877 59
92	8 464	778 688	9.591 663	30.331 50	4.514 357	9.725 888	20.953 79
93	8 649	804 357	9.643 651	30.495 90	4.530 655	9.761 000	21.029 44
94	8 836	830 584	9.695 360	30.659 42	4.546 836	9.795 861	21.104 54
95	9 025	857 375	9.745 794	30.822 07	4.562 903	9.830 476	21.179 12
96	9 216	884 736	9.795 959	30.983 87	4.578 857	9.864 848	21.253 17
97	9 409	912 673	9.845 858	31.144 82	4.594 701	9.899 983	21.325 71
98	9 604	941 192	9.899 495	31.304 95	4.610 438	9.932 884	21.399 75
99	9 801	970 299	9.949 874	31.464 27	4.626 068	9.966 555	21.472 29

Table 18 SQUARES, CUBES, SQUARE ROOTS, AND CUBE ROOTS (continued) 327

n	n^2	n^3	\sqrt{n}	$\sqrt{10n}$	$\sqrt[3]{n}$	$\sqrt[3]{10n}$	$\sqrt[3]{100n}$
100	10 000	1 000 000	10.000 00	31.622 78	4.641 589	10.000 00	21.544 26
101	10 201	1 030 301	10.049 88	31.780 50	4.657 010	10.033 22	21.615 92
102	10 404	1 061 208	10.099 50	31.937 44	4.672 329	10.066 23	21.687 03
103	10 609	1 092 727	10.148 89	32.093 81	4.687 548	10.099 02	21.757 67
104	10 816	1 124 864	10.198 04	32.249 03	4.702 669	10.131 69	21.827 88
105	11 025	1 157 625	10.246 95	32.403 70	4.717 694	10.163 06	21.897 60
106	11 236	1 191 016	10.295 63	32.557 64	4.732 623	10.194 13	21.966 80
107	11 449	1 225 043	10.344 08	32.710 83	4.747 459	10.224 09	22.035 75
108	11 664	1 259 712	10.392 30	32.863 35	4.762 203	10.253 86	22.104 19
109	11 881	1 295 029	10.440 31	33.015 15	4.776 856	10.283 42	22.172 20
110	12 100	1 331 000	10.488 09	33.166 25	4.791 420	10.322 80	22.239 80
111	12 321	1 367 631	10.535 65	33.316 66	4.805 896	10.353 09	22.306 99
112	12 544	1 404 928	10.583 01	33.466 40	4.820 285	10.384 99	22.373 78
113	12 769	1 442 887	10.630 15	33.615 47	4.834 588	10.415 80	22.440 17
114	12 996	1 481 544	10.677 08	33.763 89	4.848 808	10.446 44	22.506 17
115	13 225	1 520 875	10.723 81	33.911 65	4.862 944	10.476 90	22.571 79
116	13 456	1 560 896	10.770 33	34.058 77	4.876 990	10.507 18	22.637 02
117	13 689	1 601 613	10.816 65	34.205 26	4.890 973	10.537 28	22.701 89
118	13 924	1 643 032	10.862 78	34.351 13	4.904 868	10.567 22	22.766 38
119	14 161	1 685 159	10.908 71	34.496 38	4.918 685	10.596 99	22.830 51
120	14 400	1 728 000	10.954 45	34.641 02	4.932 424	10.626 59	22.894 28
121	14 641	1 771 561	11.000 00	34.785 05	4.946 087	10.656 02	22.957 70
122	14 884	1 815 848	11.045 36	34.928 50	4.959 676	10.685 30	23.020 78
123	15 129	1 860 867	11.090 54	35.071 36	4.973 190	10.714 41	23.083 50
124	15 376	1 906 624	11.135 53	35.213 63	4.986 631	10.743 37	23.145 89
125	15 625	1 953 125	11.180 34	35.355 34	5.000 000	10.772 17	23.207 94
126	15 876	2 000 376	11.224 97	35.496 48	5.013 298	10.800 82	23.269 67
127	16 129	2 048 383	11.269 43	35.637 06	5.026 526	10.829 32	23.331 07
128	16 384	2 097 152	11.313 71	35.777 09	5.039 684	10.857 67	23.392 14
129	16 641	2 146 689	11.357 82	35.916 57	5.052 774	10.885 87	23.452 90
130	16 900	2 197 000	11.401 75	36.055 51	5.065 797	10.913 93	23.513 35
131	17 161	2 248 091	11.445 52	36.193 92	5.078 753	10.941 84	23.573 48
132	17 424	2 299 968	11.489 13	36.331 80	5.091 643	10.969 61	23.633 32
133	17 689	2 352 637	11.532 56	36.469 17	5.104 469	10.997 24	23.692 85
134	17 956	2 406 104	11.575 84	36.606 01	5.117 230	11.024 74	23.752 08
135	18 225	2 460 375	11.618 95	36.742 35	5.129 928	11.052 09	23.811 02
136	18 496	2 515 456	11.661 90	36.878 18	5.142 563	11.079 32	23.869 66
137	18 769	2 571 333	11.704 70	37.013 51	5.155 137	11.106 41	23.928 03
138	19 044	2 628 012	11.747 34	37.148 35	5.167 649	11.133 36	23.986 10
139	19 321	2 685 499	11.789 83	37.282 70	5.180 101	11.160 19	24.043 90
140	19 600	2 744 000	11.832 16	37.416 57	5.192 494	11.186 89	24.101 42
141	19 881	2 803 221	11.874 34	37.549 97	5.204 828	11.213 46	24.158 67
142	20 164	2 863 238	11.916 38	37.682 89	5.217 103	11.239 91	24.215 65
143	20 449	2 924 207	11.958 26	37.815 34	5.229 322	11.266 43	24.272 36
144	20 736	2 985 984	12.000 00	37.947 33	5.241 483	11.292 43	24.328 81
145	21 025	3 048 625	12.041 59	38.078 87	5.253 588	11.318 51	24.384 99
146	21 316	3 112 136	12.083 05	38.209 95	5.265 637	11.344 47	24.440 92
147	21 609	3 176 523	12.124 38	38.340 58	5.277 632	11.370 31	24.496 60
148	21 904	3 241 792	12.165 53	38.470 77	5.289 572	11.396 04	24.552 02
149	22 201	3 307 949	12.206 56	38.600 52	5.301 459	11.421 68	24.607 19

Table 18 SQUARES, CUBES, SQUARE ROOTS, AND CUBE ROOTS (continued) 328

n	n^2	n^3	\sqrt{n}	$\sqrt{10n}$	$\sqrt[3]{n}$	$\sqrt[3]{10n}$	$\sqrt[3]{100n}$
150	22 500	3 375 000	12.247 45	38.729 83	5.313 293	11.447 14	24.662 12
151	22 801	3 442 951	12.268 21	38.858 72	5.325 074	11.472 52	24.716 80
152	23 104	3 511 808	12.328 83	38.987 18	5.336 803	11.497 79	24.771 25
153	23 409	3 581 577	12.369 32	39.115 21	5.348 481	11.522 95	24.825 45
154	23 716	3 652 264	12.409 67	39.242 83	5.360 108	11.548 00	24.879 42
155	24 025	3 723 875	12.449 90	39.370 04	5.371 686	11.572 95	24.933 15
156	24 336	3 796 416	12.490 00	39.496 84	5.383 213	11.597 79	24.986 06
157	24 649	3 869 893	12.529 96	39.623 23	5.394 691	11.622 51	25.039 94
158	24 964	3 944 312	12.569 81	39.749 21	5.406 120	11.647 13	25.092 98
159	25 281	4 019 679	12.609 52	39.874 80	5.417 502	11.671 65	25.145 81
160	25 600	4 096 000	12.649 11	40.000 00	5.428 835	11.696 07	25.198 42
161	25 921	4 173 281	12.688 58	40.124 81	5.440 122	11.720 39	25.250 81
162	26 244	4 251 528	12.727 92	40.249 22	5.451 362	11.744 60	25.302 96
163	26 569	4 330 747	12.767 15	40.373 26	5.462 556	11.768 72	25.354 94
164	26 895	4 410 944	12.806 25	40.496 91	5.473 704	11.792 74	25.406 88
165	27 225	4 492 125	12.845 23	40.620 19	5.484 807	11.816 66	25.458 62
166	27 556	4 574 288	12.884 10	40.743 10	5.495 865	11.840 48	25.509 54
167	27 889	4 657 433	12.922 85	40.865 63	5.506 878	11.864 21	25.560 67
168	28 224	4 741 632	12.961 48	40.987 80	5.517 848	11.887 84	25.611 56
169	28 561	4 826 809	13.000 00	41.109 61	5.528 775	11.911 38	25.662 30
170	28 900	4 913 000	13.038 40	41.231 06	5.539 658	11.934 83	25.712 82
171	29 241	5 000 211	13.078 70	41.352 15	5.550 499	11.958 19	25.763 13
172	29 584	5 088 448	13.114 88	41.472 88	5.561 298	11.981 45	25.813 26
173	29 929	5 177 717	13.152 95	41.593 27	5.572 055	12.004 63	25.863 19
174	30 276	5 268 024	13.190 91	41.713 31	5.582 770	12.027 71	25.912 92
175	30 625	5 359 375	13.228 76	41.833 00	5.593 445	12.050 71	25.962 47
176	30 976	5 451 776	13.266 50	41.952 35	5.604 079	12.073 62	26.011 83
177	31 329	5 545 233	13.304 13	42.071 37	5.614 672	12.096 45	26.061 00
178	31 684	5 639 752	13.341 66	42.190 05	5.625 226	12.119 18	26.109 99
179	32 041	5 735 339	13.379 09	42.308 39	5.635 741	12.141 84	26.158 78
180	32 400	5 832 000	13.416 41	42.426 41	5.646 218	12.164 40	26.207 41
181	32 761	5 929 741	13.453 62	42.544 09	5.656 653	12.186 89	26.255 86
182	33 124	6 028 568	13.490 74	42.661 46	5.667 051	12.209 29	26.304 12
183	33 489	6 128 487	13.527 76	42.778 50	5.677 411	12.231 61	26.352 21
184	33 856	6 229 504	13.564 66	42.895 22	5.687 734	12.253 85	26.400 12
185	34 225	6 331 625	13.601 47	43.011 63	5.698 019	12.276 01	26.447 86
186	34 596	6 434 856	13.638 18	43.127 72	5.708 267	12.298 09	26.495 43
187	34 969	6 539 203	13.674 79	43.243 50	5.718 479	12.320 09	26.542 83
188	35 344	6 644 672	13.711 31	43.358 97	5.728 654	12.342 01	26.590 06
189	35 721	6 751 269	13.747 73	43.474 13	5.738 794	12.363 86	26.637 12
190	36 100	6 859 000	13.784 05	43.588 99	5.748 897	12.385 62	26.684 02
191	36 481	6 967 871	13.820 27	43.703 55	5.758 965	12.407 31	26.730 75
192	36 864	7 077 888	13.856 41	43.817 80	5.768 998	12.428 93	26.777 32
193	37 249	7 189 057	13.892 44	43.931 77	5.778 997	12.450 47	26.823 73
194	37 636	7 301 384	13.928 39	44.045 43	5.788 960	12.471 94	26.869 97
195	38 025	7 414 875	13.964 24	44.158 80	5.798 890	12.493 33	26.916 06
196	38 416	7 529 536	14.000 00	44.271 89	5.808 786	12.514 65	26.961 98
197	38 809	7 645 373	14.035 67	44.384 68	5.818 648	12.535 90	27.007 77
198	39 204	7 762 392	14.071 25	44.497 19	5.828 477	12.557 07	27.053 39
199	39 601	7 880 589	14.106 74	44.609 42	5.838 272	12.578 18	27.098 86

Table 18 SQUARES, CUBES, SQUARE ROOTS, AND CUBE ROOTS (continued)

n	n^2	n^3	\sqrt{n}	$\sqrt{10n}$	$\sqrt[3]{n}$	$\sqrt[3]{10n}$	$\sqrt[3]{100n}$
200	40 000	8 000 000	14.142 14	44.721 36	5.848 035	12.599 21	27.144 18
201	40 401	8 120 601	14.177 45	44.833 02	5.857 766	12.620 17	27.189 34
202	40 804	8 242 408	14.212 67	44.944 41	5.867 464	12.641 07	27.234 36
203	41 209	8 365 427	14.247 81	45.055 52	5.877 131	12.661 89	27.279 22
204	41 616	8 489 654	14.282 86	45.166 35	5.886 765	12.682 65	27.323 94
205	42 025	8 615 125	14.317 82	45.276 93	5.896 369	12.703 34	27.368 52
206	42 436	8 741 816	14.352 70	45.387 22	5.905 941	12.723 99	27.412 95
207	42 848	8 869 743	14.387 49	45.497 25	5.915 482	12.744 52	27.457 23
208	43 264	8 998 912	14.422 21	45.607 02	5.924 992	12.765 01	27.501 38
209	43 681	9 129 329	14.456 83	45.716 52	5.934 472	12.785 43	27.545 38
210	44 100	9 261 000	14.491 38	45.825 75	5.943 922	12.805 79	27.589 24
211	44 521	9 393 931	14.525 84	45.934 74	5.953 342	12.826 09	27.632 96
212	44 944	9 528 128	14.560 22	46.043 46	5.962 732	12.846 32	27.676 55
213	45 369	9 663 597	14.594 52	46.151 92	5.972 093	12.866 48	27.720 00
214	45 796	9 800 344	14.628 74	46.260 13	5.981 424	12.886 59	27.763 31
215	46 225	9 938 375	14.662 86	46.368 09	5.990 726	12.906 63	27.806 49
216	46 656	10 077 696	14.696 94	46.475 80	6.000 000	12.926 61	27.849 53
217	47 089	10 218 313	14.730 92	46.583 26	6.009 245	12.946 53	27.892 44
218	47 524	10 360 232	14.764 82	46.690 47	6.018 462	12.966 38	27.935 22
219	47 961	10 503 459	14.798 65	46.797 44	6.027 650	12.986 18	27.977 87
220	48 400	10 648 000	14.832 40	46.904 16	6.036 811	13.005 91	28.020 39
221	48 841	10 793 861	14.866 07	47.010 64	6.045 944	13.025 59	28.062 78
222	49 284	10 941 048	14.899 66	47.116 89	6.055 049	13.045 21	28.105 05
223	49 729	11 089 567	14.933 18	47.222 88	6.064 127	13.064 77	28.147 18
224	50 176	11 239 424	14.966 63	47.328 64	6.073 178	13.084 27	28.189 19
225	50 625	11 390 625	15.000 00	47.434 16	6.082 202	13.103 71	28.231 08
226	51 076	11 543 176	15.033 30	47.539 46	6.091 199	13.123 09	28.272 84
227	51 529	11 697 083	15.066 52	47.644 52	6.100 170	13.142 42	28.314 48
228	51 984	11 852 352	15.099 67	47.749 35	6.109 115	13.161 69	28.356 00
229	52 441	12 008 989	15.132 75	47.853 94	6.118 033	13.180 90	28.397 39
230	52 900	12 167 000	15.165 75	47.958 32	6.126 925	13.200 06	28.438 67
231	53 361	12 326 391	15.198 68	48.062 46	6.135 792	13.219 16	28.479 83
232	53 824	12 487 168	15.231 55	48.166 38	6.144 634	13.238 21	28.520 86
233	54 289	12 649 337	15.264 34	48.270 07	6.153 440	13.257 21	28.561 78
234	54 756	12 812 904	15.297 06	48.373 55	6.162 240	13.276 14	28.602 59
235	55 225	12 977 875	15.329 71	48.476 80	6.171 006	13.295 03	28.643 27
236	55 696	13 144 256	15.362 29	48.579 83	6.179 747	13.313 86	28.683 84
237	56 169	13 312 053	15.394 80	48.682 65	6.188 453	13.332 64	28.724 30
238	56 644	13 481 272	15.427 25	48.785 24	6.197 134	13.351 36	28.764 64
239	57 121	13 651 919	15.459 62	48.887 63	6.205 822	13.370 04	28.804 87
240	57 600	13 824 000	15.491 93	48.989 79	6.214 465	13.388 66	28.844 99
241	58 081	13 997 521	15.524 17	49.091 75	6.223 084	13.407 23	28.885 00
242	58 564	14 172 488	15.556 35	49.193 50	6.231 680	13.425 75	28.924 89
243	59 049	14 348 907	15.588 46	49.295 03	6.240 251	13.444 21	28.964 68
244	59 536	14 526 784	15.620 50	49.396 36	6.248 800	13.462 63	29.004 36
245	60 025	14 706 125	15.652 48	49.497 47	6.257 325	13.481 00	29.043 93
246	60 516	14 886 936	15.684 39	49.598 39	6.265 827	13.499 31	29.083 39
247	61 009	15 069 223	15.716 23	49.699 09	6.274 305	13.517 58	29.122 75
248	61 504	15 252 992	15.748 02	49.799 60	6.282 761	13.535 80	29.161 99
249	62 001	15 438 249	15.779 73	49.899 90	6.291 198	13.553 87	29.201 14

Table 1B SQUARES, CUBES, SQUARE ROOTS, AND CUBE ROOTS (continued) 330

n	n^2	n^3	\sqrt{n}	$\sqrt{10n}$	$\sqrt[3]{n}$	$\sqrt[3]{10n}$	$\sqrt[3]{100n}$
250	62 500	15 625 000	15.811 39	50.000 00	6.299 605	13.572 09	29.240 18
251	63 001	15 813 251	15.842 98	50.099 90	6.307 994	13.590 16	29.279 11
252	63 504	16 003 008	15.874 81	50.199 60	6.316 350	13.608 18	29.317 94
253	64 009	16 194 277	15.905 97	50.299 11	6.324 704	13.625 16	29.356 67
254	64 516	16 387 054	15.937 38	50.398 41	6.333 026	13.644 09	29.395 30
255	65 025	16 581 375	15.968 72	50.497 52	6.341 326	13.661 97	29.433 83
256	65 536	16 777 216	16.000 00	50.596 44	6.349 604	13.679 81	29.472 25
257	66 049	16 974 593	16.031 22	50.695 17	6.357 861	13.697 60	29.510 58
258	66 564	17 173 512	16.062 38	50.793 70	6.366 097	13.715 34	29.548 80
259	67 081	17 373 979	16.093 48	50.892 04	6.374 311	13.733 04	29.586 93
260	67 600	17 576 000	16.124 52	50.990 20	6.382 504	13.750 69	29.624 86
261	68 121	17 779 581	16.155 49	51.088 16	6.390 677	13.768 30	29.662 89
262	68 644	17 984 728	16.186 41	51.185 94	6.398 828	13.785 86	29.700 73
263	69 169	18 191 447	16.217 27	51.283 53	6.406 959	13.803 37	29.738 47
264	69 696	18 399 744	16.248 08	51.380 93	6.415 069	13.820 85	29.776 11
265	70 225	18 609 625	16.278 82	51.478 15	6.423 158	13.838 28	29.813 66
266	70 756	18 821 096	16.309 51	51.575 19	6.431 226	13.855 66	29.851 11
267	71 289	19 034 183	16.340 13	51.672 04	6.439 277	13.873 00	29.888 47
268	71 824	19 248 832	16.370 71	51.768 72	6.447 306	13.890 30	29.925 74
269	72 361	19 465 109	16.401 22	51.865 21	6.455 315	13.907 55	29.962 92
270	72 900	19 683 000	16.431 68	51.961 52	6.463 304	13.924 77	30.000 60
271	73 441	19 902 511	16.462 08	52.057 65	6.471 274	13.941 84	30.038 89
272	73 984	20 123 648	16.492 42	52.153 62	6.479 224	13.959 06	30.077 83
273	74 529	20 346 417	16.522 71	52.249 40	6.487 154	13.976 15	30.116 70
274	75 076	20 570 824	16.552 95	52.345 01	6.495 065	13.993 19	30.154 72
275	75 625	20 796 875	16.583 12	52.440 44	6.502 957	14.010 20	30.194 05
276	76 176	21 024 576	16.613 25	52.535 70	6.510 830	14.027 16	30.232 60
277	76 729	21 253 933	16.643 32	52.630 79	6.518 684	14.044 08	30.271 05
278	77 284	21 484 952	16.673 33	52.725 71	6.526 519	14.060 96	30.293 42
279	77 841	21 717 639	16.703 28	52.820 45	6.534 335	14.077 80	30.329 70
280	78 400	21 952 000	16.733 20	52.915 03	6.542 133	14.094 60	30.365 86
281	78 961	22 188 041	16.763 05	53.009 43	6.549 912	14.111 36	30.402 00
282	79 524	22 425 768	16.792 86	53.103 67	6.557 672	14.128 08	30.438 02
283	80 089	22 665 187	16.822 60	53.197 74	6.565 414	14.144 76	30.473 85
284	80 656	22 906 304	16.852 30	53.291 65	6.573 138	14.161 40	30.509 81
285	81 225	23 149 125	16.881 94	53.385 39	6.580 844	14.178 00	30.545 57
286	81 796	23 393 656	16.911 53	53.478 97	6.588 532	14.194 56	30.581 26
287	82 369	23 639 903	16.941 07	53.572 38	6.596 202	14.211 09	30.616 86
288	82 944	23 887 872	16.970 56	53.665 63	6.603 854	14.227 57	30.652 38
289	83 521	24 137 569	17.000 00	53.758 72	6.611 489	14.244 02	30.687 81
290	84 100	24 388 900	17.029 39	53.851 65	6.619 106	14.260 43	30.723 17
291	84 681	24 642 171	17.058 72	53.944 42	6.626 705	14.276 80	30.758 44
292	85 264	24 897 288	17.088 01	54.037 02	6.634 287	14.293 14	30.793 63
293	85 849	25 153 757	17.117 24	54.129 47	6.641 852	14.309 44	30.828 75
294	86 436	25 412 184	17.146 43	54.221 77	6.649 400	14.325 70	30.863 78
295	87 025	25 672 375	17.175 56	54.313 90	6.656 930	14.341 92	30.898 73
296	87 616	25 934 336	17.204 65	54.405 88	6.664 444	14.358 11	30.933 61
297	88 209	26 198 073	17.233 69	54.497 71	6.671 940	14.374 26	30.968 40
298	88 804	26 463 592	17.262 68	54.589 38	6.679 420	14.390 37	31.003 12
299	89 401	26 730 899	17.291 52	54.680 89	6.686 883	14.406 45	31.037 76

Table 18 SQUARES, CUBES, SQUARE ROOTS, AND CUBE ROOTS (continued) 331

n	n^2	n^3	\sqrt{n}	$\sqrt{10n}$	$\sqrt[3]{n}$	$\sqrt[3]{10n}$	$\sqrt[3]{100n}$
300	90 000	27 000 000	17.320 51	54.772 26	6.594 330	14.422 50	31.072 83
301	90 601	27 270 901	17.349 35	54.863 47	6.701 759	14.438 50	31.108 81
302	91 204	27 543 608	17.378 15	54.954 53	6.709 173	14.454 47	31.144 22
303	91 809	27 818 127	17.406 90	55.046 44	6.716 570	14.470 41	31.175 86
304	92 416	28 094 464	17.435 60	55.138 20	6.723 951	14.486 31	31.209 82
305	93 025	28 372 625	17.464 25	55.226 81	6.731 315	14.502 18	31.244 00
306	93 636	28 652 616	17.492 86	55.317 27	6.738 564	14.518 01	31.278 11
307	94 249	28 934 463	17.521 42	55.407 58	6.745 997	14.533 81	31.312 14
308	94 864	29 218 112	17.549 93	55.497 75	6.753 313	14.549 57	31.346 10
309	95 481	29 503 629	17.578 40	55.587 77	6.760 614	14.565 30	31.379 99
310	96 100	29 791 000	17.606 82	55.677 64	6.767 899	14.581 00	31.413 81
311	96 721	30 080 231	17.635 19	55.767 37	6.775 169	14.596 66	31.447 55
312	97 344	30 371 328	17.663 52	55.856 96	6.782 423	14.612 29	31.481 22
313	97 969	30 664 297	17.691 81	55.946 40	6.789 661	14.627 88	31.514 82
314	98 596	30 959 144	17.720 05	56.035 70	6.796 884	14.643 44	31.548 34
315	99 225	31 256 875	17.748 24	56.124 86	6.804 092	14.658 97	31.581 80
316	99 856	31 557 496	17.776 39	56.213 88	6.811 285	14.674 47	31.615 18
317	100 489	31 860 013	17.804 49	56.302 75	6.818 462	14.689 93	31.648 50
318	101 124	32 164 432	17.832 56	56.391 49	6.825 624	14.705 36	31.681 74
319	101 761	32 470 759	17.860 57	56.480 08	6.832 771	14.720 76	31.714 92
320	102 400	32 779 000	17.888 54	56.568 64	6.839 904	14.736 13	31.748 02
321	103 041	33 089 231	17.916 47	56.656 86	6.847 021	14.751 46	31.781 06
322	103 684	33 396 248	17.944 36	56.744 84	6.854 124	14.766 76	31.814 03
323	104 329	33 698 267	17.972 20	56.833 09	6.861 212	14.782 03	31.846 93
324	104 976	34 002 224	18.000 00	56.921 00	6.868 285	14.797 27	31.879 76
325	105 625	34 308 125	18.027 76	57.008 77	6.875 344	14.812 46	31.912 52
326	106 276	34 615 976	18.055 47	57.096 41	6.882 389	14.827 56	31.945 22
327	106 929	34 925 783	18.083 14	57.183 91	6.889 419	14.842 60	31.977 85
328	107 584	35 237 552	18.110 77	57.271 28	6.896 434	14.857 59	32.010 41
329	108 241	35 551 289	18.138 36	57.358 52	6.903 435	14.873 00	32.042 91
330	108 900	35 867 000	18.165 90	57.445 63	6.910 423	14.888 06	32.075 34
331	109 561	36 184 691	18.193 41	57.532 60	6.917 396	14.903 08	32.107 71
332	110 224	36 504 368	18.220 87	57.619 44	6.924 355	14.918 07	32.140 01
333	110 889	36 826 037	18.248 29	57.706 15	6.931 301	14.933 03	32.172 25
334	111 556	37 150 704	18.275 67	57.792 73	6.938 232	14.947 97	32.204 42
335	112 225	37 478 375	18.303 01	57.879 18	6.945 159	14.962 87	32.236 53
336	112 896	37 808 056	18.330 30	57.965 51	6.952 083	14.977 74	32.268 57
337	113 569	38 139 753	18.357 56	58.051 70	6.958 943	14.992 58	32.300 55
338	114 244	38 473 472	18.384 78	58.137 77	6.965 820	15.007 40	32.332 47
339	114 921	38 809 219	18.411 95	58.223 71	6.972 683	15.022 19	32.364 33
340	115 600	39 147 000	18.439 09	58.309 52	6.979 532	15.036 95	32.396 12
341	116 281	39 486 821	18.466 19	58.395 21	6.986 368	15.051 57	32.427 85
342	116 964	40 001 688	18.493 24	58.480 77	6.993 191	15.066 37	32.459 52
343	117 649	40 363 607	18.520 26	58.566 20	7.000 000	15.081 04	32.491 12
344	118 336	40 727 584	18.547 24	58.651 51	7.006 796	15.095 68	32.522 67
345	119 025	41 093 625	18.574 18	58.736 70	7.013 579	15.110 30	32.554 15
346	119 716	41 461 736	18.601 08	58.821 78	7.020 349	15.124 80	32.585 57
347	120 409	41 831 923	18.627 94	58.906 71	7.027 105	15.139 44	32.616 94
348	121 104	42 204 192	18.654 76	58.991 52	7.033 859	15.153 97	32.648 24
349	121 801	42 578 549	18.681 54	59.076 22	7.040 581	15.168 47	32.679 48

Table 18 SQUARES, CUBES, SQUARE ROOTS, AND CUBE ROOTS (continued) 332

n	n ²	n ³	\sqrt{n}	$\sqrt{10n}$	$\sqrt[3]{n}$	$\sqrt[3]{10n}$	$\sqrt[3]{100n}$
350	122 500	42 675 000	18.708 29	59.150 90	7.047 299	15.182 94	32.710 66
351	123 201	43 243 551	18.734 99	59.245 25	7.054 004	15.197 39	32.741 79
352	123 904	43 814 208	18.761 06	59.329 59	7.060 697	15.211 81	32.772 85
353	124 609	44 386 977	18.788 29	59.413 80	7.067 377	15.226 20	32.803 86
354	125 316	44 961 864	18.814 89	59.497 90	7.074 044	15.240 57	32.834 80
355	126 025	45 538 875	18.841 44	59.581 08	7.080 699	15.254 90	32.865 69
356	126 736	46 118 016	18.867 96	59.665 74	7.087 341	15.269 21	32.896 52
357	127 449	46 699 293	18.894 44	59.749 48	7.093 971	15.283 50	32.927 30
358	128 164	47 282 712	18.920 89	59.833 10	7.100 688	15.297 75	32.958 01
359	128 881	47 868 279	18.947 30	59.916 61	7.107 194	15.311 98	32.988 67
360	129 600	48 456 000	18.973 67	60.000 00	7.113 787	15.326 19	33.019 27
361	130 321	49 046 881	19.000 00	60.083 28	7.120 367	15.340 37	33.049 82
362	131 044	49 640 928	19.026 30	60.166 44	7.126 936	15.354 52	33.080 31
363	131 769	49 238 147	19.052 56	60.249 48	7.133 492	15.368 64	33.110 74
364	132 496	49 838 544	19.078 78	60.332 41	7.140 037	15.382 74	33.141 12
365	133 225	50 441 625	19.104 97	60.415 23	7.146 569	15.396 82	33.171 44
366	133 956	51 047 396	19.131 13	60.497 93	7.153 080	15.410 87	33.201 70
367	134 689	51 655 863	19.157 24	60.580 52	7.159 599	15.424 89	33.231 91
368	135 424	52 267 032	19.183 33	60.663 00	7.166 096	15.438 89	33.262 07
369	136 161	52 880 809	19.209 37	60.745 37	7.172 581	15.452 86	33.292 17
370	136 900	53 497 200	19.235 38	60.827 63	7.179 054	15.466 80	33.322 22
371	137 641	54 116 311	19.261 36	60.909 77	7.185 516	15.480 73	33.352 21
372	138 384	54 738 148	19.287 30	60.991 80	7.191 966	15.494 62	33.382 15
373	139 129	55 362 717	19.313 21	61.073 73	7.198 405	15.508 49	33.412 04
374	139 876	55 989 924	19.339 08	61.155 54	7.204 832	15.522 34	33.441 87
375	140 625	56 619 875	19.364 92	61.237 24	7.211 248	15.536 16	33.471 65
376	141 376	57 252 576	19.390 72	61.318 84	7.217 652	15.549 96	33.501 37
377	142 129	57 887 923	19.416 49	61.400 33	7.224 045	15.563 73	33.531 06
378	142 884	58 525 928	19.442 22	61.481 70	7.230 427	15.577 48	33.560 67
379	143 641	59 166 593	19.467 92	61.562 98	7.236 797	15.591 21	33.590 24
380	144 400	59 809 900	19.493 59	61.644 14	7.243 156	15.604 91	33.619 75
381	145 161	60 455 961	19.519 22	61.725 20	7.249 505	15.618 58	33.649 22
382	145 924	61 104 788	19.544 82	61.806 15	7.255 842	15.632 24	33.678 63
383	146 689	61 756 383	19.570 39	61.886 99	7.262 167	15.645 87	33.708 00
384	147 456	62 410 744	19.595 92	61.967 73	7.268 482	15.659 47	33.737 31
385	148 225	63 067 875	19.621 42	62.048 37	7.274 786	15.673 05	33.766 57
386	148 996	63 727 786	19.646 88	62.128 90	7.281 079	15.686 61	33.795 78
387	149 769	64 390 483	19.672 32	62.209 32	7.287 362	15.700 14	33.824 94
388	150 544	65 055 872	19.697 72	62.289 63	7.293 633	15.713 66	33.854 05
389	151 321	65 723 969	19.723 08	62.369 86	7.299 894	15.727 14	33.883 10
390	152 100	66 394 700	19.748 42	62.449 98	7.306 144	15.740 61	33.912 11
391	152 881	67 068 081	19.773 72	62.529 99	7.312 383	15.754 05	33.941 07
392	153 664	67 744 112	19.799 09	62.609 90	7.318 611	15.767 47	33.969 99
393	154 449	68 422 803	19.824 43	62.689 71	7.324 829	15.780 87	33.998 85
394	155 236	69 104 154	19.849 73	62.769 42	7.331 037	15.794 24	34.027 66
395	156 025	69 788 175	19.875 00	62.849 03	7.337 234	15.807 59	34.056 42
396	156 816	70 474 888	19.899 25	62.928 53	7.343 420	15.820 92	34.085 14
397	157 609	71 164 293	19.923 46	63.007 94	7.349 597	15.834 23	34.113 81
398	158 404	71 856 392	19.947 64	63.087 24	7.355 762	15.847 51	34.142 42
399	159 201	72 551 199	19.971 78	63.166 45	7.361 918	15.860 77	34.171 00

Table 18 SQUARES, CUBES, SQUARE ROOTS, AND CUBE ROOTS (continued)

n	n^2	n^3	\sqrt{n}	$\sqrt{10n}$	$\sqrt[3]{n}$	$\sqrt[3]{10n}$	$\sqrt[3]{100n}$
400	160 000	64 000 000	20.000 00	63.245 55	7.368 063	15.874 01	34.199 52
401	160 801	64 481 201	20.024 98	63.324 56	7.374 198	15.887 23	34.227 99
402	161 604	64 964 808	20.049 94	63.403 47	7.380 323	15.900 42	34.256 42
403	162 409	65 450 827	20.074 86	63.482 28	7.386 437	15.913 60	34.284 80
404	163 216	65 939 264	20.099 75	63.560 99	7.392 542	15.926 75	34.313 14
405	164 025	66 430 125	20.124 61	63.639 51	7.398 636	15.939 88	34.341 43
406	164 836	66 923 416	20.149 44	63.718 13	7.404 721	15.952 99	34.369 67
407	165 649	67 419 143	20.174 24	63.796 55	7.410 795	15.966 07	34.397 86
408	166 464	67 917 312	20.199 01	63.874 88	7.416 860	15.979 14	34.426 01
409	167 281	68 417 929	20.223 75	63.953 11	7.422 914	15.992 18	34.454 12
410	168 100	68 921 000	20.248 46	64.031 24	7.428 959	16.005 21	34.482 17
411	168 921	69 426 531	20.273 13	64.109 28	7.434 994	16.018 21	34.510 18
412	169 744	69 934 528	20.297 78	64.187 23	7.441 019	16.031 19	34.538 15
413	170 569	70 444 997	20.322 40	64.265 08	7.447 034	16.044 15	34.566 07
414	171 396	70 957 944	20.346 99	64.342 83	7.453 040	16.057 09	34.593 95
415	172 225	71 473 375	20.371 55	64.420 49	7.459 035	16.070 01	34.621 78
416	173 056	71 991 296	20.396 08	64.498 06	7.465 022	16.082 90	34.649 58
417	173 889	72 511 713	20.420 58	64.575 54	7.470 999	16.095 78	34.677 31
418	174 724	73 034 632	20.445 05	64.652 92	7.476 966	16.108 64	34.705 00
419	175 561	73 560 059	20.469 49	64.730 21	7.482 924	16.121 47	34.732 65
420	176 400	74 088 000	20.493 90	64.807 41	7.488 872	16.134 28	34.760 27
421	177 241	74 618 461	20.518 28	64.884 51	7.494 811	16.147 09	34.787 83
422	178 084	75 151 448	20.542 64	64.961 53	7.500 741	16.159 86	34.815 35
423	178 929	75 686 967	20.566 96	65.038 45	7.506 661	16.172 61	34.842 83
424	179 776	76 225 024	20.591 26	65.115 28	7.512 572	16.185 34	34.870 27
425	180 625	76 765 625	20.615 53	65.192 02	7.518 473	16.198 06	34.897 66
426	181 476	77 308 776	20.639 77	65.268 68	7.524 365	16.210 75	34.925 01
427	182 329	77 854 483	20.663 98	65.345 24	7.530 248	16.223 43	34.952 32
428	183 184	78 402 752	20.688 16	65.421 71	7.536 122	16.236 08	34.979 58
429	184 041	78 953 589	20.712 32	65.498 09	7.541 987	16.248 72	35.006 80
430	184 900	79 507 000	20.736 44	65.574 39	7.547 842	16.261 33	35.033 98
431	185 761	80 062 991	20.760 54	65.650 59	7.553 688	16.273 93	35.061 12
432	186 624	80 621 568	20.784 61	65.726 71	7.559 526	16.286 51	35.088 21
433	187 489	81 182 737	20.808 65	65.802 74	7.565 355	16.299 06	35.115 27
434	188 356	81 746 504	20.832 67	65.878 68	7.571 174	16.311 60	35.142 28
435	189 225	82 312 875	20.856 65	65.954 53	7.576 985	16.324 12	35.169 25
436	190 096	82 881 856	20.880 61	66.030 30	7.582 787	16.336 62	35.196 18
437	190 969	83 453 453	20.904 54	66.105 98	7.588 579	16.349 10	35.223 07
438	191 844	84 027 672	20.928 45	66.181 57	7.594 363	16.361 56	35.249 91
439	192 721	84 604 519	20.952 33	66.257 08	7.600 139	16.374 00	35.276 72
440	193 600	85 184 000	20.976 18	66.332 50	7.605 905	16.386 43	35.303 48
441	194 481	85 766 121	21.000 00	66.407 83	7.611 663	16.398 83	35.330 21
442	195 364	86 350 888	21.023 89	66.483 08	7.617 412	16.411 22	35.356 89
443	196 249	86 938 307	21.047 75	66.558 25	7.623 152	16.423 58	35.383 54
444	197 136	87 528 384	21.071 57	66.633 32	7.628 884	16.435 93	35.410 14
445	198 025	88 121 125	21.095 32	66.708 32	7.634 607	16.448 26	35.436 71
446	198 916	88 716 536	21.118 77	66.783 23	7.640 321	16.460 57	35.463 23
447	199 809	89 314 623	21.142 37	66.858 06	7.646 027	16.472 87	35.489 71
448	200 704	89 915 392	21.166 01	66.932 80	7.651 725	16.485 14	35.516 16
449	201 601	90 518 849	21.189 62	67.007 46	7.657 414	16.497 40	35.542 57

Table 18 SQUARES, CUBES, SQUARE ROOTS, AND CUBE ROOTS (continued) 334

n	n^2	n^3	\sqrt{n}	$\sqrt{10n}$	$\sqrt[3]{n}$	$\sqrt[3]{10n}$	$\sqrt[3]{100n}$
450	202 500	91 125 000	21.213 20	67.082 04	7.663 084	16.509 64	35.568 93
451	203 401	91 738 851	21.236 76	67.156 53	7.663 786	16.521 86	35.595 26
452	204 304	92 345 408	21.260 29	67.230 95	7.674 430	16.534 06	35.621 56
453	205 209	92 959 677	21.283 80	67.305 27	7.680 086	16.546 24	35.647 80
454	206 116	93 576 694	21.307 28	67.379 52	7.685 733	16.558 41	35.674 01
455	207 026	94 196 375	21.330 73	67.453 69	7.691 372	16.570 56	35.700 18
456	207 936	94 818 816	21.354 16	67.527 77	7.697 002	16.582 69	35.726 32
457	208 849	95 443 993	21.377 56	67.601 78	7.702 625	16.594 80	35.752 42
458	209 764	96 071 912	21.400 93	67.675 70	7.708 239	16.606 90	35.778 48
459	210 681	96 702 579	21.424 29	67.749 54	7.713 845	16.618 97	35.804 50
460	211 600	97 336 000	21.447 61	67.823 30	7.719 443	16.631 03	35.830 48
461	212 521	97 972 181	21.470 91	67.896 98	7.725 032	16.643 08	35.856 42
462	213 444	98 611 128	21.494 19	67.970 58	7.730 614	16.655 10	35.882 33
463	214 369	99 252 847	21.517 43	68.044 10	7.736 188	16.667 11	35.908 20
464	215 296	99 897 344	21.540 66	68.117 55	7.741 753	16.679 10	35.934 04
465	216 225	100 544 625	21.563 86	68.190 91	7.747 311	16.691 06	35.960 83
466	217 156	101 194 696	21.587 03	68.264 19	7.752 861	16.703 03	35.986 69
467	218 089	101 847 563	21.610 18	68.337 40	7.758 402	16.714 97	36.011 31
468	219 024	102 503 232	21.633 31	68.410 53	7.763 936	16.726 89	36.037 00
469	219 961	103 161 709	21.656 41	68.483 57	7.769 462	16.738 80	36.062 65
470	220 900	103 823 000	21.679 48	68.556 55	7.774 980	16.750 69	36.088 26
471	221 841	104 487 111	21.702 53	68.629 44	7.780 490	16.762 56	36.113 84
472	222 784	105 154 048	21.725 56	68.702 26	7.785 993	16.774 41	36.139 38
473	223 729	105 823 817	21.748 56	68.775 00	7.791 488	16.786 25	36.164 88
474	224 676	106 496 424	21.771 54	68.847 66	7.796 976	16.798 07	36.190 35
475	225 625	107 171 875	21.794 49	68.920 24	7.802 454	16.809 88	36.215 78
476	226 576	107 850 176	21.817 42	68.992 75	7.807 925	16.821 67	36.241 18
477	227 529	108 531 323	21.840 33	69.065 19	7.813 389	16.833 44	36.266 54
478	228 484	109 215 322	21.863 21	69.137 54	7.818 846	16.845 19	36.291 87
479	229 441	109 902 269	21.886 07	69.209 83	7.824 294	16.856 93	36.317 16
480	230 400	110 592 000	21.908 90	69.282 03	7.829 735	16.868 65	36.342 41
481	231 361	111 284 641	21.931 71	69.354 16	7.835 169	16.880 35	36.367 63
482	232 324	111 980 168	21.954 50	69.426 22	7.840 599	16.892 05	36.392 82
483	233 289	112 678 587	21.977 26	69.498 20	7.846 013	16.903 72	36.417 87
484	234 256	113 379 904	22.000 00	69.570 11	7.851 424	16.915 38	36.443 06
485	235 225	114 084 125	22.022 72	69.641 94	7.856 828	16.927 02	36.468 17
486	236 196	114 791 256	22.045 41	69.713 70	7.862 224	16.938 65	36.493 21
487	237 169	115 501 303	22.068 08	69.785 39	7.867 613	16.950 26	36.518 22
488	238 144	116 214 272	22.090 72	69.857 00	7.872 994	16.961 85	36.543 20
489	239 121	116 930 169	22.113 34	69.928 53	7.878 368	16.973 43	36.568 15
490	240 100	117 649 000	22.135 94	70.000 00	7.883 735	16.984 99	36.593 06
491	241 081	118 370 771	22.158 52	70.071 39	7.889 095	16.996 54	36.617 93
492	242 064	119 095 488	22.181 07	70.142 71	7.894 447	17.008 07	36.642 78
493	243 049	119 823 157	22.203 50	70.213 96	7.899 792	17.019 59	36.667 56
494	244 036	120 553 784	22.225 91	70.285 13	7.905 129	17.031 08	36.692 36
495	245 025	121 287 375	22.248 30	70.356 24	7.910 460	17.042 57	36.717 10
496	246 016	122 023 936	22.270 66	70.427 27	7.915 783	17.054 04	36.741 81
497	247 009	122 763 473	22.292 99	70.498 23	7.921 099	17.065 49	36.766 49
498	248 004	123 505 992	22.315 31	70.569 12	7.926 408	17.076 93	36.791 13
499	249 001	124 251 499	22.337 61	70.639 93	7.931 710	17.088 35	36.815 74

Table 18 SQUARES, CUBES, SQUARE ROOTS, AND CUBE ROOTS (continued)

n	n^2	n^3	\sqrt{n}	$\sqrt{10n}$	$\sqrt[3]{n}$	$\sqrt[3]{10n}$	$\sqrt[3]{100n}$
500	250 000	125 000 000	22.360 68	70.710 68	7.937 005	17.099 76	36.840 31
501	251 001	126 751 501	22.388 03	70.781 86	7.942 293	17.111 15	36.854 86
502	252 004	128 506 008	22.405 36	70.851 96	7.947 574	17.122 53	36.869 37
503	253 009	127 263 527	22.427 66	70.922 49	7.952 848	17.133 89	36.883 85
504	254 016	128 024 064	22.449 94	70.992 96	7.958 114	17.145 24	36.898 30
505	255 025	128 787 525	22.472 21	71.063 35	7.963 374	17.156 57	36.912 71
506	256 036	129 554 216	22.494 44	71.133 68	7.968 627	17.167 89	36.927 09
507	257 049	130 323 843	22.516 65	71.203 93	7.973 873	17.179 19	37.011 44
508	258 064	131 096 512	22.538 85	71.274 12	7.979 112	17.190 48	37.035 76
509	259 081	131 872 229	22.561 03	71.344 24	7.984 344	17.201 75	37.050 04
510	260 100	132 651 000	22.583 18	71.414 28	7.989 570	17.213 01	37.064 30
511	261 121	133 432 831	22.605 31	71.484 26	7.994 788	17.224 25	37.105 52
512	262 144	134 217 728	22.627 42	71.554 18	8.000 000	17.235 48	37.132 71
513	263 169	135 005 867	22.649 50	71.624 02	8.005 255	17.246 89	37.155 87
514	264 196	135 796 744	22.671 57	71.693 79	8.010 403	17.257 89	37.181 00
515	265 225	136 590 875	22.693 61	71.763 50	8.015 595	17.269 08	37.205 09
516	266 256	137 388 096	22.715 63	71.833 14	8.020 779	17.280 25	37.229 16
517	267 289	138 188 413	22.737 63	71.902 71	8.025 957	17.291 40	37.253 19
518	268 324	138 991 832	22.759 61	71.972 22	8.031 129	17.302 54	37.277 20
519	269 361	139 798 359	22.781 57	72.041 65	8.036 293	17.313 67	37.301 17
520	270 400	140 608 000	22.803 51	72.111 03	8.041 452	17.324 78	37.325 11
521	271 441	141 420 761	22.825 42	72.180 33	8.046 603	17.335 88	37.349 02
522	272 484	142 236 648	22.847 32	72.249 57	8.051 748	17.346 96	37.372 90
523	273 529	143 055 867	22.869 19	72.318 74	8.056 886	17.358 04	37.396 76
524	274 576	143 877 824	22.891 05	72.387 84	8.062 018	17.369 09	37.420 57
525	275 625	144 703 125	22.912 88	72.456 88	8.067 143	17.380 13	37.444 36
526	276 676	145 531 976	22.934 69	72.525 86	8.072 262	17.391 16	37.468 12
527	277 729	146 363 383	22.956 48	72.594 77	8.077 374	17.402 18	37.491 85
528	278 784	147 197 952	22.978 25	72.663 61	8.082 480	17.413 18	37.515 55
529	279 841	148 035 869	23.000 00	72.732 39	8.087 579	17.424 16	37.539 22
530	280 900	148 877 000	23.021 73	72.801 10	8.092 672	17.435 13	37.562 88
531	281 961	149 721 291	23.043 44	72.869 75	8.097 759	17.446 09	37.586 47
532	283 024	150 568 768	23.065 13	72.938 33	8.102 839	17.457 04	37.610 05
533	284 089	151 419 437	23.086 79	73.006 85	8.107 913	17.467 97	37.633 60
534	285 156	152 273 304	23.108 44	73.075 30	8.112 980	17.478 89	37.657 12
535	286 225	153 130 375	23.130 07	73.143 69	8.118 041	17.489 79	37.680 61
536	287 296	153 990 656	23.151 67	73.212 02	8.123 096	17.500 68	37.704 07
537	288 369	154 854 153	23.173 26	73.280 28	8.128 145	17.511 56	37.727 51
538	289 444	155 720 872	23.194 83	73.348 48	8.133 187	17.522 42	37.750 91
539	290 521	156 590 819	23.216 37	73.416 62	8.138 223	17.533 27	37.774 29
540	291 600	157 464 000	23.237 90	73.484 69	8.143 253	17.544 11	37.797 63
541	292 681	158 340 421	23.259 41	73.552 70	8.148 276	17.554 93	37.820 95
542	293 764	159 220 088	23.280 99	73.620 65	8.153 294	17.565 74	37.844 24
543	294 849	160 103 007	23.302 36	73.688 53	8.158 305	17.576 54	37.867 50
544	295 936	160 989 184	23.323 61	73.756 36	8.163 310	17.587 32	37.890 73
545	297 025	161 878 625	23.345 24	73.824 12	8.168 309	17.598 09	37.913 93
546	298 116	162 771 336	23.366 84	73.891 81	8.173 302	17.608 85	37.937 11
547	299 209	163 667 323	23.388 03	73.959 45	8.178 289	17.619 59	37.960 25
548	300 304	164 566 592	23.409 40	74.027 02	8.183 269	17.630 32	37.983 37
549	301 401	165 469 149	23.430 75	74.094 53	8.188 244	17.641 04	38.006 46

Table 18 SQUARES, CUBES, SQUARE ROOTS, AND CUBE ROOTS (continued) 336

n	n^2	n^3	\sqrt{n}	$\sqrt{10n}$	$\sqrt[3]{n}$	$\sqrt[3]{10n}$	$\sqrt[3]{100n}$
550	302 600	166 376 000	23.452 08	74.161 98	8.193 213	17.651 74	38.029 52
551	303 601	167 284 151	23.473 39	74.229 37	8.198 175	17.662 43	38.052 56
552	304 704	168 196 508	23.494 68	74.296 70	8.203 132	17.673 11	38.075 57
553	305 809	169 112 377	23.515 95	74.363 97	8.208 082	17.683 78	38.098 54
554	306 916	170 031 464	23.537 20	74.431 18	8.213 027	17.694 43	38.121 49
555	308 025	170 953 875	23.558 44	74.498 32	8.217 966	17.705 07	38.144 42
556	309 136	171 879 516	23.579 65	74.565 41	8.222 899	17.715 70	38.167 31
557	310 249	172 808 593	23.600 85	74.632 43	8.227 825	17.726 31	38.190 18
558	311 364	173 741 112	23.622 02	74.699 40	8.232 746	17.736 91	38.213 02
559	312 481	174 676 879	23.643 18	74.766 30	8.237 661	17.747 50	38.235 84
560	313 600	175 616 000	23.664 32	74.833 16	8.242 571	17.758 08	38.258 62
561	314 721	176 558 481	23.685 44	74.899 93	8.247 474	17.768 64	38.281 38
562	315 844	177 504 328	23.706 54	74.966 66	8.252 372	17.779 20	38.304 12
563	316 969	178 453 547	23.727 62	75.033 33	8.257 263	17.789 73	38.326 82
564	318 096	179 406 144	23.748 68	75.099 93	8.262 149	17.800 26	38.349 50
565	319 225	180 362 125	23.769 73	75.166 48	8.267 029	17.810 77	38.372 16
566	320 356	181 321 496	23.790 75	75.232 97	8.271 904	17.821 28	38.394 77
567	321 489	182 284 263	23.811 76	75.299 40	8.276 773	17.831 77	38.417 37
568	322 624	183 250 432	23.832 75	75.365 77	8.281 635	17.842 24	38.439 95
569	323 761	184 220 009	23.853 72	75.432 09	8.286 493	17.852 71	38.462 48
570	324 900	185 193 000	23.874 67	75.498 34	8.291 344	17.863 18	38.485 01
571	326 041	186 169 411	23.895 61	75.564 54	8.296 190	17.873 60	38.507 50
572	327 184	187 149 248	23.916 52	75.630 68	8.301 031	17.884 03	38.529 97
573	328 329	188 132 517	23.937 42	75.696 76	8.305 865	17.894 44	38.552 41
574	329 476	189 119 224	23.958 30	75.762 79	8.310 694	17.904 85	38.574 82
575	330 625	190 109 375	23.979 16	75.828 75	8.315 517	17.915 24	38.597 21
576	331 776	191 102 976	24.000 00	75.894 66	8.320 335	17.925 62	38.619 58
577	332 929	192 100 033	24.020 82	75.960 52	8.325 148	17.935 99	38.641 91
578	334 084	193 100 562	24.041 63	76.026 31	8.329 954	17.946 34	38.664 22
579	335 241	194 104 539	24.062 42	76.092 05	8.334 755	17.956 69	38.686 51
580	336 400	195 112 000	24.083 19	76.157 73	8.339 551	17.967 02	38.708 77
581	337 561	196 122 941	24.103 94	76.223 36	8.344 341	17.977 34	38.731 00
582	338 724	197 137 368	24.124 68	76.288 92	8.349 126	17.987 65	38.753 21
583	339 889	198 155 287	24.145 39	76.354 44	8.353 905	17.997 94	38.775 39
584	341 056	199 176 704	24.166 09	76.419 89	8.358 678	18.008 23	38.797 55
585	342 225	200 201 625	24.186 77	76.485 29	8.363 447	18.018 50	38.819 58
586	343 396	201 230 056	24.207 44	76.550 64	8.368 209	18.028 76	38.841 79
587	344 569	202 262 003	24.228 09	76.615 93	8.372 967	18.039 01	38.863 87
588	345 744	203 297 472	24.248 71	76.681 16	8.377 719	18.049 25	38.885 93
589	346 921	204 336 469	24.269 32	76.746 34	8.382 465	18.059 47	38.907 96
590	348 100	205 379 000	24.289 92	76.811 46	8.387 207	18.069 69	38.929 96
591	349 281	206 425 071	24.310 49	76.876 52	8.391 942	18.079 89	38.951 95
592	350 464	207 474 688	24.331 05	76.941 54	8.396 673	18.090 08	38.973 90
593	351 649	208 527 857	24.351 58	77.006 49	8.401 398	18.100 26	38.995 84
594	352 836	209 584 584	24.372 12	77.071 40	8.406 118	18.110 43	39.017 74
595	354 025	210 644 875	24.392 62	77.136 24	8.410 833	18.120 59	39.039 63
596	355 216	211 708 736	24.413 11	77.201 04	8.415 542	18.130 74	39.061 49
597	356 409	212 776 173	24.433 58	77.265 76	8.420 245	18.140 87	39.083 32
598	357 604	213 847 192	24.454 04	77.330 46	8.424 945	18.150 99	39.105 13
599	358 801	214 921 799	24.474 48	77.395 09	8.429 638	18.161 11	39.126 92

Table 18 SQUARES, CUBES, SQUARE ROOTS, AND CUBE ROOTS (continued)

n	n^2	n^3	\sqrt{n}	$\sqrt{10n}$	$\sqrt[3]{n}$	$\sqrt[3]{10n}$	$\sqrt[3]{100n}$
600	360 000	216 000 000	24.494 90	77.459 67	8.434 327	18.171 21	89.148 68
601	361 201	217 081 801	24.515 30	77.524 19	8.439 010	18.181 30	89.170 41
602	362 404	218 167 208	24.535 69	77.588 66	8.443 688	18.191 37	89.192 13
603	363 609	219 256 227	24.556 06	77.653 07	8.448 361	18.201 44	89.213 82
604	364 816	220 348 864	24.576 41	77.717 44	8.453 028	18.211 50	89.235 48
605	366 025	221 445 125	24.596 75	77.781 75	8.457 691	18.221 54	89.257 12
606	367 236	222 545 016	24.617 07	77.846 00	8.462 346	18.231 58	89.278 74
607	368 449	223 648 543	24.637 37	77.910 20	8.467 000	18.241 60	89.300 33
608	369 664	224 755 712	24.657 66	77.974 35	8.471 647	18.251 61	89.321 90
609	370 881	225 866 529	24.677 93	78.038 45	8.476 289	18.261 61	89.343 45
610	372 100	226 981 000	24.698 18	78.102 50	8.480 926	18.271 60	89.364 97
611	373 321	228 099 131	24.718 41	78.166 49	8.485 558	18.281 58	89.386 47
612	374 544	229 220 928	24.738 63	78.230 43	8.490 185	18.291 55	89.407 95
613	375 769	230 346 397	24.758 84	78.294 32	8.494 807	18.301 51	89.429 40
614	376 996	231 475 544	24.779 02	78.358 15	8.499 423	18.311 45	89.450 83
615	378 225	232 608 375	24.799 19	78.421 94	8.504 035	18.321 39	89.472 23
616	379 456	233 744 896	24.819 35	78.485 67	8.508 642	18.331 31	89.493 62
617	380 689	234 885 113	24.839 48	78.549 35	8.513 243	18.341 23	89.514 98
618	381 924	236 029 032	24.859 51	78.612 98	8.517 840	18.351 13	89.536 31
619	383 161	237 176 660	24.879 51	78.676 55	8.522 432	18.361 02	89.557 63
620	384 400	238 328 000	24.899 50	78.740 08	8.527 019	18.370 91	89.578 92
621	385 641	239 483 061	24.919 37	78.803 55	8.531 601	18.380 78	89.600 19
622	386 884	240 641 848	24.939 33	78.866 98	8.536 178	18.390 64	89.621 43
623	388 129	241 804 367	24.959 37	78.930 35	8.540 750	18.400 49	89.642 65
624	389 376	242 970 624	24.979 39	78.993 57	8.545 317	18.410 33	89.663 85
625	390 625	244 140 625	25.000 00	79.056 64	8.549 880	18.420 16	89.685 03
626	391 876	245 314 376	25.019 99	79.120 16	8.554 437	18.429 98	89.706 19
627	393 129	246 491 883	25.039 97	79.183 33	8.558 989	18.439 78	89.727 31
628	394 384	247 673 152	25.059 93	79.246 45	8.563 538	18.449 58	89.748 42
629	395 641	248 858 189	25.079 87	79.309 52	8.568 081	18.459 37	89.769 51
630	396 900	250 047 000	25.099 80	79.372 54	8.572 619	18.469 15	89.790 57
631	398 161	251 239 591	25.119 71	79.435 51	8.577 152	18.478 91	89.811 61
632	399 424	252 435 968	25.139 61	79.498 43	8.581 681	18.488 67	89.832 63
633	400 689	253 636 137	25.159 49	79.561 30	8.586 205	18.498 42	89.853 63
634	401 956	254 840 104	25.179 36	79.624 12	8.590 724	18.508 15	89.874 61
635	403 225	256 047 875	25.199 21	79.688 89	8.595 239	18.517 88	89.895 56
636	404 496	257 259 456	25.219 04	79.753 61	8.599 748	18.527 59	89.916 49
637	405 769	258 474 853	25.238 86	79.818 28	8.604 252	18.537 30	89.937 40
638	407 044	259 694 072	25.258 66	79.882 90	8.608 753	18.547 00	89.958 29
639	408 321	260 917 119	25.278 45	79.937 48	8.613 248	18.556 68	89.979 16
640	409 600	262 144 000	25.298 22	80.000 00	8.617 739	18.566 36	40.000 00
641	410 881	263 374 721	25.317 98	80.062 48	8.622 225	18.576 02	40.020 82
642	412 164	264 609 288	25.337 72	80.124 90	8.626 706	18.585 68	40.041 62
643	413 449	265 847 707	25.357 44	80.187 28	8.631 183	18.595 32	40.062 40
644	414 736	267 089 984	25.377 16	80.249 51	8.635 655	18.604 95	40.083 16
645	416 025	268 336 125	25.396 85	80.311 89	8.640 123	18.614 58	40.103 90
646	417 316	269 586 136	25.416 53	80.374 13	8.644 585	18.624 19	40.124 61
647	418 609	270 840 023	25.436 19	80.436 31	8.649 044	18.633 80	40.145 30
648	419 904	272 097 792	25.455 84	80.498 45	8.653 497	18.643 40	40.165 98
649	421 201	273 359 449	25.475 48	80.560 54	8.657 947	18.652 98	40.186 63

Table 18 SQUARES, CUBES, SQUARE ROOTS, AND CUBE ROOTS (continued) 333

n	n^2	n^3	\sqrt{n}	$\sqrt{10n}$	$\sqrt[3]{n}$	$\sqrt[3]{10n}$	$\sqrt[3]{100n}$
650	422 500	274 625 000	25.495 10	80.622 58	8.662 391	18.662 56	40.267 26
651	423 801	275 894 451	25.514 70	80.684 57	8.666 831	18.672 12	40.227 87
652	425 104	277 187 808	25.534 28	80.746 52	8.671 268	18.681 68	40.248 46
653	426 409	278 446 077	25.553 86	80.808 42	8.675 697	18.691 22	40.269 02
654	427 716	279 726 264	25.573 42	80.870 27	8.680 124	18.700 76	40.289 57
655	429 025	281 011 375	25.592 97	80.932 07	8.684 546	18.710 29	40.310 09
656	430 336	282 300 416	25.612 50	80.993 83	8.688 963	18.719 80	40.330 59
657	431 649	283 593 393	25.632 01	81.055 54	8.693 375	18.729 31	40.351 08
658	432 964	284 890 312	25.651 51	81.117 20	8.697 784	18.738 81	40.371 54
659	434 281	286 191 179	25.671 00	81.178 81	8.702 188	18.748 30	40.391 98
660	435 600	287 496 000	25.690 47	81.240 38	8.706 588	18.757 77	40.412 40
661	436 921	288 804 781	25.709 92	81.301 91	8.710 983	18.767 24	40.432 80
662	438 244	290 117 528	25.729 36	81.363 38	8.715 373	18.776 70	40.453 18
663	439 569	291 434 247	25.748 79	81.424 81	8.719 760	18.786 15	40.473 54
664	440 896	292 754 944	25.768 20	81.486 20	8.724 141	18.795 69	40.493 88
665	442 225	294 079 625	25.787 59	81.547 55	8.728 519	18.805 02	40.514 20
666	443 556	295 408 296	25.806 98	81.608 92	8.732 892	18.814 44	40.534 49
667	444 889	296 740 963	25.826 34	81.670 07	8.737 260	18.823 86	40.554 77
668	446 224	298 077 632	25.845 70	81.731 27	8.741 625	18.833 26	40.575 03
669	447 561	299 418 303	25.865 03	81.792 42	8.745 985	18.842 65	40.595 26
670	448 900	300 763 000	25.884 36	81.853 53	8.750 340	18.852 04	40.615 48
671	450 241	302 111 711	25.903 67	81.914 69	8.754 691	18.861 41	40.635 68
672	451 584	303 464 448	25.922 96	81.975 81	8.759 038	18.870 78	40.655 85
673	452 929	304 821 217	25.942 24	82.036 98	8.763 381	18.880 13	40.676 01
674	454 276	306 182 024	25.961 51	82.097 50	8.767 719	18.889 48	40.696 15
675	455 625	307 546 875	25.980 76	82.158 38	8.772 053	18.898 82	40.716 26
676	456 976	308 915 776	25.999 99	82.219 22	8.776 383	18.908 14	40.736 36
677	458 329	310 288 733	26.019 22	82.280 01	8.780 708	18.917 46	40.756 44
678	459 684	311 665 752	26.038 43	82.340 76	8.785 030	18.926 77	40.776 50
679	461 041	313 046 839	26.057 63	82.401 46	8.789 347	18.936 07	40.796 53
680	462 400	314 432 000	26.076 81	82.462 11	8.793 659	18.945 36	40.816 55
681	463 761	315 821 241	26.095 98	82.522 72	8.797 968	18.954 65	40.836 55
682	465 124	317 214 563	26.115 13	82.583 29	8.802 272	18.963 92	40.856 53
683	466 489	318 611 967	26.134 27	82.643 81	8.806 572	18.973 18	40.876 49
684	467 856	320 013 504	26.153 39	82.704 29	8.810 866	18.982 44	40.896 43
685	469 225	321 419 125	26.172 50	82.764 73	8.815 160	18.991 69	40.916 35
686	470 596	322 828 888	26.191 60	82.825 12	8.819 447	19.000 92	40.936 25
687	471 969	324 242 703	26.210 68	82.885 46	8.823 731	19.010 15	40.956 13
688	473 344	325 660 672	26.229 75	82.945 77	8.828 010	19.019 37	40.975 99
689	474 721	327 082 769	26.248 81	83.006 02	8.832 285	19.028 58	40.995 84
690	476 100	328 509 000	26.267 85	83.066 24	8.836 556	19.037 78	41.015 66
691	477 481	329 939 371	26.286 88	83.126 41	8.840 822	19.046 98	41.035 46
692	478 864	331 373 888	26.305 89	83.186 54	8.845 085	19.056 16	41.055 25
693	480 249	332 812 567	26.324 89	83.246 62	8.849 344	19.065 33	41.075 02
694	481 636	334 255 384	26.343 88	83.306 66	8.853 599	19.074 50	41.094 78
695	483 025	335 702 375	26.362 85	83.366 66	8.857 849	19.083 65	41.114 49
696	484 416	337 153 536	26.381 81	83.426 61	8.862 095	19.092 81	41.134 20
697	485 809	338 608 873	26.400 76	83.486 53	8.866 338	19.101 95	41.153 89
698	487 204	340 068 392	26.419 69	83.546 39	8.870 576	19.111 08	41.173 57
699	488 601	341 532 099	26.438 61	83.606 22	8.874 810	19.120 20	41.193 22

Table 1B SQUARES, CUBES, SQUARE ROOTS, AND CUBE ROOTS (continued)

n	n^2	n^3	\sqrt{n}	$\sqrt{10n}$	$\sqrt[3]{n}$	$\sqrt[3]{10n}$	$\sqrt[3]{100n}$
700	490 000	343 000 000	26.457 51	83.665 00	8.879 040	19.126 31	41.212 85
701	491 401	344 472 101	26.476 40	83.725 74	8.883 286	19.138 42	41.232 47
702	492 804	345 948 408	26.495 28	83.785 44	8.887 488	19.147 51	41.252 07
703	494 209	347 428 927	26.514 15	83.845 10	8.891 706	19.156 60	41.271 54
704	495 616	348 913 664	26.533 00	83.904 71	8.895 920	19.165 68	41.291 20
705	497 025	350 402 625	26.551 84	83.964 28	8.900 130	19.174 75	41.310 75
706	498 436	351 895 816	26.570 66	84.023 81	8.904 337	19.183 81	41.330 47
707	499 849	353 393 243	26.589 47	84.083 29	8.908 539	19.192 86	41.349 77
708	501 264	354 894 912	26.608 27	84.142 74	8.912 737	19.201 91	41.369 26
709	502 681	356 400 829	26.627 05	84.202 14	8.916 931	19.210 95	41.388 73
710	504 100	357 911 000	26.645 83	84.261 50	8.921 121	19.219 97	41.408 10
711	505 521	359 425 431	26.664 58	84.320 82	8.925 308	19.228 99	41.427 61
712	506 944	360 944 128	26.683 33	84.380 09	8.929 490	19.238 00	41.447 02
713	508 369	362 467 087	26.702 06	84.439 33	8.933 669	19.247 01	41.466 42
714	509 796	363 994 344	26.720 78	84.498 52	8.937 843	19.256 00	41.485 79
715	511 225	365 525 875	26.739 48	84.557 67	8.942 014	19.264 99	41.505 15
716	512 656	367 061 696	26.758 18	84.616 78	8.946 181	19.273 96	41.524 49
717	514 089	368 601 813	26.776 86	84.675 85	8.950 344	19.282 93	41.543 82
718	515 524	370 146 232	26.795 52	84.734 88	8.954 503	19.291 89	41.563 12
719	516 961	371 694 959	26.814 18	84.793 87	8.958 658	19.300 84	41.582 41
720	518 400	373 248 000	26.832 82	84.852 81	8.962 809	19.309 79	41.601 68
721	519 841	374 806 361	26.851 44	84.911 72	8.966 957	19.318 72	41.620 93
722	521 284	376 369 048	26.870 06	84.970 58	8.971 101	19.327 65	41.640 16
723	522 729	377 937 067	26.888 66	85.029 41	8.975 241	19.336 57	41.659 38
724	524 176	379 510 424	26.907 25	85.088 19	8.979 377	19.345 48	41.678 57
725	525 625	381 089 125	26.925 82	85.146 93	8.983 509	19.354 38	41.697 75
726	527 076	382 673 178	26.944 39	85.205 63	8.987 637	19.363 28	41.716 92
727	528 529	384 262 583	26.962 94	85.264 29	8.991 762	19.372 16	41.736 06
728	529 984	385 857 352	26.981 48	85.322 92	8.995 883	19.381 04	41.755 19
729	531 441	387 457 489	27.000 00	85.381 50	9.000 000	19.389 91	41.774 30
730	532 900	389 062 900	27.018 51	85.440 04	9.004 113	19.398 77	41.793 39
731	534 361	390 673 691	27.037 01	85.498 54	9.008 223	19.407 63	41.812 47
732	535 824	392 289 868	27.055 50	85.557 00	9.012 329	19.416 47	41.831 52
733	537 289	393 912 437	27.073 97	85.615 42	9.016 431	19.425 31	41.850 56
734	538 756	395 540 504	27.092 43	85.673 80	9.020 529	19.434 14	41.869 59
735	540 225	397 174 175	27.110 88	85.732 14	9.024 624	19.442 96	41.888 59
736	541 696	398 813 456	27.129 32	85.790 44	9.028 716	19.451 78	41.907 58
737	543 169	400 457 453	27.147 74	85.848 70	9.032 802	19.460 58	41.926 55
738	544 644	402 107 272	27.166 16	85.906 93	9.036 886	19.469 38	41.945 51
739	546 121	403 761 919	27.184 55	85.965 11	9.040 966	19.478 17	41.964 44
740	547 600	405 421 400	27.202 94	86.023 25	9.045 042	19.486 95	41.983 36
741	549 081	407 085 821	27.221 32	86.081 36	9.049 114	19.495 78	42.002 27
742	550 564	408 755 188	27.239 68	86.139 42	9.053 183	19.504 49	42.021 15
743	552 049	410 429 507	27.258 03	86.197 45	9.057 248	19.513 25	42.040 02
744	553 536	412 108 784	27.276 38	86.255 43	9.061 310	19.522 00	42.058 87
745	555 025	413 793 025	27.294 69	86.313 38	9.065 368	19.530 74	42.077 71
746	556 516	415 483 236	27.313 00	86.371 29	9.069 422	19.539 48	42.096 53
747	558 009	417 178 513	27.331 30	86.429 16	9.073 473	19.548 20	42.115 33
748	559 504	418 878 862	27.349 59	86.486 99	9.077 520	19.556 92	42.134 11
749	561 001	420 584 289	27.367 88	86.544 79	9.081 563	19.565 63	42.152 88

Table 18 SQUARES, CUBES, SQUARE ROOTS, AND CUBE ROOTS (continued) 340

n	n^2	n^3	\sqrt{n}	$\sqrt{10n}$	$\sqrt[3]{n}$	$\sqrt[3]{10n}$	$\sqrt[3]{100n}$
750	562 500	421 875 000	27.386 13	86.602 54	9.085 603	19.574 34	42.171 63
751	564 001	423 564 751	27.404 38	86.660 26	9.089 639	19.582 03	42.190 87
752	565 504	425 269 008	27.422 62	86.717 93	9.093 672	19.591 72	42.209 09
753	567 009	426 987 777	27.440 85	86.775 57	9.097 701	19.600 40	42.227 79
754	568 516	428 761 064	27.459 06	86.833 17	9.101 727	19.609 08	42.246 47
755	570 025	430 588 375	27.477 26	86.890 74	9.105 748	19.617 74	42.265 14
756	571 536	432 481 216	27.495 45	86.948 25	9.109 767	19.626 40	42.283 79
757	573 049	434 438 093	27.513 63	87.005 75	9.113 782	19.635 05	42.302 43
758	574 564	436 459 512	27.531 80	87.063 20	9.117 793	19.643 69	42.321 05
759	576 081	438 546 479	27.549 95	87.120 61	9.121 801	19.652 32	42.339 65
760	577 600	438 976 000	27.568 10	87.177 98	9.125 805	19.660 95	42.358 24
761	579 121	440 711 081	27.586 23	87.235 31	9.129 806	19.669 57	42.376 81
762	580 644	442 460 728	27.604 35	87.292 61	9.133 803	19.678 18	42.395 36
763	582 169	444 324 947	27.622 45	87.349 87	9.137 797	19.686 79	42.413 90
764	583 696	445 943 744	27.640 55	87.407 09	9.141 787	19.695 38	42.432 42
765	585 225	447 627 125	27.658 63	87.464 27	9.145 774	19.703 97	42.450 92
766	586 756	449 465 096	27.676 71	87.521 43	9.149 758	19.712 55	42.469 41
767	588 289	451 457 653	27.694 76	87.578 54	9.153 738	19.721 13	42.487 89
768	589 824	453 514 832	27.712 81	87.635 61	9.157 714	19.729 70	42.506 34
769	591 361	455 636 609	27.730 85	87.692 65	9.161 687	19.738 26	42.524 78
770	592 900	455 533 000	27.748 87	87.749 64	9.165 656	19.746 81	42.543 21
771	594 441	458 314 011	27.766 89	87.806 61	9.169 623	19.755 35	42.561 62
772	595 984	460 099 048	27.784 89	87.863 53	9.173 585	19.763 89	42.580 01
773	597 529	461 889 917	27.802 88	87.920 42	9.177 544	19.772 42	42.598 39
774	599 076	463 684 824	27.820 86	87.977 27	9.181 500	19.780 94	42.616 75
775	600 625	465 484 375	27.838 82	88.034 08	9.185 453	19.789 46	42.635 09
776	602 176	467 288 576	27.856 78	88.090 85	9.189 402	19.797 97	42.653 42
777	603 729	469 097 433	27.874 72	88.147 60	9.193 347	19.806 47	42.671 74
778	605 284	470 910 952	27.892 65	88.204 31	9.197 290	19.814 96	42.690 04
779	606 841	472 729 139	27.910 57	88.260 98	9.201 229	19.823 45	42.708 32
780	608 400	474 552 000	27.928 48	88.317 61	9.205 164	19.831 92	42.726 59
781	609 961	476 379 541	27.946 38	88.374 20	9.209 096	19.840 40	42.744 84
782	611 524	478 211 766	27.964 26	88.430 76	9.213 025	19.848 86	42.763 07
783	613 089	480 048 687	27.982 14	88.487 29	9.216 950	19.857 32	42.781 29
784	614 656	481 890 304	28.000 00	88.543 77	9.220 873	19.865 77	42.799 50
785	616 225	483 736 825	28.017 85	88.600 23	9.224 791	19.874 21	42.817 69
786	617 796	485 587 656	28.035 69	88.656 64	9.228 707	19.882 65	42.835 86
787	619 369	487 443 403	28.053 52	88.713 02	9.232 619	19.891 07	42.854 02
788	620 944	489 303 872	28.071 34	88.769 36	9.236 528	19.899 50	42.872 16
789	622 521	491 169 069	28.089 14	88.825 67	9.240 433	19.907 91	42.890 29
790	624 100	493 039 000	28.106 94	88.881 94	9.244 335	19.916 32	42.908 40
791	625 681	494 913 671	28.124 72	88.938 18	9.248 234	19.924 72	42.926 50
792	627 264	496 793 088	28.142 49	88.994 38	9.252 130	19.933 11	42.944 58
793	628 849	498 677 257	28.160 26	89.050 55	9.256 022	19.941 50	42.962 65
794	630 436	500 566 184	28.178 01	89.106 68	9.259 911	19.949 87	42.980 70
795	632 025	502 459 875	28.195 74	89.162 77	9.263 797	19.958 25	42.998 74
796	633 616	504 368 336	28.213 47	89.218 83	9.267 680	19.966 61	43.016 76
797	635 209	506 281 573	28.231 19	89.274 86	9.271 559	19.974 97	43.034 77
798	636 804	508 199 592	28.248 89	89.330 85	9.275 435	19.983 32	43.052 76
799	638 401	510 082 399	28.266 59	89.386 80	9.279 308	19.991 66	43.070 73

Table 18 SQUARES, CUBES, SQUARE ROOTS, AND CUBE ROOTS (continued)

n	n^2	n^3	\sqrt{n}	$\sqrt{10n}$	$\sqrt[3]{n}$	$\sqrt[3]{10n}$	$\sqrt[3]{100n}$
800	640 000	512 000 000	28.284 27	89.442 72	9.283 178	20.000 00	43.089 69
801	641 601	513 922 461	28.301 94	89.498 60	9.287 044	20.008 33	43.106 84
802	643 204	515 849 608	28.319 60	89.554 45	9.290 907	20.016 65	43.124 57
803	644 809	517 781 627	28.337 25	89.610 27	9.294 767	20.024 97	43.142 49
804	646 416	519 718 464	28.354 89	89.666 05	9.298 624	20.033 28	43.160 39
805	648 025	521 660 125	28.372 52	89.721 79	9.302 477	20.041 58	43.178 28
806	649 636	523 606 616	28.390 14	89.777 50	9.306 328	20.049 88	43.196 15
807	651 249	525 557 943	28.407 75	89.833 18	9.310 175	20.058 16	43.214 00
808	652 864	527 514 112	28.425 34	89.888 82	9.314 019	20.066 45	43.231 85
809	654 481	529 475 129	28.442 93	89.944 43	9.317 860	20.074 72	43.249 67
810	656 100	531 441 000	28.460 50	90.000 00	9.321 698	20.082 99	43.267 46
811	657 721	533 411 731	28.478 06	90.055 54	9.325 532	20.091 25	43.285 23
812	659 344	535 387 328	28.495 61	90.111 04	9.329 363	20.099 50	43.303 07
813	660 969	537 367 797	28.513 15	90.166 51	9.333 192	20.107 75	43.320 84
814	662 596	539 353 144	28.530 69	90.221 95	9.337 017	20.116 99	43.338 59
815	664 225	541 343 375	28.548 20	90.277 35	9.340 839	20.124 23	43.356 33
816	665 856	543 338 496	28.565 71	90.332 72	9.344 657	20.132 45	43.374 06
817	667 489	545 338 513	28.583 21	90.388 05	9.348 473	20.140 67	43.391 77
818	669 124	547 343 432	28.600 70	90.443 35	9.352 286	20.148 89	43.409 47
819	670 761	549 353 259	28.618 18	90.498 62	9.356 095	20.157 10	43.427 15
820	672 400	551 368 000	28.635 64	90.553 85	9.359 902	20.165 30	43.444 81
821	674 041	553 387 661	28.653 10	90.609 05	9.363 705	20.173 49	43.462 47
822	675 684	555 412 248	28.670 54	90.664 22	9.367 505	20.181 68	43.480 11
823	677 329	557 441 767	28.687 98	90.719 36	9.371 302	20.189 85	43.497 73
824	678 976	559 476 224	28.705 40	90.774 45	9.375 095	20.198 03	43.515 34
825	680 625	561 515 625	28.722 81	90.829 51	9.378 887	20.206 20	43.532 94
826	682 276	563 559 976	28.740 22	90.884 54	9.382 675	20.214 36	43.550 52
827	683 929	565 609 283	28.757 61	90.939 54	9.386 460	20.222 52	43.568 09
828	685 584	567 663 552	28.774 99	90.994 51	9.390 242	20.230 66	43.585 64
829	687 241	569 722 789	28.792 36	91.049 44	9.394 021	20.238 80	43.603 18
830	688 900	571 787 000	28.809 72	91.104 34	9.397 796	20.246 94	43.620 71
831	690 561	573 856 191	28.827 07	91.159 20	9.401 569	20.255 07	43.638 22
832	692 224	575 930 368	28.844 41	91.214 03	9.405 339	20.263 19	43.655 72
833	693 889	578 009 537	28.861 74	91.268 83	9.409 105	20.271 30	43.673 20
834	695 556	580 093 704	28.879 06	91.323 60	9.412 869	20.279 41	43.690 67
835	697 225	582 182 875	28.896 37	91.378 33	9.416 630	20.287 51	43.708 12
836	698 896	584 277 056	28.913 66	91.433 04	9.420 387	20.295 61	43.725 56
837	700 569	586 376 253	28.930 95	91.487 70	9.424 142	20.303 70	43.742 99
838	702 244	588 480 472	28.948 23	91.542 34	9.427 894	20.311 78	43.760 41
839	703 921	590 589 719	28.965 50	91.596 94	9.431 642	20.319 86	43.777 81
840	705 600	592 704 000	28.982 75	91.651 51	9.435 388	20.327 93	43.795 19
841	707 281	594 823 321	29.000 00	91.706 05	9.439 131	20.336 99	43.812 56
842	708 964	596 947 588	29.017 24	91.760 56	9.442 870	20.344 05	43.829 92
843	710 649	599 077 107	29.034 46	91.815 03	9.446 607	20.352 10	43.847 27
844	712 336	601 211 584	29.051 68	91.869 47	9.450 341	20.360 14	43.864 60
845	714 025	603 351 125	29.068 88	91.923 88	9.454 072	20.368 18	43.881 91
846	715 716	605 495 736	29.086 08	91.978 26	9.457 800	20.376 21	43.899 22
847	717 409	607 645 423	29.103 26	92.032 60	9.461 525	20.384 24	43.916 51
848	719 104	609 800 192	29.120 44	92.086 92	9.465 247	20.392 26	43.933 78
849	720 801	611 960 049	29.137 60	92.141 20	9.468 965	20.400 27	43.951 05

Table 18 SQUARES, CUBES, SQUARE ROOTS, AND CUBE ROOTS (continued) 342

n	n ²	n ³	\sqrt{n}	$\sqrt{10n}$	$\sqrt[3]{n}$	$\sqrt[3]{10n}$	$\sqrt[4]{100n}$
850	722 500	614 125 000	29.154 76	52.195 44	9.472 682	20.408 28	43.956 30
851	724 201	616 295 051	29.171 90	52.249 66	9.476 396	20.416 28	43.985 53
852	725 904	618 470 208	29.189 04	52.303 85	9.480 106	20.424 27	44.002 75
853	727 609	620 650 477	29.206 16	52.358 00	9.483 814	20.432 26	44.019 95
854	729 316	622 835 864	29.223 28	52.412 12	9.487 518	20.440 24	44.037 16
855	731 026	625 026 375	29.240 36	52.466 21	9.491 226	20.448 21	44.054 34
856	732 736	627 222 016	29.257 48	52.520 27	9.494 919	20.456 18	44.071 51
857	734 449	629 422 793	29.274 56	52.574 29	9.498 615	20.464 15	44.088 66
858	736 164	631 628 712	29.291 64	52.628 28	9.502 308	20.472 10	44.105 81
859	737 881	633 839 779	29.308 70	52.682 25	9.505 998	20.480 06	44.122 93
860	739 600	636 056 000	29.325 76	52.736 18	9.509 685	20.488 00	44.140 05
861	741 321	638 277 381	29.342 80	52.790 09	9.513 370	20.495 93	44.157 15
862	743 044	640 503 928	29.359 84	52.843 96	9.517 052	20.503 87	44.174 24
863	744 769	642 735 647	29.376 86	52.897 79	9.520 730	20.511 79	44.191 32
864	746 496	644 972 644	29.393 86	52.951 60	9.524 406	20.519 71	44.208 38
865	748 225	647 214 925	29.410 85	53.005 38	9.528 079	20.527 62	44.225 43
866	749 956	649 461 396	29.427 83	53.059 12	9.531 750	20.535 53	44.242 46
867	751 689	651 713 063	29.444 81	53.112 83	9.535 417	20.543 43	44.259 49
868	753 424	653 970 032	29.461 84	53.166 52	9.539 082	20.551 33	44.276 50
869	755 161	656 234 309	29.478 81	53.220 17	9.542 744	20.559 22	44.293 49
870	756 900	658 505 000	29.495 76	53.273 79	9.546 403	20.567 10	44.310 48
871	758 641	660 781 311	29.512 71	53.327 38	9.550 059	20.574 96	44.327 45
872	760 384	663 064 348	29.529 65	53.380 94	9.553 712	20.582 85	44.344 40
873	762 129	665 354 117	29.546 57	53.434 47	9.557 363	20.590 71	44.361 35
874	763 876	667 650 724	29.563 48	53.487 97	9.561 011	20.598 57	44.378 28
875	765 625	669 954 275	29.580 40	53.541 43	9.564 656	20.606 43	44.395 20
876	767 376	672 264 776	29.597 30	53.594 87	9.568 298	20.614 27	44.412 11
877	769 129	674 581 323	29.614 19	53.648 28	9.571 938	20.622 11	44.429 00
878	770 884	676 904 022	29.631 06	53.701 65	9.575 574	20.629 95	44.445 88
879	772 641	679 233 079	29.647 93	53.755 00	9.579 208	20.637 78	44.462 75
880	774 400	681 568 500	29.664 79	53.808 32	9.582 840	20.645 60	44.479 60
881	776 161	683 910 381	29.681 64	53.861 60	9.586 468	20.653 42	44.496 44
882	777 924	686 258 728	29.698 48	53.914 86	9.590 094	20.661 23	44.513 27
883	779 689	688 613 647	29.715 32	53.968 08	9.593 717	20.669 04	44.530 09
884	781 456	690 975 154	29.732 14	54.021 27	9.597 337	20.676 84	44.546 89
885	783 225	693 343 275	29.748 95	54.074 44	9.600 955	20.684 65	44.563 68
886	784 996	695 718 016	29.765 76	54.127 57	9.604 570	20.692 42	44.580 46
887	786 769	698 099 483	29.782 55	54.180 68	9.608 182	20.700 20	44.597 23
888	788 544	700 487 692	29.799 33	54.233 75	9.611 791	20.707 98	44.613 98
889	790 321	702 882 659	29.816 10	54.286 80	9.615 398	20.715 75	44.630 72
890	792 100	704 969 000	29.832 87	54.339 81	9.619 002	20.723 51	44.647 45
891	793 881	707 347 971	29.849 62	54.392 80	9.622 603	20.731 27	44.664 17
892	795 664	709 732 488	29.866 37	54.445 75	9.626 202	20.739 02	44.680 87
893	797 449	712 121 557	29.883 11	54.498 68	9.629 797	20.746 77	44.697 56
894	799 236	714 516 184	29.899 83	54.551 57	9.633 391	20.754 51	44.714 24
895	801 025	716 917 375	29.916 55	54.604 44	9.636 981	20.762 25	44.730 90
896	802 816	719 324 736	29.933 26	54.657 28	9.640 569	20.769 98	44.747 56
897	804 609	721 738 273	29.949 96	54.710 08	9.644 154	20.777 70	44.764 20
898	806 404	724 158 092	29.966 65	54.762 86	9.647 737	20.785 42	44.780 83
899	808 201	726 572 699	29.983 33	54.815 61	9.651 317	20.793 13	44.797 44

Table 18 SQUARES, CUBES, SQUARE ROOTS, AND CUBE ROOTS (continued)

n	n^2	n^3	\sqrt{n}	$\sqrt{10n}$	$\sqrt[3]{n}$	$\sqrt[3]{10n}$	$\sqrt[3]{100n}$
900	810 000	729 000 000	30.000 00	94.868 33	9.654 894	20.909 84	44.814 05
901	811 801	731 432 701	30.016 66	94.921 02	9.658 468	20.908 94	44.830 64
902	813 604	733 870 808	30.033 31	94.973 68	9.662 040	20.916 23	44.847 23
903	815 409	736 314 327	30.049 96	95.026 31	9.665 610	20.923 52	44.863 79
904	817 216	738 763 264	30.066 59	95.079 91	9.669 176	20.931 61	44.880 34
905	819 025	741 217 628	30.083 22	95.131 49	9.672 740	20.939 29	44.896 88
906	820 836	743 677 416	30.099 83	95.184 03	9.676 302	20.949 96	44.913 41
907	822 649	746 142 643	30.116 44	95.236 55	9.679 860	20.954 93	44.929 93
908	824 464	748 613 312	30.133 04	95.289 03	9.683 417	20.962 29	44.946 44
909	826 281	751 089 429	30.149 63	95.341 49	9.686 970	20.969 94	44.962 93
910	828 100	753 571 000	30.166 21	95.393 92	9.690 521	20.977 59	44.979 41
911	829 921	756 058 031	30.182 78	95.446 32	9.694 069	20.985 24	44.995 89
912	831 744	758 550 528	30.199 34	95.498 69	9.697 615	20.992 88	45.012 34
913	833 569	761 048 487	30.215 89	95.551 03	9.701 158	20.990 51	45.028 79
914	835 396	763 551 944	30.232 43	95.603 35	9.704 699	20.998 14	45.045 22
915	837 225	766 060 876	30.248 97	95.655 63	9.708 237	20.915 76	45.061 64
916	839 056	768 575 295	30.265 49	95.707 89	9.711 772	20.923 38	45.078 05
917	840 889	771 095 213	30.282 01	95.760 12	9.715 306	20.930 99	45.094 45
918	842 724	773 620 632	30.298 51	95.812 32	9.718 836	20.938 60	45.110 84
919	844 561	776 151 559	30.315 01	95.864 49	9.722 363	20.946 20	45.127 21
920	846 400	778 688 000	30.331 50	95.916 63	9.725 888	20.953 79	45.143 57
921	848 241	781 229 961	30.347 98	95.968 74	9.729 411	20.961 38	45.159 92
922	850 084	783 777 448	30.364 45	96.020 83	9.732 931	20.968 96	45.176 26
923	851 929	786 330 467	30.380 92	96.072 89	9.736 448	20.976 54	45.192 59
924	853 776	788 889 024	30.397 37	96.124 92	9.739 963	20.984 11	45.208 91
925	855 625	791 453 125	30.413 81	96.176 92	9.743 476	20.991 68	45.225 21
926	857 476	794 022 776	30.430 25	96.228 89	9.746 986	20.999 24	45.241 50
927	859 329	796 597 983	30.446 67	96.280 84	9.750 493	21.006 80	45.257 79
928	861 184	799 178 752	30.463 09	96.332 76	9.753 998	21.014 35	45.274 05
929	863 041	801 765 089	30.479 50	96.384 65	9.757 500	21.021 90	45.290 30
930	864 900	804 357 000	30.495 90	96.436 51	9.761 000	21.029 44	45.306 55
931	866 761	806 954 491	30.512 29	96.488 34	9.764 497	21.035 97	45.322 78
932	868 624	809 557 468	30.528 68	96.540 15	9.767 992	21.044 50	45.339 00
933	870 489	812 165 937	30.545 05	96.591 93	9.771 485	21.052 03	45.355 21
934	872 356	814 780 904	30.561 41	96.643 68	9.774 974	21.059 54	45.371 41
935	874 225	817 400 376	30.577 77	96.695 40	9.778 462	21.067 08	45.387 60
936	876 096	820 025 356	30.594 12	96.747 09	9.781 946	21.074 56	45.403 77
937	877 969	822 655 843	30.610 46	96.798 76	9.785 429	21.082 07	45.419 94
938	879 844	825 291 872	30.626 79	96.850 40	9.788 909	21.089 56	45.436 09
939	881 721	827 933 419	30.643 11	96.902 01	9.792 386	21.097 06	45.452 23
940	883 600	830 584 000	30.659 42	96.953 60	9.795 861	21.104 64	45.468 36
941	885 481	833 237 621	30.675 72	97.005 15	9.799 334	21.112 02	45.484 48
942	887 364	835 896 288	30.692 02	97.056 68	9.802 804	21.119 50	45.500 58
943	889 249	838 561 007	30.708 31	97.108 19	9.806 271	21.126 97	45.516 68
944	891 136	841 232 384	30.724 58	97.159 68	9.809 736	21.134 44	45.532 76
945	893 025	843 908 625	30.740 85	97.211 11	9.813 199	21.141 90	45.548 83
946	894 916	846 590 536	30.757 11	97.262 53	9.816 659	21.149 35	45.564 89
947	896 809	849 278 123	30.773 37	97.313 93	9.820 117	21.156 80	45.580 95
948	898 704	851 971 392	30.789 61	97.365 29	9.823 572	21.164 24	45.596 99
949	900 601	854 670 349	30.805 84	97.416 63	9.827 025	21.171 68	45.613 01

Table 13 SQUARES, CUBES, SQUARE ROOTS, AND CUBE ROOTS (continued) 344

n	n^2	n^3	\sqrt{n}	$\sqrt{10n}$	$\sqrt[3]{n}$	$\sqrt[3]{10n}$	$\sqrt[3]{100n}$
950	902 500	857 375 000	30.822 07	97.467 94	9.830 476	21.179 12	45.829 03
951	904 401	860 085 351	30.836 29	97.519 23	9.833 924	21.186 55	45.845 03
952	906 304	862 801 408	30.854 50	97.570 49	9.837 369	21.193 97	45.861 02
953	908 209	865 523 177	30.870 70	97.621 72	9.840 813	21.201 39	45.877 01
954	910 116	868 250 664	30.886 89	97.672 92	9.844 254	21.208 80	45.892 98
955	912 025	870 983 375	30.903 07	97.724 10	9.847 692	21.216 21	45.708 94
956	913 936	873 722 816	30.919 25	97.775 25	9.851 128	21.223 61	45.724 89
957	915 849	876 467 493	30.935 42	97.826 38	9.854 562	21.231 01	45.740 82
958	917 764	879 217 912	30.951 58	97.877 47	9.857 993	21.238 40	45.756 75
959	919 681	881 974 079	30.967 73	97.928 55	9.861 422	21.245 79	45.772 67
960	921 600	884 736 000	30.983 87	97.979 59	9.864 848	21.253 17	45.788 57
961	923 521	887 503 681	31.000 00	98.030 61	9.868 272	21.260 55	45.804 46
962	925 444	890 277 128	31.016 12	98.081 60	9.871 694	21.267 92	45.820 35
963	927 369	893 056 347	31.032 24	98.132 56	9.875 113	21.275 29	45.836 22
964	929 296	895 841 344	31.048 35	98.183 50	9.878 530	21.282 65	45.852 08
965	931 225	898 632 125	31.064 45	98.234 41	9.881 945	21.290 01	45.867 93
966	933 156	901 428 696	31.080 54	98.285 30	9.885 357	21.297 36	45.883 76
967	935 089	904 231 053	31.096 62	98.336 16	9.888 767	21.304 70	45.899 59
968	937 024	907 039 232	31.112 70	98.386 99	9.892 175	21.312 04	45.915 41
969	938 961	909 853 209	31.128 76	98.437 80	9.895 580	21.319 38	45.931 21
970	940 900	912 673 000	31.144 82	98.488 58	9.898 983	21.326 71	45.947 01
971	942 841	915 498 611	31.160 87	98.539 33	9.902 384	21.334 04	45.962 79
972	944 784	918 330 048	31.176 91	98.590 05	9.905 782	21.341 36	45.978 57
973	946 729	921 167 317	31.192 95	98.640 76	9.909 178	21.348 68	45.994 33
974	948 676	924 010 424	31.208 97	98.691 44	9.912 571	21.355 99	46.010 08
975	950 625	926 859 375	31.224 99	98.742 09	9.915 962	21.363 29	46.025 82
976	952 576	929 714 176	31.241 00	98.792 71	9.919 351	21.370 59	46.041 55
977	954 529	932 574 833	31.257 00	98.843 31	9.922 738	21.377 89	46.057 27
978	956 484	935 441 352	31.272 99	98.893 88	9.926 122	21.385 18	46.072 98
979	958 441	938 313 739	31.288 98	98.944 43	9.929 504	21.392 47	46.088 68
980	960 400	941 192 000	31.304 95	98.994 95	9.932 884	21.399 75	46.104 36
981	962 361	944 076 141	31.320 92	99.045 44	9.936 261	21.407 03	46.120 04
982	964 324	946 966 168	31.336 88	99.095 91	9.939 636	21.414 30	46.135 71
983	966 289	949 862 087	31.352 83	99.146 36	9.943 009	21.421 56	46.151 36
984	968 256	952 763 904	31.368 77	99.196 77	9.946 380	21.428 83	46.167 00
985	970 225	955 671 625	31.384 71	99.247 17	9.949 748	21.436 09	46.182 64
986	972 196	958 585 256	31.400 64	99.297 53	9.953 114	21.443 33	46.198 25
987	974 169	961 504 803	31.416 56	99.347 87	9.956 478	21.450 56	46.213 87
988	976 144	964 430 272	31.432 47	99.398 19	9.959 839	21.457 78	46.229 47
989	978 121	967 361 669	31.448 37	99.448 48	9.963 198	21.465 00	46.245 03
990	980 100	970 299 000	31.464 27	99.498 74	9.966 555	21.472 20	46.260 55
991	982 081	973 242 271	31.480 15	99.548 98	9.969 910	21.479 39	46.276 05
992	984 064	976 191 488	31.496 03	99.599 20	9.973 262	21.486 54	46.291 53
993	986 049	979 146 657	31.511 90	99.649 39	9.976 612	21.493 66	46.307 03
994	988 036	982 107 784	31.527 77	99.699 55	9.979 960	21.500 77	46.322 47
995	990 025	985 074 875	31.543 62	99.749 68	9.983 305	21.507 86	46.338 00
996	992 016	988 047 936	31.559 47	99.799 80	9.986 640	21.515 08	46.353 62
997	994 009	991 026 973	31.575 31	99.849 89	9.989 990	21.522 28	46.359 43
998	996 004	994 011 992	31.591 14	99.899 95	9.993 329	21.529 97	46.384 92
999	998 001	997 002 999	31.606 96	99.949 99	9.996 666	21.537 16	46.400 41

Table 19 RECIPROCAL, CIRCUMFERENCES, AND AREAS OF CIRCLES

<i>n</i>	1000/ <i>n</i>	Circum. of circle πn	Area of circle $\pi n^2/4$	<i>n</i>	1000/ <i>n</i>	Circum. of circle πn	Area of circle $\pi n^2/4$
1	1000.000	3.141 593	.785 3982	50	20.000 00	157.079 6	1 963.495
2	500.000 0	6.283 185	3.141 593	51	19.607 84	150.221 2	2 042.821
3	333.333 3	9.424 778	7.068 583	52	19.230 77	163.362 8	2 123.717
4	250.000 0	12.566 37	12.566 37	53	18.857 92	166.504 4	2 206.183
5	200.000 0	15.707 96	19.634 95	54	18.518 52	169.646 0	2 290.221
6	166.666 7	18.849 56	28.274 33	55	18.181 82	172.787 6	2 375.829
7	142.857 1	21.991 15	38.484 51	56	17.857 14	175.929 2	2 463.009
8	125.000 0	25.132 74	50.265 48	57	17.543 86	179.070 8	2 551.759
9	111.111 1	28.274 33	62.617 25	58	17.241 38	182.212 4	2 642.079
10	100.000 0	31.415 93	78.539 82	59	16.949 16	185.354 0	2 733.371
11	90.909 09	34.557 52	95.033 18	60	16.666 67	188.495 6	2 827.433
12	83.333 33	37.699 11	113.097 3	61	16.393 44	191.637 2	2 922.467
13	76.923 08	40.840 70	132.732 3	62	16.129 03	194.778 7	3 019.071
14	71.428 57	43.982 30	153.938 0	63	15.873 02	197.920 3	3 117.245
15	66.666 67	47.123 89	176.714 6	64	15.625 00	201.061 9	3 216.961
16	62.500 00	50.265 48	201.061 9	65	15.384 62	204.203 5	3 318.307
17	58.823 53	53.407 08	226.980 1	66	15.151 52	207.345 1	3 421.194
18	55.555 56	56.548 67	254.469 0	67	14.925 37	210.486 7	3 525.652
19	52.631 58	59.690 26	283.528 7	68	14.705 88	213.628 3	3 631.681
20	50.000 00	62.831 85	314.159 3	69	14.492 75	216.769 9	3 739.281
21	47.619 05	65.973 45	346.360 6	70	14.285 71	219.911 5	3 848.451
22	45.454 55	69.115 04	380.132 7	71	14.084 51	223.053 1	3 959.192
23	43.478 26	72.256 63	415.475 8	72	13.888 89	226.194 7	4 071.504
24	41.666 67	75.398 22	452.389 3	73	13.698 63	229.336 3	4 185.387
25	40.000 00	78.539 82	490.873 9	74	13.513 51	232.477 9	4 300.840
26	38.461 54	81.681 41	530.929 2	75	13.333 33	235.619 4	4 417.865
27	37.037 04	84.823 00	572.555 3	76	13.157 89	238.761 0	4 536.460
28	35.714 29	87.964 59	615.752 2	77	12.987 01	241.902 6	4 656.626
29	34.482 76	91.106 19	660.519 9	78	12.820 51	245.044 2	4 778.362
30	33.333 33	94.247 78	706.858 3	79	12.658 23	248.185 8	4 901.670
31	32.258 06	97.389 37	754.767 6	80	12.500 00	251.327 4	5 026.548
32	31.250 00	100.531 0	804.247 7	81	12.345 68	254.469 0	5 152.997
33	30.303 03	103.672 6	855.298 8	82	12.195 12	257.610 6	5 281.017
34	29.411 76	106.814 2	907.920 3	83	12.048 19	260.752 2	5 410.608
35	28.571 43	109.955 7	962.112 8	84	11.904 78	263.893 8	5 541.769
36	27.777 78	113.097 3	1 017.876	85	11.764 71	267.035 4	5 674.502
37	27.027 03	116.238 9	1 075.210	86	11.627 91	270.177 0	5 808.905
38	26.315 79	119.380 5	1 134.115	87	11.494 25	273.318 6	5 944.879
39	25.641 03	122.522 1	1 194.591	88	11.363 64	276.460 2	6 082.123
40	25.000 00	125.663 7	1 256.637	89	11.235 96	279.601 7	6 221.139
41	24.380 24	128.805 3	1 320.254	90	11.111 11	282.743 3	6 361.725
42	23.809 52	131.946 9	1 385.442	91	10.989 01	285.884 9	6 503.382
43	23.255 81	135.088 5	1 452.201	92	10.869 57	289.026 5	6 647.510
44	22.727 27	138.230 1	1 520.531	93	10.752 69	292.168 1	6 792.909
45	22.222 22	141.371 7	1 590.431	94	10.638 30	295.309 7	6 939.778
46	21.739 13	144.513 3	1 661.903	95	10.526 32	298.451 3	7 088.218
47	21.276 60	147.654 9	1 734.945	96	10.416 67	301.592 9	7 238.229
48	20.833 33	150.796 4	1 809.557	97	10.309 26	304.734 5	7 389.811
49	20.408 16	153.938 0	1 885.741	98	10.204 06	307.876 1	7 542.964
				99	10.101 01	311.017 7	7 697.687

Table 19 RECIPROCAL, CIRCUMFERENCES, AND AREAS OF CIRCLES (continued)

n	1000/n	Circum. of circle πn	Area of circle $\pi n^2/4$	n	1000/n	Circum. of circle πn	Area of circle $\pi n^2/4$
100	10.000 000	314.159 3	7 853.982	150	6.666 667	471.238 9	17 671.46
101	9.900 990	317.300 9	8 011.847	151	6.622 517	474.380 5	17 907.06
102	9.803 922	320.442 5	8 171.262	152	6.578 947	477.522 1	18 145.64
103	9.708 738	323.584 0	8 332.289	153	6.535 948	480.663 7	18 385.39
104	9.615 385	326.725 6	8 494.867	154	6.493 506	483.805 3	18 626.50
105	9.523 810	329.867 2	8 659.015	155	6.451 613	486.946 9	18 869.19
106	9.433 962	333.008 8	8 824.784	156	6.410 256	490.088 5	19 113.45
107	9.345 794	336.150 4	8 992.024	157	6.369 427	493.230 0	19 359.28
108	9.259 269	339.292 0	9 160.884	158	6.329 114	496.371 5	19 606.68
109	9.174 312	342.433 6	9 331.316	159	6.289 306	499.513 2	19 855.65
110	9.090 909	345.575 2	9 503.318	160	6.250 000	502.654 8	20 106.19
111	9.009 009	348.716 8	9 676.891	161	6.211 180	505.796 4	20 358.31
112	8.928 571	351.858 4	9 852.036	162	6.172 840	508.938 0	20 611.99
113	8.849 558	355.000 0	10 028.76	163	6.134 969	512.079 6	20 867.24
114	8.771 950	358.141 6	10 207.03	164	6.097 561	515.221 2	21 124.07
115	8.695 652	361.283 2	10 386.89	165	6.060 606	518.362 8	21 382.43
116	8.620 693	364.424 7	10 568.32	166	6.024 096	521.504 4	21 642.43
117	8.547 009	367.566 3	10 751.32	167	5.988 024	524.646 0	21 903.97
118	8.474 576	370.707 9	10 935.88	168	5.952 381	527.787 6	22 167.08
119	8.403 367	373.849 5	11 122.02	169	5.917 160	530.929 2	22 431.76
120	8.333 333	376.991 1	11 309.73	170	5.882 353	534.070 8	22 698.01
121	8.264 463	380.132 7	11 499.01	171	5.847 953	537.212 3	22 965.83
122	8.196 721	383.274 3	11 689.87	172	5.813 963	540.353 9	23 235.22
123	8.130 061	386.415 9	11 882.29	173	5.780 347	543.495 5	23 506.18
124	8.064 516	389.557 5	12 076.28	174	5.747 126	546.637 1	23 778.71
125	8.000 000	392.699 1	12 271.85	175	5.714 286	549.778 7	24 052.82
126	7.936 508	395.840 7	12 468.98	176	5.681 818	552.920 3	24 328.49
127	7.874 016	398.982 3	12 667.69	177	5.649 718	556.061 9	24 605.74
128	7.812 500	402.123 9	12 867.96	178	5.617 978	559.203 5	24 884.56
129	7.751 928	405.265 5	13 069.81	179	5.586 592	562.345 1	25 164.94
130	7.692 308	408.407 0	13 273.23	180	5.555 556	565.486 7	25 446.90
131	7.633 588	411.548 6	13 478.22	181	5.524 862	568.628 3	25 730.43
132	7.575 758	414.690 2	13 684.78	182	5.494 505	571.769 9	26 015.53
133	7.518 797	417.831 8	13 892.91	183	5.464 481	574.911 5	26 302.20
134	7.462 697	420.973 4	14 102.61	184	5.434 783	578.053 0	26 590.44
135	7.407 407	424.115 0	14 313.88	185	5.405 405	581.194 6	26 880.26
136	7.352 941	427.256 6	14 526.72	186	5.376 344	584.336 2	27 171.63
137	7.299 270	430.398 2	14 741.14	187	5.347 594	587.477 8	27 464.59
138	7.246 377	433.539 8	14 957.12	188	5.319 149	590.619 4	27 759.11
139	7.194 245	436.681 4	15 174.68	189	5.291 005	593.761 0	28 055.21
140	7.142 857	439.823 0	15 393.80	190	5.263 138	596.902 6	28 352.87
141	7.092 196	442.964 6	15 614.50	191	5.235 502	600.044 2	28 652.11
142	7.042 254	446.106 2	15 836.77	192	5.208 333	603.185 8	28 952.92
143	6.993 007	449.247 7	16 060.61	193	5.181 347	606.327 4	29 255.30
144	6.944 444	452.389 3	16 286.02	194	5.154 639	609.469 0	29 559.26
145	6.896 562	455.530 9	16 513.00	195	5.128 205	612.610 6	29 864.77
146	6.849 315	458.672 5	16 741.56	196	5.102 041	615.752 2	30 171.86
147	6.802 721	461.814 1	16 971.67	197	5.076 142	618.893 8	30 480.52
148	6.756 787	464.955 7	17 203.36	198	5.050 505	622.035 3	30 790.75
149	6.711 409	468.097 3	17 436.62	199	5.025 126	625.176 9	31 102.56

Table 19 RECIPROCAL, CIRCUMFERENCES, AND AREAS OF CIRCLES (continued)

n	$1000/n$	Circum. of circle πn	Area of circle $\pi n^2/4$	n	$1000/n$	Circum. of circle πn	Area of circle $\pi n^2/4$
200	5.000 000	628.318 5	31 415.93	250	4.000 000	785.398 2	49 087.39
201	4.975 124	631.460 1	31 730.87	251	3.984 064	788.639 8	49 480.87
202	4.950 495	634.601 7	32 047.39	252	3.968 254	791.681 3	49 875.92
203	4.926 106	637.743 3	32 365.47	253	3.952 569	794.822 9	50 272.85
204	4.901 951	640.884 9	32 685.13	254	3.937 008	797.964 5	50 670.75
205	4.878 049	644.026 5	33 006.36	255	3.921 569	801.106 1	51 070.52
206	4.854 369	647.168 1	33 329.16	256	3.906 250	804.247 7	51 471.85
207	4.830 918	650.309 7	33 653.53	257	3.891 051	807.389 3	51 874.78
208	4.807 692	653.451 3	33 979.47	258	3.875 969	810.530 9	52 279.24
209	4.784 689	656.592 9	34 306.98	259	3.861 004	813.672 5	52 685.29
210	4.761 805	659.734 5	34 636.06	260	3.846 154	816.814 1	53 092.92
211	4.739 136	662.876 0	34 966.71	261	3.831 418	819.955 7	53 502.11
212	4.716 681	666.017 5	35 298.94	262	3.816 794	823.097 3	53 912.87
213	4.694 436	669.159 2	35 632.73	263	3.802 281	826.238 9	54 325.21
214	4.672 397	672.300 8	35 968.09	264	3.787 879	829.380 5	54 739.11
215	4.650 463	675.442 4	36 305.03	265	3.773 585	832.522 1	55 154.59
216	4.628 630	678.584 0	36 643.54	266	3.759 398	835.663 6	55 571.63
217	4.606 895	681.725 6	36 983.61	267	3.745 318	838.805 2	55 990.25
218	4.585 256	684.867 2	37 325.26	268	3.731 343	841.946 8	56 410.44
219	4.563 710	688.008 8	37 668.48	269	3.717 472	845.088 4	56 832.20
220	4.542 265	691.150 4	38 013.27	270	3.703 704	848.230 0	57 256.53
221	4.520 917	694.292 0	38 359.63	271	3.690 037	851.371 6	57 682.43
222	4.500 665	697.433 6	38 707.56	272	3.676 471	854.513 2	58 109.90
223	4.480 505	700.575 2	39 057.07	273	3.663 004	857.654 8	58 539.84
224	4.460 436	703.716 8	39 408.14	274	3.649 635	860.796 4	58 971.55
225	4.440 464	706.858 3	39 760.78	275	3.636 364	863.938 0	59 405.74
226	4.420 588	709.999 9	40 115.00	276	3.623 193	867.079 6	59 841.49
227	4.400 805	713.141 5	40 470.78	277	3.610 122	870.221 2	60 278.82
228	4.381 112	716.283 1	40 828.14	278	3.597 152	873.362 8	60 717.71
229	4.361 507	719.424 7	41 187.07	279	3.584 281	876.504 4	61 158.13
230	4.342 086	722.566 3	41 547.59	280	3.571 509	879.646 0	61 600.22
231	4.322 747	725.707 9	41 909.63	281	3.558 837	882.787 5	62 043.82
232	4.303 488	728.849 5	42 273.27	282	3.546 265	885.929 1	62 488.90
233	4.284 305	731.991 1	42 638.48	283	3.533 793	889.070 7	62 935.55
234	4.265 195	735.132 7	43 005.26	284	3.521 421	892.212 3	63 383.77
235	4.246 155	738.274 3	43 373.61	285	3.509 149	895.353 9	63 833.57
236	4.227 183	741.415 9	43 743.54	286	3.496 977	898.495 5	64 284.92
237	4.208 277	744.557 5	44 115.03	287	3.484 905	901.637 1	64 737.82
238	4.189 434	747.699 1	44 488.09	288	3.472 933	904.778 7	65 192.27
239	4.170 652	750.840 6	44 862.73	289	3.461 061	907.920 3	65 648.24
240	4.151 927	753.982 2	45 238.93	290	3.449 289	911.061 9	66 105.79
241	4.133 265	757.123 8	45 616.71	291	3.437 617	914.203 5	66 564.83
242	4.114 654	760.265 4	45 996.09	292	3.426 045	917.345 1	67 025.35
243	4.096 091	763.407 0	46 376.98	293	3.414 573	920.486 6	67 487.45
244	4.077 576	766.548 6	46 759.47	294	3.403 201	923.628 2	67 951.11
245	4.059 107	769.690 2	47 143.52	295	3.391 929	926.769 8	68 416.32
246	4.040 681	772.831 8	47 529.15	296	3.380 757	929.911 4	68 883.08
247	4.022 295	775.973 4	47 916.36	297	3.369 685	933.053 0	69 351.39
248	4.003 956	779.115 0	48 305.13	298	3.358 713	936.194 6	69 821.24
249	3.985 661	782.256 6	48 695.47	299	3.347 841	939.336 2	70 292.63

Table 19 RECIPROCAL, CIRCUMFERENCES, AND AREAS OF CIRCLES (continued)

n	$1000/n$	Circum. of circle πn	Area of circle $\pi n^2/4$	n	$1000/n$	Circum. of circle πn	Area of circle $\pi n^2/4$
300	3.333 333	942.477 8	70 685.83	350	2.857 143	1 099.557	96 211.28
301	3.322 259	945.619 4	71 157.86	351	2.849 003	1 102.899	96 761.84
302	3.311 258	948.761 0	71 631.45	352	2.840 909	1 106.841	97 313.97
303	3.300 330	951.902 5	72 106.62	353	2.832 861	1 108.982	97 867.68
304	3.289 474	956.044 2	72 583.36	354	2.824 859	1 112.124	98 422.96
305	3.278 689	958.185 8	73 061.66	355	2.816 901	1 115.265	98 979.80
306	3.267 974	961.327 4	73 541.54	356	2.808 989	1 118.407	99 538.22
307	3.257 329	964.468 9	74 022.89	357	2.801 120	1 121.549	100 098.2
308	3.246 753	967.610 5	74 506.01	358	2.793 296	1 124.690	100 659.8
309	3.236 246	970.752 1	74 990.60	359	2.785 515	1 127.832	101 222.9
310	3.225 806	973.893 7	75 476.76	360	2.777 778	1 130.973	101 787.0
311	3.215 434	977.035 3	75 964.50	361	2.770 083	1 134.115	102 353.9
312	3.205 128	980.176 9	76 453.80	362	2.762 431	1 137.257	102 921.7
313	3.194 888	983.318 5	76 944.67	363	2.754 821	1 140.398	103 491.1
314	3.184 713	986.460 1	77 437.12	364	2.747 253	1 143.540	104 062.1
315	3.174 603	989.601 7	77 931.13	365	2.739 726	1 146.681	104 634.7
316	3.164 557	992.743 3	78 426.72	366	2.732 240	1 149.823	105 208.8
317	3.154 574	995.884 9	78 923.88	367	2.724 796	1 152.965	105 784.5
318	3.144 654	999.026 5	79 422.60	368	2.717 391	1 156.106	106 361.8
319	3.134 796	1 002.168	79 922.90	369	2.710 027	1 159.248	106 940.6
320	3.125 000	1 005.310	80 424.77	370	2.702 703	1 162.389	107 521.0
321	3.115 265	1 008.451	80 928.21	371	2.695 418	1 165.531	108 103.0
322	3.105 590	1 011.593	81 433.22	372	2.688 172	1 168.672	108 686.5
323	3.095 975	1 014.734	81 939.80	373	2.680 965	1 171.814	109 271.7
324	3.086 420	1 017.876	82 447.86	374	2.673 797	1 174.956	109 858.4
325	3.076 923	1 021.018	82 957.68	375	2.666 667	1 178.097	110 446.6
326	3.067 485	1 024.159	83 468.97	376	2.659 574	1 181.239	111 036.5
327	3.058 104	1 027.301	83 981.84	377	2.652 520	1 184.380	111 627.9
328	3.048 780	1 030.442	84 496.28	378	2.645 503	1 187.522	112 220.8
329	3.039 514	1 033.584	85 012.28	379	2.638 522	1 190.664	112 815.4
330	3.030 303	1 036.726	85 529.86	380	2.631 579	1 193.805	113 411.5
331	3.021 148	1 039.867	86 049.01	381	2.624 672	1 196.947	114 009.2
332	3.012 048	1 043.009	86 569.73	382	2.617 801	1 200.088	114 608.4
333	3.003 003	1 046.150	87 092.02	383	2.610 966	1 203.230	115 209.3
334	2.994 012	1 049.292	87 615.88	384	2.604 167	1 206.372	115 811.7
335	2.985 075	1 052.434	88 141.31	385	2.597 403	1 209.513	116 415.6
336	2.976 190	1 055.575	88 668.31	386	2.590 674	1 212.655	117 021.2
337	2.967 359	1 058.717	89 196.88	387	2.583 979	1 215.796	117 628.3
338	2.958 580	1 061.858	89 727.03	388	2.577 320	1 218.938	118 237.0
339	2.949 853	1 065.000	90 258.74	389	2.570 694	1 222.080	118 847.2
340	2.941 176	1 068.142	90 792.03	390	2.564 103	1 225.221	119 459.1
341	2.932 551	1 071.283	91 326.88	391	2.557 546	1 228.363	120 072.5
342	2.923 977	1 074.425	91 863.31	392	2.551 020	1 231.504	120 687.4
343	2.915 452	1 077.566	92 401.31	393	2.544 529	1 234.646	121 304.0
344	2.906 977	1 080.708	92 940.88	394	2.538 071	1 237.788	121 922.1
345	2.898 551	1 083.849	93 482.02	395	2.531 646	1 240.929	122 541.7
346	2.890 173	1 086.991	94 024.73	396	2.525 253	1 244.071	123 163.0
347	2.881 844	1 090.133	94 569.01	397	2.518 892	1 247.212	123 785.8
348	2.873 563	1 093.274	95 114.86	398	2.512 563	1 250.354	124 410.2
349	2.865 330	1 096.416	95 662.28	399	2.506 266	1 253.495	125 036.2

Table 19 RECIPROCAL, CIRCUMFERENCES, AND AREAS OF CIRCLES (continued)

n	$1000/n$	Circum. of circle πn	Area of circle $\pi n^2/4$	n	$1000/n$	Circum. of circle πn	Area of circle $\pi n^2/4$
400	2.500 000	1 256.637	129 653.7	450	2.222 222	1 413.717	159 043.1
401	2.493 766	1 259.779	126 292.8	451	2.217 295	1 416.858	159 750.8
402	2.487 562	1 262.920	126 923.5	452	2.212 389	1 420.000	160 460.0
403	2.481 390	1 266.062	127 555.7	453	2.207 506	1 423.141	161 170.8
404	2.475 248	1 269.203	128 189.5	454	2.202 643	1 426.283	161 883.1
405	2.469 136	1 272.345	128 824.9	455	2.197 802	1 429.425	162 597.1
406	2.463 054	1 275.487	129 461.9	456	2.192 982	1 432.566	163 312.6
407	2.457 002	1 278.628	130 100.4	457	2.188 184	1 435.708	164 029.6
408	2.450 980	1 281.770	130 740.8	458	2.183 406	1 438.849	164 748.3
409	2.444 988	1 284.911	131 382.2	459	2.178 649	1 441.991	165 468.5
410	2.439 024	1 288.053	132 025.4	460	2.173 913	1 445.133	166 190.3
411	2.433 090	1 291.195	132 670.2	461	2.169 197	1 448.274	166 913.6
412	2.427 184	1 294.338	133 316.6	462	2.164 502	1 451.416	167 638.5
413	2.421 306	1 297.478	133 964.6	463	2.159 827	1 454.557	168 365.0
414	2.415 459	1 300.619	134 614.1	464	2.155 172	1 457.699	169 093.1
415	2.409 639	1 303.761	135 265.2	465	2.150 538	1 460.841	169 822.7
416	2.403 846	1 306.903	135 917.9	466	2.145 923	1 463.982	170 553.9
417	2.398 082	1 310.044	136 572.1	467	2.141 328	1 467.124	171 286.7
418	2.392 344	1 313.186	137 227.9	468	2.136 752	1 470.265	172 021.0
419	2.386 635	1 316.327	137 885.3	469	2.132 196	1 473.407	172 757.0
420	2.380 952	1 319.469	138 544.2	470	2.127 660	1 476.549	173 494.5
421	2.375 297	1 322.611	139 204.6	471	2.123 142	1 479.690	174 233.5
422	2.369 668	1 325.752	139 866.8	472	2.118 644	1 482.832	174 974.1
423	2.364 066	1 328.894	140 530.6	473	2.114 165	1 485.973	175 716.3
424	2.358 491	1 332.035	141 195.7	474	2.109 705	1 489.115	176 460.1
425	2.352 941	1 335.177	141 862.5	475	2.105 263	1 492.257	177 205.5
426	2.347 418	1 338.318	142 530.9	476	2.100 840	1 495.398	177 952.4
427	2.341 920	1 341.460	143 200.9	477	2.096 436	1 498.540	178 700.9
428	2.336 449	1 344.602	143 872.4	478	2.092 050	1 501.681	179 450.9
429	2.331 002	1 347.743	144 545.5	479	2.087 683	1 504.823	180 202.5
430	2.325 581	1 350.885	145 220.1	480	2.083 333	1 507.964	180 955.7
431	2.320 186	1 354.026	145 896.3	481	2.078 902	1 511.106	181 710.5
432	2.314 816	1 357.168	146 574.1	482	2.074 489	1 514.248	182 466.8
433	2.309 469	1 360.310	147 253.5	483	2.070 093	1 517.389	183 224.8
434	2.304 147	1 363.451	147 934.5	484	2.065 715	1 520.531	183 984.2
435	2.298 851	1 366.593	148 617.0	485	2.061 356	1 523.672	184 745.3
436	2.293 579	1 369.734	149 301.0	486	2.057 013	1 526.814	185 507.9
437	2.288 330	1 372.876	149 986.7	487	2.052 688	1 529.955	186 272.1
438	2.283 105	1 376.018	150 673.9	488	2.048 380	1 533.097	187 037.9
439	2.277 904	1 379.159	151 362.7	489	2.044 090	1 536.239	187 805.2
440	2.272 727	1 382.301	152 053.1	490	2.040 816	1 539.380	188 574.1
441	2.267 574	1 385.442	152 745.0	491	2.036 660	1 542.522	189 344.6
442	2.262 443	1 388.584	153 438.5	492	2.032 520	1 545.664	190 116.6
443	2.257 336	1 391.726	154 133.6	493	2.028 398	1 548.805	190 890.2
444	2.252 252	1 394.867	154 830.3	494	2.024 291	1 551.947	191 665.4
445	2.247 191	1 398.009	155 528.5	495	2.020 202	1 555.088	192 442.2
446	2.242 152	1 401.150	156 228.3	496	2.016 129	1 558.230	193 220.5
447	2.237 136	1 404.292	156 929.6	497	2.012 072	1 561.372	194 000.4
448	2.232 143	1 407.434	157 632.6	498	2.008 032	1 564.513	194 781.9
449	2.227 171	1 410.575	158 337.1	499	2.004 008	1 567.655	195 564.9

Table 19 RECIPROCAL, CIRCUMFERENCES, AND AREAS OF CIRCLES (continued)

n	$1000/n$	Circum. of circle πn	Area of circle $\pi n^2/4$	n	$1000/n$	Circum. of circle πn	Area of circle $\pi n^2/4$
500	2.000 000	1 570.796	195 349.5	550	1.818 182	1 727.876	237 582.9
501	1.996 008	1 578.938	197 135.7	551	1.814 882	1 731.018	238 447.7
502	1.992 032	1 577.080	197 923.5	552	1.811 594	1 734.159	239 314.0
503	1.988 072	1 580.221	198 712.8	553	1.808 318	1 737.301	240 181.3
504	1.984 127	1 583.363	199 503.7	554	1.805 054	1 740.442	241 051.3
505	1.980 198	1 586.504	200 295.2	555	1.801 802	1 743.584	241 922.3
506	1.976 285	1 589.646	201 087.2	556	1.798 561	1 746.726	242 794.8
507	1.972 387	1 592.787	201 885.8	557	1.795 332	1 749.867	243 669.0
508	1.968 504	1 595.929	202 683.0	558	1.792 115	1 753.009	244 544.7
509	1.964 637	1 599.071	203 481.7	559	1.788 909	1 756.150	245 422.0
510	1.960 784	1 602.212	204 282.1	560	1.785 714	1 759.292	246 300.9
511	1.956 947	1 605.354	205 084.0	561	1.782 531	1 762.433	247 181.3
512	1.953 125	1 608.495	205 887.4	562	1.779 359	1 765.575	248 063.3
513	1.949 318	1 611.637	206 692.4	563	1.776 199	1 768.717	248 946.9
514	1.945 525	1 614.779	207 499.1	564	1.773 050	1 771.858	249 832.0
515	1.941 748	1 617.920	208 307.2	565	1.769 912	1 775.000	250 718.7
516	1.937 984	1 621.062	209 117.0	566	1.766 784	1 778.141	251 607.0
517	1.934 236	1 624.203	209 928.3	567	1.763 668	1 781.283	252 496.9
518	1.930 502	1 627.345	210 741.2	568	1.760 563	1 784.425	253 388.3
519	1.926 782	1 630.487	211 555.6	569	1.757 469	1 787.566	254 281.3
520	1.923 077	1 633.628	212 371.7	570	1.754 386	1 790.708	255 175.9
521	1.919 386	1 636.770	213 189.3	571	1.751 313	1 793.849	256 072.0
522	1.915 709	1 639.911	214 008.4	572	1.748 252	1 796.991	256 969.7
523	1.912 046	1 643.053	214 829.2	573	1.745 201	1 800.133	257 869.0
524	1.908 397	1 646.195	215 651.5	574	1.742 160	1 803.274	258 769.8
525	1.904 762	1 649.336	216 475.4	575	1.739 130	1 806.416	259 672.3
526	1.901 141	1 652.478	217 300.8	576	1.736 111	1 809.557	260 576.3
527	1.897 533	1 655.619	218 127.8	577	1.733 102	1 812.699	261 481.8
528	1.893 939	1 658.761	218 956.4	578	1.730 104	1 815.841	262 388.0
529	1.890 359	1 661.903	219 786.6	579	1.727 116	1 818.982	263 297.7
530	1.886 792	1 665.044	220 618.3	580	1.724 138	1 822.124	264 207.9
531	1.883 239	1 668.185	221 451.7	581	1.721 170	1 825.265	265 119.8
532	1.879 699	1 671.327	222 285.5	582	1.718 213	1 828.407	266 033.2
533	1.876 173	1 674.469	223 123.0	583	1.715 265	1 831.549	266 948.2
534	1.872 658	1 677.610	223 961.0	584	1.712 329	1 834.690	267 864.8
535	1.869 159	1 680.752	224 800.6	585	1.709 402	1 837.832	268 782.9
536	1.865 672	1 683.894	225 641.8	586	1.706 485	1 840.973	269 702.6
537	1.862 197	1 687.035	226 484.5	587	1.703 578	1 844.115	270 623.9
538	1.858 736	1 690.177	227 328.8	588	1.700 680	1 847.256	271 546.7
539	1.855 288	1 693.318	228 174.7	589	1.697 793	1 850.398	272 471.1
540	1.851 852	1 696.460	229 022.1	590	1.694 915	1 853.540	273 397.1
541	1.848 429	1 699.602	229 871.1	591	1.692 047	1 856.681	274 324.7
542	1.845 018	1 702.743	230 721.7	592	1.689 189	1 859.823	275 253.8
543	1.841 621	1 705.885	231 573.9	593	1.686 341	1 862.964	276 184.5
544	1.838 235	1 709.026	232 427.6	594	1.683 502	1 866.105	277 116.7
545	1.834 862	1 712.168	233 282.9	595	1.680 672	1 869.248	278 050.6
546	1.831 502	1 715.310	234 139.8	596	1.677 852	1 872.389	278 986.0
547	1.828 154	1 718.451	234 998.2	597	1.675 042	1 875.531	279 923.0
548	1.824 818	1 721.593	235 858.2	598	1.672 241	1 878.672	280 861.5
549	1.821 494	1 724.734	236 719.5	599	1.669 449	1 881.814	281 801.6

Table 19 RECIPROCAL, CIRCUMFERENCES, AND AREAS OF CIRCLES (continued)

n	$1000/n$	Circum. of circle πn	Area of circle $\pi n^2/4$	n	$1000/n$	Circum. of circle πn	Area of circle $\pi n^2/4$
600	1.666 667	1 884.966	282 743.3	650	1.538 462	2 042.035	321 839.7
601	1.663 894	1 888.097	283 686.6	651	1.536 098	2 045.177	322 852.6
602	1.661 130	1 891.239	284 631.4	652	1.533 742	2 048.318	323 875.9
603	1.658 375	1 894.380	285 577.8	653	1.531 394	2 051.460	324 900.8
604	1.655 629	1 897.522	286 525.8	654	1.529 052	2 054.602	325 927.4
605	1.652 893	1 900.664	287 475.4	655	1.526 718	2 057.743	326 955.4
606	1.650 165	1 903.805	288 426.5	656	1.524 390	2 060.885	327 984.1
607	1.647 446	1 906.947	289 379.2	657	1.522 070	2 064.026	329 015.3
608	1.644 737	1 910.088	290 333.4	658	1.519 757	2 067.168	330 049.1
609	1.642 035	1 913.230	291 289.3	659	1.517 451	2 070.310	331 085.6
610	1.639 344	1 916.372	292 246.7	660	1.515 152	2 073.451	332 119.4
611	1.636 661	1 919.513	293 205.6	661	1.512 859	2 076.593	333 157.0
612	1.633 987	1 922.655	294 166.2	662	1.510 574	2 079.734	334 196.0
613	1.631 321	1 925.796	295 128.3	663	1.508 295	2 082.876	335 236.7
614	1.628 664	1 928.938	296 092.0	664	1.506 024	2 086.018	336 278.9
615	1.626 016	1 932.079	297 057.2	665	1.503 759	2 089.159	337 322.7
616	1.623 377	1 935.221	298 024.0	666	1.501 502	2 092.301	338 368.1
617	1.620 746	1 938.363	298 992.4	667	1.499 250	2 095.442	339 415.0
618	1.618 123	1 941.504	299 962.4	668	1.497 005	2 098.584	350 463.5
619	1.615 509	1 944.645	300 933.9	669	1.494 763	2 101.725	351 513.6
620	1.612 903	1 947.787	301 907.1	670	1.492 527	2 104.867	352 565.2
621	1.610 305	1 950.929	302 881.7	671	1.490 297	2 108.009	353 618.5
622	1.607 717	1 954.071	303 858.0	672	1.488 075	2 111.150	354 673.2
623	1.605 136	1 957.212	304 835.8	673	1.485 864	2 114.292	355 729.6
624	1.602 564	1 960.354	305 815.2	674	1.483 660	2 117.433	356 787.6
625	1.600 000	1 963.495	306 796.2	675	1.481 461	2 120.575	357 847.0
626	1.597 444	1 966.637	307 778.7	676	1.479 269	2 123.717	358 908.1
627	1.594 896	1 969.779	308 762.8	677	1.477 085	2 126.858	359 970.8
628	1.592 357	1 972.920	309 748.5	678	1.474 925	2 130.000	361 035.0
629	1.589 825	1 976.062	310 735.7	679	1.472 754	2 133.141	362 100.8
630	1.587 302	1 979.203	311 724.5	680	1.470 583	2 136.283	363 168.1
631	1.584 786	1 982.345	312 714.9	681	1.468 429	2 139.425	364 237.0
632	1.582 278	1 985.487	313 706.9	682	1.466 276	2 142.568	365 307.5
633	1.579 779	1 988.628	314 700.4	683	1.464 129	2 145.708	366 379.6
634	1.577 287	1 991.770	315 695.5	684	1.461 985	2 148.849	367 453.2
635	1.574 803	1 994.911	316 692.2	685	1.459 854	2 151.991	368 528.5
636	1.572 327	1 998.053	317 690.4	686	1.457 728	2 155.133	369 605.2
637	1.569 859	2 001.195	318 690.2	687	1.455 604	2 158.274	370 683.6
638	1.567 388	2 004.336	319 691.6	688	1.453 483	2 161.415	371 763.6
639	1.564 945	2 007.478	320 694.6	689	1.451 379	2 164.557	372 845.0
640	1.562 500	2 010.619	321 699.1	690	1.449 275	2 167.699	373 928.1
641	1.560 062	2 013.761	322 705.2	691	1.447 178	2 170.841	375 012.7
642	1.557 632	2 016.902	323 712.8	692	1.445 087	2 173.982	376 098.9
643	1.555 210	2 020.044	324 722.1	693	1.443 001	2 177.124	377 186.7
644	1.552 795	2 023.186	325 732.9	694	1.440 922	2 180.265	378 276.0
645	1.550 388	2 026.327	326 745.3	695	1.438 849	2 183.407	379 366.9
646	1.547 988	2 029.469	327 759.2	696	1.436 782	2 186.548	380 459.4
647	1.545 595	2 032.610	328 774.7	697	1.434 720	2 189.689	381 553.5
648	1.543 210	2 035.752	329 791.8	698	1.432 665	2 192.832	382 649.1
649	1.540 832	2 038.894	330 810.5	699	1.430 615	2 195.973	383 746.3

Table 19 RECIPROALS, CIRCUMFERENCES, AND AREAS OF CIRCLES (continued)

n	1000/n	Circum. of circle πn	Area of circle $\pi n^2/4$	n	1000/n	Circum. of circle πn	Area of circle $\pi n^2/4$
700	1.428 571	2 199.115	384 845.1	750	1.333 333	2 356.184	441 786.5
701	1.426 684	2 202.256	395 945.4	751	1.331 559	2 359.336	442 965.3
702	1.424 801	2 205.398	387 047.4	752	1.329 787	2 362.478	444 145.8
703	1.422 475	2 206.540	388 150.8	753	1.328 021	2 365.619	445 327.8
704	1.420 455	2 211.661	389 255.9	754	1.326 260	2 368.761	446 511.4
705	1.418 440	2 214.823	390 362.5	755	1.324 503	2 371.902	447 698.6
706	1.416 431	2 217.964	391 470.7	756	1.322 751	2 375.044	448 888.3
707	1.414 427	2 221.106	392 580.5	757	1.321 004	2 378.186	450 071.6
708	1.412 429	2 224.248	393 691.8	758	1.319 261	2 381.327	451 261.5
709	1.410 437	2 227.389	394 804.7	759	1.317 523	2 384.469	462 453.0
710	1.408 451	2 230.531	395 919.2	760	1.315 789	2 387.610	453 646.0
711	1.406 470	2 233.672	397 035.3	761	1.314 060	2 390.752	454 840.6
712	1.404 494	2 236.814	398 152.0	762	1.312 336	2 393.894	456 036.7
713	1.402 523	2 239.956	399 272.1	763	1.310 616	2 397.035	457 234.5
714	1.400 560	2 243.097	400 392.8	764	1.308 901	2 400.177	458 433.8
715	1.398 601	2 246.239	401 515.2	765	1.307 190	2 403.318	459 634.6
716	1.396 648	2 249.380	402 639.1	766	1.305 483	2 406.460	460 837.1
717	1.394 700	2 252.522	403 764.6	767	1.303 781	2 409.602	462 041.1
718	1.392 768	2 255.664	404 891.6	768	1.302 083	2 412.743	463 245.7
719	1.390 821	2 258.805	406 020.2	769	1.300 390	2 415.885	464 453.8
720	1.388 889	2 261.947	407 150.4	770	1.298 701	2 419.026	465 662.6
721	1.386 963	2 265.088	408 282.2	771	1.297 017	2 422.168	466 872.9
722	1.385 042	2 268.230	409 415.5	772	1.295 337	2 425.310	468 084.7
723	1.383 126	2 271.371	410 550.4	773	1.293 661	2 428.451	469 298.2
724	1.381 215	2 274.513	411 686.9	774	1.291 990	2 431.593	470 513.2
725	1.379 310	2 277.655	412 824.9	775	1.290 323	2 434.734	471 729.8
726	1.377 410	2 280.796	413 964.5	776	1.288 660	2 437.876	472 947.9
727	1.375 516	2 283.938	415 105.7	777	1.287 001	2 441.017	474 167.6
728	1.373 626	2 287.079	416 248.5	778	1.285 347	2 444.159	475 388.9
729	1.371 742	2 290.221	417 392.8	779	1.283 697	2 447.301	476 611.8
730	1.369 863	2 293.363	418 538.7	780	1.282 051	2 450.442	477 836.2
731	1.367 989	2 296.504	419 686.1	781	1.280 410	2 453.584	479 062.2
732	1.366 120	2 299.646	420 835.2	782	1.278 772	2 456.725	480 289.8
733	1.364 268	2 302.787	421 985.8	783	1.277 139	2 459.867	481 519.0
734	1.362 398	2 305.929	423 138.0	784	1.275 510	2 463.009	482 749.7
735	1.360 544	2 309.071	424 291.7	785	1.273 885	2 466.150	483 982.0
736	1.358 696	2 312.212	425 447.0	786	1.272 266	2 469.292	485 215.8
737	1.356 852	2 315.354	426 603.9	787	1.270 648	2 472.433	486 451.3
738	1.355 014	2 318.495	427 762.4	788	1.269 035	2 475.575	487 688.5
739	1.353 180	2 321.637	428 922.4	789	1.267 427	2 478.717	488 926.9
740	1.351 351	2 324.779	430 084.0	790	1.265 823	2 481.858	490 167.0
741	1.349 528	2 327.920	431 247.2	791	1.264 223	2 485.000	491 408.7
742	1.347 709	2 331.062	432 412.0	792	1.262 626	2 488.141	492 652.0
743	1.345 885	2 334.203	433 578.3	793	1.261 034	2 491.283	493 896.8
744	1.344 065	2 337.345	434 746.2	794	1.259 446	2 494.425	495 143.3
745	1.342 242	2 340.487	435 915.8	795	1.257 862	2 497.566	496 391.3
746	1.340 423	2 343.628	437 086.6	796	1.256 281	2 500.708	497 640.8
747	1.338 608	2 346.770	438 259.2	797	1.254 705	2 503.849	498 892.0
748	1.336 788	2 349.911	439 433.4	798	1.253 133	2 506.991	500 144.7
749	1.335 113	2 353.053	440 609.2	799	1.251 564	2 510.133	501 399.0

Table 19 RECIPROCAL, CIRCUMFERENCES, AND AREAS OF CIRCLES (continued)

n	$1000/n$	Circum. of circle πn	Area of circle $\pi n^2/4$	n	$1000/n$	Circum. of circle πn	Area of circle $\pi n^2/4$
800	1.250 000	2 513.274	502 654.8	850	1.176 471	2 670.364	567 450.2
801	1.248 439	2 516.476	503 912.2	851	1.175 068	2 673.495	568 786.1
802	1.246 883	2 519.567	505 171.2	852	1.173 709	2 676.637	570 123.7
803	1.245 330	2 522.699	506 431.8	853	1.172 333	2 679.779	571 462.8
804	1.243 781	2 525.840	507 693.9	854	1.170 960	2 682.920	572 803.4
805	1.242 236	2 528.982	508 957.6	855	1.169 591	2 686.062	574 145.7
806	1.240 695	2 532.124	510 222.9	856	1.168 224	2 689.203	575 489.5
807	1.239 157	2 535.266	511 489.8	857	1.166 861	2 692.345	576 834.9
808	1.237 624	2 538.407	512 758.2	858	1.165 501	2 695.486	578 181.9
809	1.236 094	2 541.548	514 028.2	859	1.164 144	2 698.628	579 530.4
810	1.234 568	2 544.690	515 299.7	860	1.162 791	2 701.770	580 880.5
811	1.233 046	2 547.832	516 572.9	861	1.161 440	2 704.911	582 232.2
812	1.231 527	2 550.973	517 847.6	862	1.160 093	2 708.053	583 585.4
813	1.230 012	2 554.115	519 123.8	863	1.158 749	2 711.194	584 940.2
814	1.228 501	2 557.258	520 401.7	864	1.157 407	2 714.336	586 296.6
815	1.226 994	2 560.398	521 681.1	865	1.156 060	2 717.478	587 654.5
816	1.225 490	2 563.540	522 962.1	866	1.154 734	2 720.619	589 014.1
817	1.223 990	2 566.681	524 244.6	867	1.153 403	2 723.761	590 375.2
818	1.222 494	2 569.823	525 528.8	868	1.152 074	2 726.902	591 737.8
819	1.221 001	2 572.984	526 814.5	869	1.150 748	2 730.044	593 102.1
820	1.219 512	2 576.106	528 101.7	870	1.149 425	2 733.186	594 467.9
821	1.218 027	2 579.248	529 390.6	871	1.148 106	2 736.327	595 835.2
822	1.216 545	2 582.389	530 681.0	872	1.146 789	2 739.469	597 204.2
823	1.215 067	2 585.531	531 973.0	873	1.145 475	2 742.610	598 574.7
824	1.213 592	2 588.672	533 266.5	874	1.144 165	2 745.752	599 946.8
825	1.212 121	2 591.814	534 561.6	875	1.142 857	2 748.894	601 320.5
826	1.210 654	2 594.956	535 858.3	876	1.141 553	2 752.035	602 695.7
827	1.209 190	2 598.097	537 156.6	877	1.140 251	2 755.177	604 072.5
828	1.207 728	2 601.238	538 456.4	878	1.138 952	2 758.318	605 450.9
829	1.206 273	2 604.380	539 757.8	879	1.137 656	2 761.460	606 830.8
830	1.204 819	2 607.522	541 060.8	880	1.136 364	2 764.602	608 212.3
831	1.203 369	2 610.663	542 365.3	881	1.135 074	2 767.743	609 595.4
832	1.201 923	2 613.805	543 671.5	882	1.133 787	2 770.885	610 980.1
833	1.200 480	2 616.947	544 979.1	883	1.132 503	2 774.026	612 366.3
834	1.199 041	2 620.088	546 288.4	884	1.131 222	2 777.168	613 754.1
835	1.197 605	2 623.230	547 599.2	885	1.129 944	2 780.309	615 143.5
836	1.196 172	2 626.371	548 911.6	886	1.128 668	2 783.451	616 534.4
837	1.194 743	2 629.513	550 225.8	887	1.127 396	2 786.593	617 926.9
838	1.193 317	2 632.655	551 541.1	888	1.126 126	2 789.734	619 321.0
839	1.191 895	2 635.796	552 858.3	889	1.124 859	2 792.876	620 716.7
840	1.190 476	2 638.938	554 176.9	890	1.123 596	2 796.017	622 113.9
841	1.189 061	2 642.079	555 497.2	891	1.122 334	2 799.159	623 512.7
842	1.187 648	2 645.221	556 819.0	892	1.121 076	2 802.301	624 913.0
843	1.186 240	2 648.363	558 142.4	893	1.119 821	2 805.442	626 315.0
844	1.184 834	2 651.504	559 467.4	894	1.118 568	2 808.584	627 718.5
845	1.183 432	2 654.646	560 793.9	895	1.117 318	2 811.725	629 123.6
846	1.182 033	2 657.787	562 122.0	896	1.116 071	2 814.867	630 530.2
847	1.180 638	2 660.929	563 451.7	897	1.114 827	2 818.009	631 938.4
848	1.179 245	2 664.071	564 783.0	898	1.113 586	2 821.150	633 348.2
849	1.177 856	2 667.212	566 115.8	899	1.112 347	2 824.292	634 759.6

Table 19 RECIPROCAL, CIRCUMFERENCES, AND AREAS OF CIRCLES (continued)

n	$1000/n$	Circum. of circle πn	Area of circle $\pi n^2/4$	n	$1000/n$	Circum. of circle πn	Area of circle $\pi n^2/4$
900	1.111 111	2 627.433	636 172.5	950	1.052 632	2 984.513	708 821.8
901	1.109 878	2 630.575	637 537.0	951	1.051 525	2 987.655	710 314.9
902	1.108 647	2 633.717	639 003.1	952	1.050 420	2 990.796	711 809.5
903	1.107 420	2 636.858	640 420.7	953	1.049 318	2 993.938	713 305.7
904	1.106 195	2 640.000	641 839.9	954	1.048 218	2 997.079	714 803.4
905	1.104 972	2 643.141	643 260.7	955	1.047 120	3 000.221	716 302.8
906	1.103 753	2 646.283	644 683.1	956	1.046 025	3 003.362	717 803.7
907	1.102 536	2 649.425	646 107.0	957	1.044 932	3 006.504	719 306.1
908	1.101 322	2 652.566	647 532.5	958	1.043 841	3 009.646	720 810.2
909	1.100 110	2 655.708	648 959.6	959	1.042 753	3 012.787	722 315.8
910	1.098 901	2 658.849	650 388.2	960	1.041 667	3 015.929	723 822.9
911	1.097 695	2 661.991	651 818.4	961	1.040 583	3 019.071	725 331.7
912	1.096 491	2 665.133	653 250.2	962	1.039 501	3 022.212	726 842.0
913	1.095 290	2 668.274	654 683.6	963	1.038 422	3 025.354	728 353.9
914	1.094 092	2 671.416	656 118.5	964	1.037 344	3 028.496	729 867.4
915	1.092 896	2 674.557	657 555.0	965	1.036 269	3 031.637	731 382.4
916	1.091 703	2 677.699	658 993.0	966	1.035 197	3 034.779	732 899.0
917	1.090 513	2 680.840	660 432.7	967	1.034 126	3 037.920	734 417.2
918	1.089 325	2 683.982	661 873.9	968	1.033 058	3 041.062	735 936.9
919	1.088 139	2 687.124	663 316.7	969	1.031 992	3 044.203	737 458.2
920	1.086 957	2 690.265	664 761.0	970	1.030 928	3 047.345	738 981.1
921	1.085 778	2 693.407	666 206.9	971	1.029 866	3 050.486	740 505.6
922	1.084 599	2 696.548	667 654.4	972	1.028 807	3 053.628	742 031.6
923	1.083 424	2 699.690	669 103.5	973	1.027 749	3 056.770	743 559.2
924	1.082 251	2 702.832	670 554.1	974	1.026 694	3 059.911	745 088.4
925	1.081 081	2 705.973	672 006.3	975	1.025 641	3 063.053	746 619.1
926	1.079 914	2 709.115	673 460.1	976	1.024 590	3 066.194	748 151.4
927	1.078 749	2 712.256	674 915.4	977	1.023 541	3 069.336	749 685.3
928	1.077 585	2 715.398	676 372.3	978	1.022 495	3 072.478	751 220.8
929	1.076 426	2 718.540	677 830.8	979	1.021 450	3 075.619	752 757.8
930	1.075 269	2 721.681	679 290.9	980	1.020 408	3 078.761	754 296.4
931	1.074 114	2 724.823	680 752.5	981	1.019 368	3 081.902	755 836.6
932	1.072 961	2 727.964	682 215.7	982	1.018 330	3 085.044	757 378.3
933	1.071 811	2 731.106	683 680.5	983	1.017 294	3 088.186	758 921.5
934	1.070 664	2 734.248	685 146.8	984	1.016 260	3 091.327	760 466.5
935	1.069 519	2 737.389	686 614.7	985	1.015 228	3 094.469	762 012.9
936	1.068 376	2 740.531	688 084.2	986	1.014 199	3 097.610	763 561.0
937	1.067 236	2 743.672	689 555.2	987	1.013 171	3 100.752	765 110.5
938	1.066 098	2 746.814	691 027.9	988	1.012 146	3 103.894	766 661.7
939	1.064 963	2 749.956	692 502.1	989	1.011 122	3 107.035	768 214.4
940	1.063 830	2 753.097	693 977.8	990	1.010 101	3 110.177	769 768.7
941	1.062 699	2 756.239	695 455.2	991	1.009 082	3 113.318	771 324.6
942	1.061 571	2 759.380	696 934.1	992	1.008 065	3 116.460	772 882.1
943	1.060 445	2 762.522	698 414.5	993	1.007 049	3 119.602	774 441.1
944	1.059 322	2 765.663	699 896.6	994	1.006 036	3 122.743	776 001.7
945	1.058 201	2 768.805	701 380.2	995	1.005 026	3 125.885	777 563.8
946	1.057 082	2 771.947	702 865.4	996	1.004 016	3 129.026	779 127.5
947	1.055 966	2 775.088	704 352.1	997	1.003 009	3 132.168	780 692.8
948	1.054 852	2 778.230	705 840.5	998	1.002 004	3 135.309	782 259.7
949	1.053 741	2 781.371	707 330.4	999	1.001 001	3 138.451	783 828.2

STATISTICS AND PROBABILITY



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Table 20 LOGARITHMS OF FACTORIAL *n*

<i>n</i>	log <i>n!</i>	<i>n</i>	log <i>n!</i>	<i>n</i>	log <i>n!</i>	<i>n</i>	log <i>n!</i>
1	0.00000	50	64.48307	100	157.97000	150	262.75689
2	0.30103	51	66.19084	101	159.97432	151	264.93587
3	0.77815	52	67.90666	102	161.98293	152	267.11771
4	1.38021	53	69.63092	103	163.99576	153	269.30241
5	2.07918	54	71.36332	104	166.01260	154	271.48993
6	2.85733	55	73.10368	105	168.03399	155	273.68026
7	3.70243	56	74.85187	106	170.05929	156	275.87336
8	4.60552	57	76.60774	107	172.08867	157	278.06926
9	5.55976	58	78.37117	108	174.12210	158	280.26794
10	6.55976	59	80.14202	109	176.15952	159	282.46934
11	7.60110	60	81.92017	110	178.20092	160	284.67346
12	8.69304	61	83.70550	111	180.24624	161	286.88028
13	9.79428	62	85.49790	112	182.29546	162	289.08990
14	10.94041	63	87.29724	113	184.34854	163	291.30198
15	12.11650	64	89.10342	114	186.40544	164	293.51683
16	13.32052	65	90.91633	115	188.46614	165	295.73431
17	14.55107	66	92.73587	116	190.53060	166	297.95442
18	15.80834	67	94.56195	117	192.59878	167	300.17714
19	17.08506	68	96.39446	118	194.67067	168	302.40246
20	18.38612	69	98.23331	119	196.74621	169	304.63033
21	19.70834	70	100.07840	120	198.82539	170	306.86078
22	21.05077	71	101.92906	121	200.90818	171	309.09378
23	22.41249	72	103.78700	122	202.99454	172	311.32931
24	23.79271	73	105.65032	123	205.08444	173	313.56736
25	25.19065	74	107.51955	124	207.17787	174	315.80790
26	26.60562	75	109.39461	125	209.27478	175	318.05094
27	28.03698	76	111.27543	126	211.37515	176	320.29644
28	29.48414	77	113.16192	127	213.47895	177	322.54443
29	30.94664	78	115.05401	128	215.58618	178	324.79486
30	32.42366	79	116.95164	129	217.69675	179	327.04770
31	33.91502	80	118.85473	130	219.81069	180	329.30297
32	35.42017	81	120.76321	131	221.92796	181	331.56056
33	36.93869	82	122.67703	132	224.04854	182	333.82072
34	38.47016	83	124.59610	133	226.17239	183	336.08317
35	40.01423	84	126.52038	134	228.29949	184	338.34799
36	41.57054	85	128.44980	135	230.42983	185	340.61516
37	43.13874	86	130.38430	136	232.56337	186	342.88468
38	44.71852	87	132.32382	137	234.70009	187	345.15652
39	46.30969	88	134.26830	138	236.83997	188	347.43057
40	47.91165	89	136.21769	139	238.98298	189	349.70714
41	49.52443	90	138.17194	140	241.12911	190	351.98589
42	51.14768	91	140.13098	141	243.27833	191	354.26692
43	52.78116	92	142.09476	142	245.43062	192	356.55022
44	54.42460	93	144.06325	143	247.58695	193	358.83576
45	56.07761	94	146.03638	144	249.74632	194	361.12358
46	57.74057	95	148.01410	145	251.90868	195	363.41362
47	59.41267	96	149.99637	146	254.07504	196	365.70587
48	61.09391	97	151.98314	147	256.24535	197	368.00034
49	62.78410	98	153.97437	148	258.41972	198	370.29701
		99	155.97000	149	260.59800	199	372.59586

FACTORIALS AND THEIR RECIPROCAL

<i>n</i>	<i>n!</i>	<i>n</i>	<i>n!</i>	<i>n</i>	1/ <i>n!</i>	<i>n</i>	1/ <i>n!</i>
1	1	11	39916800	11	.25052 × 10 ⁻⁷		
2	2	12	479001600	12	.20877 × 10 ⁻⁸		
3	6	13	6227020800	13	.16559 × 10 ⁻⁹		
4	24	14	87178291200	14	.11471 × 10 ⁻¹⁰		
5	120	15	1307674368000	15	.76472 × 10 ⁻¹²		
6	720	16	20922783888000	16	.47795 × 10 ⁻¹³		
7	5040	17	355687428096000	17	.28115 × 10 ⁻¹⁴		
8	40320	18	6402373705728000	18	.15619 × 10 ⁻¹⁵		
9	362880	19	121645100408832000	19	.82206 × 10 ⁻¹⁷		
10	3628800	20	2432902008176640000	20	.41103 × 10 ⁻¹⁸		

Table 21 BINOMIAL COEFFICIENTS

n	$\binom{n}{0}$	$\binom{n}{1}$	$\binom{n}{2}$	$\binom{n}{3}$	$\binom{n}{4}$	$\binom{n}{5}$	$\binom{n}{6}$	$\binom{n}{7}$	$\binom{n}{8}$	$\binom{n}{9}$	$\binom{n}{10}$
0	1										
1	1	1									
2	1	2	1								
3	1	3	3	1							
4	1	4	6	4	1						
5	1	5	10	10	5	1					
6	1	6	15	20	15	6	1				
7	1	7	21	35	35	21	7	1			
8	1	8	28	56	70	56	28	8	1		
9	1	9	36	84	126	126	84	36	9	1	
10	1	10	45	120	210	252	210	120	45	10	1
11	1	11	55	165	330	462	462	330	165	55	11
12	1	12	66	220	495	792	924	792	495	220	66
13	1	13	78	286	715	1287	1716	1716	1287	715	286
14	1	14	91	364	1001	2002	3003	3432	3003	2002	1001
15	1	15	105	455	1365	3003	5005	6435	6435	5005	3003
16	1	16	120	560	1820	4368	8008	11440	12870	11440	8008
17	1	17	136	680	2380	6188	12876	19448	24310	24310	19448
18	1	18	153	816	3060	8568	18564	31824	43758	43758	31824
19	1	19	171	969	3876	11628	27132	50368	75562	92376	92376
20	1	20	190	1140	4845	15504	38760	77520	125970	167960	184756

$${}^nC_m = \binom{n}{m} = \frac{n!}{[(n-m)!m!]} = \binom{n}{n-m} \quad \binom{n}{0} = 1$$

$$(p + q)^n = p^n + \binom{n}{1} p^{n-1} q + \dots + \binom{n}{s} p^s q^t + \dots + q^n \quad s + t = n$$

PROBABILITY

Let p be the probability that an event E will happen in a single trial and $q = 1 - p$ the probability that the event will fail in a single trial. The probability that E will happen

exactly r times in n trials is $\binom{n}{r} p^r q^{n-r}$. The probability that E will occur at least r times in

n trials is $\sum_{i=r}^n \binom{n}{i} p^i q^{n-i}$; at most r times in n trials is $\sum_{i=0}^r \binom{n}{i} p^i q^{n-i}$.

In a *binomial (Bernoulli) distribution* $(q + p)^n$, the mean number \bar{x} of favorable events is np ; the mean number of unfavorable events is nq , where $q + p = 1$; the *standard deviation* is $\sigma = \sqrt{pqn}$.

If a variable y is *normally distributed (Gaussian distribution)*, the probability P that y will lie between $y = y_1$ and $y = y_2$ is

$$P = \int_{y_1}^{y_2} p(y) dy \quad \text{where } p(y) = \frac{1}{\sigma\sqrt{2\pi}} e^{-(y-a)^2/2\sigma^2}$$

a is the *mean* of the distribution, and σ is the *standard deviation (root mean square)* of the distribution. If $x = (y - a)/\sigma$, $P = \int_{x_1}^{x_2} \Phi(x) dx$, where $\Phi(x) = \frac{1}{\sqrt{2\pi}} e^{-x^2/2}$ is the *normal function*, and x_1 is the value of x when $y = y_1$.

For further details see Richard S. Burington and Donald C. May, *Handbook of Probability and Statistics with Tables*, McGraw-Hill Book Company, New York.

Table 22 PROBABILITY FUNCTIONS
(NORMAL DISTRIBUTION)

x	$\frac{1}{2}(1+\alpha)$	$\Phi(x)$	$\Phi^{(2)}(x)$	$\Phi^{(3)}(x)$	$\Phi^{(4)}(x)$	x	$\frac{1}{2}(1+\alpha)$	$\Phi(x)$	$\Phi^{(2)}(x)$	$\Phi^{(3)}(x)$	$\Phi^{(4)}(x)$
0.00	.5000	.3989	-.3989	.0000	1.1968	0.50	.5015	.3521	-.2641	.4641	.5501
0.01	.5040	.3989	-.3989	.0120	1.1965	0.51	.5050	.3503	-.2592	.4695	.5279
0.02	.5080	.3989	-.3987	.0239	1.1956	0.52	.5085	.3485	-.2543	.4747	.5058
0.03	.5120	.3988	-.3984	.0359	1.1941	0.53	.5119	.3467	-.2493	.4796	.4831
0.04	.5160	.3986	-.3980	.0478	1.1920	0.54	.5154	.3448	-.2443	.4843	.4605
0.05	.5199	.3984	-.3975	.0597	1.1894	0.55	.5188	.3429	-.2392	.4888	.4378
0.06	.5238	.3982	-.3968	.0716	1.1861	0.56	.5223	.3410	-.2341	.4931	.4150
0.07	.5277	.3980	-.3960	.0834	1.1822	0.57	.5257	.3391	-.2289	.4971	.3921
0.08	.5315	.3977	-.3951	.0952	1.1778	0.58	.5290	.3372	-.2238	.5009	.3691
0.09	.5353	.3973	-.3941	.1070	1.1727	0.59	.5324	.3352	-.2185	.5045	.3461
0.10	.5391	.3970	-.3930	.1187	1.1671	0.60	.5357	.3332	-.2133	.5078	.3231
0.11	.5428	.3965	-.3917	.1303	1.1609	0.61	.5391	.3312	-.2080	.5109	.3000
0.12	.5465	.3961	-.3904	.1419	1.1541	0.62	.5424	.3292	-.2027	.5138	.2770
0.13	.5502	.3956	-.3889	.1534	1.1468	0.63	.5457	.3271	-.1973	.5165	.2538
0.14	.5538	.3951	-.3873	.1648	1.1389	0.64	.5489	.3251	-.1919	.5190	.2306
0.15	.5574	.3945	-.3855	.1762	1.1304	0.65	.5522	.3230	-.1865	.5211	.2074
0.16	.5609	.3938	-.3838	.1875	1.1214	0.66	.5554	.3209	-.1811	.5231	.1842
0.17	.5644	.3932	-.3819	.1988	1.1118	0.67	.5586	.3187	-.1757	.5248	.1610
0.18	.5679	.3925	-.3798	.2099	1.1017	0.68	.5617	.3166	-.1702	.5263	.1379
0.19	.5713	.3918	-.3777	.2206	1.0911	0.69	.5649	.3144	-.1647	.5276	.1148
0.20	.5747	.3910	-.3754	.2315	1.0799	0.70	.5680	.3123	-.1593	.5286	.0917
0.21	.5781	.3902	-.3730	.2422	1.0682	0.71	.5711	.3101	-.1538	.5293	.0686
0.22	.5814	.3894	-.3706	.2526	1.0560	0.72	.5742	.3079	-.1483	.5298	.0454
0.23	.5847	.3886	-.3680	.2634	1.0434	0.73	.5773	.3056	-.1428	.5299	.0223
0.24	.5880	.3876	-.3653	.2737	1.0302	0.74	.5804	.3034	-.1373	.5298	.0000
0.25	.5912	.3867	-.3625	.2840	1.0165	0.75	.5834	.3011	-.1318	.5294	-.0176
0.26	.5944	.3857	-.3598	.2941	1.0024	0.76	.5864	.2988	-.1262	.5288	-.0352
0.27	.5975	.3847	-.3568	.3040	0.9878	0.77	.5894	.2966	-.1207	.5279	-.0528
0.28	.6006	.3836	-.3538	.3138	0.9727	0.78	.5923	.2943	-.1153	.5268	-.0704
0.29	.6036	.3825	-.3504	.3235	0.9572	0.79	.5952	.2920	-.1098	.5254	-.0880
0.30	.6065	.3814	-.3471	.3330	0.9413	0.80	.5981	.2897	-.1043	.5238	-.1056
0.31	.6094	.3802	-.3437	.3423	0.9250	0.81	.6010	.2874	-.0988	.5219	-.1232
0.32	.6122	.3790	-.3402	.3515	0.9082	0.82	.6038	.2850	-.0934	.5198	-.1408
0.33	.6150	.3778	-.3367	.3606	0.8910	0.83	.6066	.2827	-.0880	.5175	-.1584
0.34	.6177	.3765	-.3330	.3693	0.8735	0.84	.6094	.2803	-.0825	.5150	-.1760
0.35	.6204	.3752	-.3293	.3779	0.8556	0.85	.6122	.2780	-.0771	.5124	-.1936
0.36	.6231	.3739	-.3255	.3864	0.8373	0.86	.6150	.2756	-.0716	.5096	-.2112
0.37	.6257	.3725	-.3216	.3947	0.8186	0.87	.6177	.2732	-.0664	.5066	-.2288
0.38	.6283	.3712	-.3175	.4028	0.7996	0.88	.6204	.2709	-.0611	.5034	-.2464
0.39	.6309	.3697	-.3135	.4107	0.7803	0.89	.6231	.2685	-.0558	.5000	-.2640
0.40	.6334	.3683	-.3094	.4184	0.7607	0.90	.6257	.2661	-.0505	.4964	-.2816
0.41	.6359	.3668	-.3053	.4259	0.7408	0.91	.6283	.2637	-.0453	.4926	-.2992
0.42	.6384	.3653	-.3008	.4332	0.7206	0.92	.6309	.2613	-.0401	.4887	-.3168
0.43	.6408	.3637	-.2965	.4403	0.7001	0.93	.6334	.2589	-.0350	.4846	-.3344
0.44	.6432	.3621	-.2920	.4472	0.6793	0.94	.6359	.2565	-.0299	.4804	-.3520
0.45	.6455	.3605	-.2875	.4539	0.6583	0.95	.6384	.2541	-.0248	.4760	-.3696
0.46	.6478	.3589	-.2830	.4603	0.6371	0.96	.6408	.2516	-.0197	.4715	-.3872
0.47	.6501	.3572	-.2783	.4665	0.6158	0.97	.6432	.2492	-.0147	.4669	-.4048
0.48	.6524	.3555	-.2736	.4727	0.5940	0.98	.6455	.2468	-.0098	.4622	-.4224
0.49	.6547	.3538	-.2688	.4785	0.5721	0.99	.6478	.2444	-.0049	.4574	-.4400

$$\frac{1}{2}(1 + \alpha) = \int_{-\infty}^x \Phi(x) dx = \text{area under } \Phi(x) \text{ from } -\infty \text{ to } x$$

$$\alpha = \int_{-x}^x \Phi(x) dx \quad \Phi(x) = \frac{1}{\sqrt{2\pi}} e^{-x^2/2} = \text{normal function}$$

$$\Phi^{(2)}(x) = (x^2 - 1) \Phi(x) = \text{second derivative of } \Phi(x)$$

$$\Phi^{(3)}(x) = (3x - x^3) \Phi(x) = \text{third derivative of } \Phi(x)$$

$$\Phi^{(4)}(x) = (x^4 - 6x^2 + 3) \Phi(x) = \text{fourth derivative of } \Phi(x)$$

Table 22 PROBABILITY FUNCTIONS (NORMAL DISTRIBUTION) (continued) 359

x	$\frac{1}{2}(1+\alpha)$	$\Phi(x)$	$\Phi^{(2)}(x)$	$\Phi^{(3)}(x)$	$\Phi^{(4)}(x)$	x	$\frac{1}{2}(1+\alpha)$	$\Phi(x)$	$\Phi^{(2)}(x)$	$\Phi^{(3)}(x)$	$\Phi^{(4)}(x)$
1.00	.8413	.2420	.0000	.4839	-.4839	1.80	.9332	.1295	.1819	.1457	-.7043
1.01	.8438	.2396	.0048	.4790	-.4983	1.81	.9345	.1276	.1833	.1387	-.6994
1.02	.8461	.2371	.0096	.4740	-.5122	1.82	.9357	.1257	.1847	.1317	-.6942
1.03	.8485	.2347	.0143	.4688	-.5257	1.83	.9370	.1238	.1860	.1248	-.6888
1.04	.8508	.2323	.0190	.4635	-.5389	1.84	.9382	.1219	.1872	.1180	-.6831
1.05	.8531	.2299	.0236	.4580	-.5518	1.85	.9394	.1200	.1883	.1111	-.6772
1.06	.8554	.2275	.0281	.4524	-.5639	1.86	.9405	.1182	.1894	.1044	-.6710
1.07	.8577	.2251	.0326	.4467	-.5759	1.87	.9418	.1163	.1704	.0977	-.6646
1.08	.8599	.2227	.0371	.4409	-.5873	1.88	.9429	.1145	.1714	.0911	-.6580
1.09	.8621	.2203	.0414	.4350	-.5984	1.89	.9441	.1127	.1722	.0846	-.6511
1.10	.8643	.2179	.0458	.4290	-.6091	1.90	.9452	.1109	.1730	.0781	-.6441
1.11	.8665	.2155	.0500	.4228	-.6193	1.91	.9463	.1092	.1738	.0717	-.6368
1.12	.8686	.2131	.0542	.4166	-.6292	1.92	.9474	.1074	.1746	.0654	-.6293
1.13	.8708	.2107	.0583	.4102	-.6388	1.93	.9484	.1057	.1751	.0591	-.6216
1.14	.8729	.2083	.0624	.4038	-.6476	1.94	.9495	.1040	.1757	.0529	-.6138
1.15	.8749	.2059	.0664	.3973	-.6561	1.95	.9505	.1023	.1762	.0468	-.6057
1.16	.8770	.2035	.0704	.3907	-.6643	1.96	.9515	.1006	.1766	.0408	-.5974
1.17	.8790	.2012	.0742	.3840	-.6720	1.97	.9525	.0989	.1770	.0349	-.5889
1.18	.8810	.1989	.0780	.3772	-.6792	1.98	.9535	.0973	.1773	.0290	-.5805
1.19	.8830	.1965	.0818	.3704	-.6861	1.99	.9545	.0957	.1776	.0233	-.5720
1.20	.8849	.1942	.0854	.3635	-.6928	1.70	.9554	.0940	.1778	.0176	-.5632
1.21	.8869	.1919	.0890	.3566	-.6986	1.71	.9563	.0925	.1779	.0120	-.5542
1.22	.8888	.1895	.0926	.3496	-.7042	1.72	.9573	.0909	.1780	.0065	-.5452
1.23	.8907	.1872	.0960	.3425	-.7094	1.73	.9582	.0893	.1780	.0011	-.5360
1.24	.8925	.1849	.0994	.3354	-.7141	1.74	.9591	.0878	.1780	-.0042	-.5267
1.25	.8944	.1826	.1027	.3282	-.7185	1.75	.9599	.0863	.1780	-.0094	-.5173
1.26	.8962	.1804	.1060	.3210	-.7224	1.76	.9608	.0848	.1778	-.0146	-.5079
1.27	.8980	.1781	.1092	.3138	-.7259	1.77	.9616	.0833	.1777	-.0198	-.4983
1.28	.8997	.1758	.1123	.3065	-.7291	1.78	.9625	.0818	.1774	-.0245	-.4887
1.29	.9015	.1736	.1153	.2992	-.7318	1.79	.9633	.0804	.1772	-.0294	-.4789
1.30	.9032	.1714	.1182	.2918	-.7341	1.80	.9641	.0790	.1769	-.0341	-.4692
1.31	.9049	.1691	.1211	.2845	-.7361	1.81	.9649	.0775	.1765	-.0388	-.4593
1.32	.9066	.1669	.1239	.2771	-.7376	1.82	.9656	.0761	.1761	-.0433	-.4494
1.33	.9082	.1647	.1267	.2697	-.7388	1.83	.9664	.0748	.1756	-.0477	-.4395
1.34	.9099	.1626	.1293	.2624	-.7395	1.84	.9671	.0734	.1751	-.0521	-.4295
1.35	.9115	.1604	.1319	.2550	-.7399	1.85	.9678	.0721	.1746	-.0563	-.4195
1.36	.9131	.1582	.1344	.2476	-.7400	1.86	.9686	.0707	.1740	-.0605	-.4095
1.37	.9147	.1561	.1369	.2402	-.7398	1.87	.9693	.0694	.1734	-.0645	-.3995
1.38	.9162	.1539	.1392	.2328	-.7393	1.88	.9699	.0681	.1727	-.0685	-.3894
1.39	.9177	.1518	.1415	.2254	-.7378	1.89	.9706	.0669	.1720	-.0723	-.3793
1.40	.9192	.1497	.1437	.2180	-.7364	1.90	.9713	.0656	.1713	-.0761	-.3693
1.41	.9207	.1476	.1459	.2107	-.7347	1.91	.9719	.0644	.1705	-.0797	-.3592
1.42	.9222	.1456	.1480	.2033	-.7326	1.92	.9726	.0632	.1697	-.0832	-.3492
1.43	.9237	.1435	.1500	.1960	-.7301	1.93	.9732	.0620	.1688	-.0867	-.3392
1.44	.9251	.1415	.1519	.1887	-.7274	1.94	.9738	.0608	.1679	-.0900	-.3292
1.45	.9265	.1394	.1537	.1815	-.7243	1.95	.9744	.0596	.1670	-.0933	-.3192
1.46	.9279	.1374	.1555	.1742	-.7209	1.96	.9750	.0584	.1661	-.0964	-.3093
1.47	.9292	.1354	.1572	.1670	-.7172	1.97	.9756	.0573	.1651	-.0994	-.2994
1.48	.9306	.1334	.1588	.1599	-.7132	1.98	.9761	.0562	.1641	-.1024	-.2895
1.49	.9319	.1315	.1604	.1528	-.7089	1.99	.9767	.0551	.1630	-.1052	-.2797

The sum of those terms of $(p + q)^n = \sum_{t=0}^n \binom{n}{t} p^{n-t} q^t$, $p + q = 1$, in which t ranges from a to b inclusive, a and b being integers ($a \leq t \leq b$), is (if n is large enough) approximately

$$\int_{x_1}^{x_2} \Phi(x) dx + \left[\frac{q-p}{6\sigma} \Phi^{(2)}(x) + \frac{1}{2!} \left(\frac{1}{\sigma^2} - \frac{6}{n} \right) \Phi^{(3)}(x) \right]_{x_1}^{x_2}$$

where $x_1 = (a - \frac{1}{2} - nq)/\sigma$ $x_2 = (b + \frac{1}{2} - nq)/\sigma$ $\sigma = \sqrt{npq}$

Table 22 PROBABILITY FUNCTIONS (NORMAL DISTRIBUTION) (continued) 360

x	$\frac{1}{2}(1+\alpha)$	$\Phi(x)$	$\Phi^{(2)}(x)$	$\Phi^{(3)}(x)$	$\Phi^{(4)}(x)$	x	$\frac{1}{2}(1+\alpha)$	$\Phi(x)$	$\Phi^{(2)}(x)$	$\Phi^{(3)}(x)$	$\Phi^{(4)}(x)$
2.00	.9772	.0540	.1620	-.1080	-.2700	2.50	.9938	.0175	-.0920	-.1424	-.6860
2.01	.9778	.0529	.1609	-.1105	-.2603	2.51	.9940	.0171	-.0906	-.1416	-.6836
2.02	.9783	.0519	.1598	-.1132	-.2506	2.52	.9941	.0167	-.0892	-.1408	-.6811
2.03	.9788	.0508	.1586	-.1157	-.2411	2.53	.9943	.0163	-.0878	-.1399	-.6785
2.04	.9793	.0498	.1575	-.1180	-.2316	2.54	.9945	.0158	-.0864	-.1389	-.6757
2.05	.9798	.0488	.1563	-.1203	-.2222	2.55	.9946	.0154	-.0850	-.1380	-.6728
2.06	.9803	.0478	.1550	-.1225	-.2129	2.56	.9948	.0151	-.0836	-.1370	-.6698
2.07	.9808	.0468	.1538	-.1245	-.2036	2.57	.9949	.0147	-.0823	-.1360	-.6667
2.08	.9812	.0458	.1526	-.1265	-.1945	2.58	.9951	.0143	-.0809	-.1350	-.6635
2.09	.9817	.0449	.1513	-.1284	-.1854	2.59	.9952	.0139	-.0796	-.1339	-.6602
2.10	.9821	.0440	.1500	-.1302	-.1765	2.60	.9953	.0136	-.0782	-.1328	-.6568
2.11	.9826	.0431	.1487	-.1320	-.1676	2.61	.9955	.0132	-.0769	-.1317	-.6533
2.12	.9830	.0422	.1474	-.1336	-.1588	2.62	.9956	.0129	-.0756	-.1305	-.6497
2.13	.9834	.0413	.1460	-.1351	-.1502	2.63	.9957	.0126	-.0743	-.1294	-.6460
2.14	.9838	.0404	.1446	-.1366	-.1416	2.64	.9959	.0122	-.0730	-.1282	-.6422
2.15	.9842	.0395	.1433	-.1380	-.1332	2.65	.9960	.0119	-.0717	-.1270	-.6383
2.16	.9846	.0387	.1419	-.1393	-.1249	2.66	.9961	.0116	-.0705	-.1258	-.6343
2.17	.9850	.0378	.1405	-.1405	-.1167	2.67	.9962	.0112	-.0692	-.1246	-.6302
2.18	.9854	.0371	.1391	-.1416	-.1086	2.68	.9963	.0110	-.0680	-.1233	-.6260
2.19	.9857	.0363	.1377	-.1426	-.1006	2.69	.9964	.0107	-.0668	-.1220	-.6217
2.20	.9861	.0355	.1362	-.1436	-.0927	2.70	.9965	.0104	-.0656	-.1207	-.6173
2.21	.9864	.0347	.1348	-.1445	-.0850	2.71	.9966	.0101	-.0644	-.1194	-.6128
2.22	.9868	.0339	.1333	-.1453	-.0774	2.72	.9967	.0099	-.0632	-.1181	-.6082
2.23	.9871	.0332	.1319	-.1460	-.0700	2.73	.9968	.0096	-.0620	-.1168	-.6035
2.24	.9875	.0325	.1304	-.1467	-.0626	2.74	.9969	.0093	-.0608	-.1154	-.5987
2.25	.9878	.0317	.1289	-.1473	-.0554	2.75	.9970	.0091	-.0597	-.1141	-.5938
2.26	.9881	.0310	.1275	-.1478	-.0484	2.76	.9971	.0088	-.0585	-.1127	-.5888
2.27	.9884	.0303	.1260	-.1483	-.0414	2.77	.9972	.0086	-.0574	-.1114	-.5837
2.28	.9887	.0297	.1245	-.1486	-.0346	2.78	.9973	.0084	-.0563	-.1100	-.5785
2.29	.9890	.0290	.1230	-.1490	-.0279	2.79	.9974	.0081	-.0552	-.1087	-.5732
2.30	.9893	.0283	.1215	-.1492	-.0214	2.80	.9974	.0079	-.0541	-.1073	-.5678
2.31	.9895	.0277	.1200	-.1494	-.0150	2.81	.9975	.0077	-.0531	-.1059	-.5623
2.32	.9898	.0270	.1185	-.1496	-.0086	2.82	.9976	.0075	-.0520	-.1045	-.5567
2.33	.9901	.0264	.1170	-.1498	-.0027	2.83	.9977	.0073	-.0510	-.1031	-.5510
2.34	.9904	.0258	.1155	-.1496	-.0033	2.84	.9977	.0071	-.0500	-.1017	-.5452
2.35	.9906	.0252	.1141	-.1495	-.0092	2.85	.9978	.0069	-.0490	-.1003	-.5393
2.36	.9909	.0246	.1126	-.1494	-.0149	2.86	.9979	.0067	-.0480	-.0989	-.5333
2.37	.9911	.0241	.1111	-.1492	-.0204	2.87	.9979	.0065	-.0470	-.0976	-.5272
2.38	.9913	.0235	.1096	-.1490	-.0258	2.88	.9980	.0063	-.0460	-.0962	-.5210
2.39	.9916	.0229	.1081	-.1487	-.0311	2.89	.9981	.0061	-.0451	-.0948	-.5147
2.40	.9918	.0224	.1066	-.1483	-.0362	2.90	.9981	.0060	-.0441	-.0934	-.5083
2.41	.9920	.0219	.1051	-.1480	-.0412	2.91	.9982	.0058	-.0432	-.0920	-.5018
2.42	.9922	.0213	.1036	-.1475	-.0461	2.92	.9982	.0056	-.0423	-.0906	-.4952
2.43	.9925	.0208	.1022	-.1470	-.0508	2.93	.9983	.0055	-.0414	-.0893	-.4885
2.44	.9927	.0203	.1007	-.1465	-.0554	2.94	.9984	.0053	-.0405	-.0879	-.4817
2.45	.9929	.0198	.0992	-.1459	-.0598	2.95	.9984	.0051	-.0396	-.0865	-.4748
2.46	.9931	.0194	.0978	-.1453	-.0641	2.96	.9985	.0050	-.0388	-.0852	-.4678
2.47	.9932	.0189	.0963	-.1446	-.0683	2.97	.9985	.0048	-.0379	-.0838	-.4607
2.48	.9934	.0184	.0949	-.1439	-.0723	2.98	.9986	.0047	-.0371	-.0825	-.4535
2.49	.9936	.0180	.0935	-.1432	-.0762	2.99	.9986	.0046	-.0363	-.0811	-.4462

The sum of the first $(t + 1)$ terms of $(p + q)^n \equiv \sum_{t=0}^n \binom{n}{t} p^{n-t} q^t$, $p + q = 1$, is approximately

$$\int_x^\infty \Phi(x) dx + \frac{q-p}{6\sigma} \Phi^{(2)}(x) - \frac{1}{24} \left(\frac{1}{\sigma^2} - \frac{6}{n} \right) \Phi^{(3)}(x)$$

where $x = (s - \frac{1}{2} - np)/\sigma$, $s = n - t$. The sum of the last $(s + 1)$ terms is approximately

$$\int_x^\infty \Phi(x) dx - \frac{q-p}{6\sigma} \Phi^{(2)}(x) - \frac{1}{24} \left(\frac{1}{\sigma^2} - \frac{6}{n} \right) \Phi^{(3)}(x)$$

where $x = (t - \frac{1}{2} - nq)/\sigma$, $t = n - s$, $\sigma = \sqrt{npq}$

Table 22 PROBABILITY FUNCTIONS (NORMAL DISTRIBUTION) (continued) 361

x	$\frac{1}{2}(1+\alpha)$	$\Phi(x)$	$\phi^{(2)}(x)$	$\phi^{(3)}(x)$	$\phi^{(4)}(x)$	x	$\frac{1}{2}(1+\alpha)$	$\Phi(x)$	$\phi^{(2)}(x)$	$\phi^{(3)}(x)$	$\phi^{(4)}(x)$
3.00	.9987	.0044	.0355	-.0798	.1330	3.50	.9998	.0009	.0008	-.0283	.0694
3.01	.9987	.0043	.0347	-.0785	.1321	3.51	.9998	.0008	.0009	-.0276	.0681
3.02	.9987	.0042	.0339	-.0771	.1313	3.52	.9998	.0008	.0009	-.0269	.0669
3.03	.9988	.0040	.0331	-.0758	.1304	3.53	.9998	.0008	.0009	-.0262	.0656
3.04	.9988	.0039	.0324	-.0745	.1294	3.54	.9998	.0008	.0007	-.0255	.0643
3.05	.9989	.0038	.0316	-.0732	.1285	3.55	.9998	.0007	.0008	-.0249	.0631
3.06	.9989	.0037	.0309	-.0720	.1275	3.56	.9998	.0007	.0008	-.0243	.0618
3.07	.9989	.0036	.0302	-.0707	.1264	3.57	.9998	.0007	.0008	-.0237	.0606
3.08	.9990	.0035	.0295	-.0694	.1254	3.58	.9998	.0007	.0007	-.0231	.0594
3.09	.9990	.0034	.0288	-.0682	.1243	3.59	.9998	.0006	.0007	-.0225	.0582
3.10	.9990	.0033	.0281	-.0669	.1231	3.60	.9998	.0006	.0007	-.0219	.0570
3.11	.9991	.0032	.0275	-.0657	.1220	3.61	.9998	.0006	.0007	-.0214	.0559
3.12	.9991	.0031	.0268	-.0645	.1208	3.62	.9999	.0006	.0006	-.0208	.0547
3.13	.9991	.0030	.0262	-.0633	.1196	3.63	.9999	.0005	.0006	-.0203	.0536
3.14	.9992	.0029	.0256	-.0621	.1184	3.64	.9999	.0005	.0006	-.0198	.0524
3.15	.9992	.0028	.0249	-.0609	.1171	3.65	.9999	.0005	.0006	-.0192	.0513
3.16	.9992	.0027	.0243	-.0598	.1159	3.66	.9999	.0005	.0006	-.0187	.0502
3.17	.9992	.0026	.0237	-.0586	.1146	3.67	.9999	.0005	.0005	-.0182	.0492
3.18	.9993	.0025	.0232	-.0575	.1133	3.68	.9999	.0005	.0005	-.0177	.0481
3.19	.9993	.0025	.0226	-.0564	.1120	3.69	.9999	.0004	.0005	-.0173	.0470
3.20	.9993	.0024	.0220	-.0552	.1107	3.70	.9999	.0004	.0005	-.0168	.0460
3.21	.9993	.0023	.0215	-.0541	.1093	3.71	.9999	.0004	.0005	-.0164	.0450
3.22	.9994	.0022	.0210	-.0531	.1080	3.72	.9999	.0004	.0005	-.0159	.0440
3.23	.9994	.0022	.0204	-.0520	.1066	3.73	.9999	.0004	.0004	-.0155	.0430
3.24	.9994	.0021	.0199	-.0509	.1053	3.74	.9999	.0004	.0004	-.0150	.0420
3.25	.9994	.0020	.0194	-.0499	.1039	3.75	.9999	.0004	.0004	-.0146	.0410
3.26	.9994	.0020	.0189	-.0488	.1025	3.76	.9999	.0003	.0004	-.0142	.0401
3.27	.9995	.0019	.0184	-.0478	.1011	3.77	.9999	.0003	.0004	-.0138	.0392
3.28	.9995	.0018	.0180	-.0468	.0997	3.78	.9999	.0003	.0004	-.0134	.0382
3.29	.9995	.0018	.0175	-.0458	.0983	3.79	.9999	.0003	.0004	-.0131	.0373
3.30	.9995	.0017	.0170	-.0448	.0969	3.80	.9999	.0003	.0003	-.0127	.0365
3.31	.9995	.0017	.0166	-.0439	.0955	3.81	.9999	.0003	.0003	-.0123	.0356
3.32	.9996	.0016	.0162	-.0429	.0941	3.82	.9999	.0003	.0003	-.0120	.0347
3.33	.9996	.0016	.0157	-.0420	.0927	3.83	.9999	.0003	.0003	-.0116	.0339
3.34	.9996	.0015	.0153	-.0411	.0913	3.84	.9999	.0003	.0003	-.0113	.0331
3.35	.9996	.0015	.0149	-.0402	.0899	3.85	.9999	.0002	.0003	-.0110	.0323
3.36	.9996	.0014	.0145	-.0393	.0885	3.86	.9999	.0002	.0003	-.0107	.0315
3.37	.9996	.0014	.0141	-.0384	.0871	3.87	.9999	.0002	.0003	-.0104	.0307
3.38	.9996	.0013	.0138	-.0376	.0857	3.88	.9999	.0002	.0003	-.0100	.0299
3.39	.9997	.0013	.0134	-.0367	.0843	3.89	.9999	.0002	.0003	-.0098	.0292
3.40	.9997	.0012	.0130	-.0359	.0829	3.90	1.0000	.0002	.0002	-.0095	.0284
3.41	.9997	.0012	.0127	-.0350	.0815	3.91	1.0000	.0002	.0002	-.0092	.0277
3.42	.9997	.0012	.0123	-.0342	.0801	3.92	1.0000	.0002	.0002	-.0089	.0270
3.43	.9997	.0011	.0120	-.0334	.0788	3.93	1.0000	.0002	.0002	-.0086	.0263
3.44	.9997	.0011	.0116	-.0327	.0774	3.94	1.0000	.0002	.0002	-.0084	.0256
3.45	.9997	.0010	.0113	-.0319	.0761	3.95	1.0000	.0002	.0002	-.0081	.0250
3.46	.9997	.0010	.0110	-.0311	.0747	3.96	1.0000	.0002	.0002	-.0079	.0243
3.47	.9997	.0010	.0107	-.0304	.0734	3.97	1.0000	.0002	.0002	-.0076	.0237
3.48	.9997	.0009	.0104	-.0297	.0721	3.98	1.0000	.0001	.0002	-.0074	.0230
3.49	.9998	.0009	.0101	-.0290	.0707	3.99	1.0000	.0001	.0002	-.0072	.0224

4.00	1.0000	.0001	.0020	-.0070	.0218	4.50	1.0000	.0000	.0003	-.0012	.0047
4.05	1.0000	.0001	.0017	-.0059	.0190	4.55	1.0000	.0000	.0003	-.0010	.0039
4.10	1.0000	.0001	.0014	-.0051	.0165	4.60	1.0000	.0000	.0002	-.0009	.0033
4.15	1.0000	.0001	.0012	-.0043	.0143	4.65	1.0000	.0000	.0002	-.0007	.0027
4.20	1.0000	.0001	.0010	-.0038	.0123	4.70	1.0000	.0000	.0001	-.0006	.0023
4.25	1.0000	.0000	.0008	-.0031	.0105	4.75	1.0000	.0000	.0001	-.0005	.0019
4.30	1.0000	.0000	.0007	-.0026	.0090	4.80	1.0000	.0000	.0001	-.0004	.0016
4.35	1.0000	.0000	.0006	-.0022	.0077	4.85	1.0000	.0000	.0001	-.0003	.0013
4.40	1.0000	.0000	.0005	-.0018	.0065	4.90	1.0000	.0000	.0001	-.0003	.0011
4.45	1.0000	.0000	.0004	-.0015	.0055	4.95	1.0000	.0000	.0000	-.0002	.0009

Table 23 FACTORS FOR COMPUTING PROBABLE ERRORS

n	$\frac{1}{\sqrt{n}}$	$\frac{1}{\sqrt{n(n-1)}}$	$\frac{.6745}{\sqrt{n-1}}$	$\frac{.6745}{\sqrt{n(n-1)}}$	$\frac{.8453}{n\sqrt{n-1}}$	$\frac{.8453}{\sqrt{n(n-1)}}$
	2	.707 107	.707 107	.6745	.4769	.4227
3	.577 350	.408 248	.4769	.2754	.1993	.3451
4	.500 000	.288 675	.3894	.1947	.1220	.2440
5	.447 214	.223 607	.3372	.1508	.0845	.1890
6	.408 248	.182 574	.3016	.1231	.0630	.1543
7	.377 964	.154 303	.2754	.1041	.0493	.1304
8	.353 563	.133 631	.2549	.0901	.0399	.1130
9	.333 333	.117 851	.2385	.0795	.0332	.0996
10	.316 228	.105 409	.2248	.0711	.0282	.0891
11	.301 511	.095 345	.2133	.0643	.0243	.0806
12	.288 675	.087 039	.2034	.0587	.0212	.0736
13	.277 350	.080 054	.1947	.0540	.0188	.0677
14	.267 261	.074 125	.1871	.0500	.0167	.0627
15	.258 199	.069 007	.1803	.0465	.0151	.0583
16	.250 000	.064 550	.1742	.0435	.0136	.0545
17	.242 535	.060 634	.1686	.0409	.0124	.0513
18	.235 702	.057 166	.1635	.0386	.0114	.0483
19	.229 416	.054 074	.1589	.0365	.0105	.0457
20	.223 607	.051 299	.1547	.0345	.0097	.0434
21	.218 218	.048 795	.1508	.0329	.0090	.0412
22	.213 201	.046 524	.1472	.0314	.0084	.0393
23	.208 514	.044 455	.1438	.0300	.0078	.0375
24	.204 124	.042 563	.1406	.0287	.0073	.0360
25	.200 000	.040 825	.1377	.0275	.0069	.0345
26	.195 116	.039 223	.1349	.0265	.0065	.0332
27	.192 450	.037 743	.1323	.0255	.0061	.0319
28	.188 982	.036 370	.1298	.0245	.0058	.0307
29	.185 695	.035 093	.1275	.0237	.0055	.0297
30	.182 574	.033 903	.1252	.0229	.0052	.0287
31	.179 605	.032 791	.1231	.0221	.0050	.0277
32	.176 777	.031 750	.1211	.0214	.0047	.0268
33	.174 078	.030 773	.1192	.0208	.0045	.0260
34	.171 499	.029 854	.1174	.0201	.0043	.0252
35	.169 031	.028 989	.1157	.0196	.0041	.0245
36	.166 667	.028 172	.1140	.0190	.0040	.0238
37	.164 399	.027 400	.1124	.0185	.0038	.0232
38	.162 221	.026 668	.1109	.0180	.0037	.0225
39	.160 128	.025 976	.1094	.0175	.0036	.0220
40	.158 114	.025 318	.1080	.0171	.0034	.0214
41	.156 174	.024 693	.1066	.0167	.0033	.0209
42	.154 303	.024 098	.1053	.0163	.0031	.0204
43	.152 499	.023 531	.1041	.0159	.0030	.0199
44	.150 756	.022 990	.1029	.0155	.0029	.0194
45	.149 071	.022 473	.1017	.0152	.0028	.0190
46	.147 442	.021 979	.1005	.0148	.0027	.0186
47	.145 865	.021 507	.0994	.0145	.0027	.0182
48	.144 338	.021 054	.0984	.0142	.0026	.0178
49	.142 857	.020 620	.0974	.0139	.0025	.0174

The probable error e of a single observation in a series of n measurements, t_1, t_2, \dots, t_n , the arithmetic mean of which is $m = \sum_{i=1}^n t_i/n$, is

$$e = 0.6745\sigma = \frac{0.6745}{\sqrt{n-1}} \sqrt{(m-t_1)^2 + (m-t_2)^2 + \dots + (m-t_n)^2}$$

σ is the standard deviation. The standard deviation σ_m of the arithmetic mean m is $\sigma_m = \sigma/\sqrt{n}$. The probable error E of the arithmetic mean is

$$E = 0.6745\sigma_m = \frac{0.6745}{\sqrt{n(n-1)}} \sqrt{(m-t_1)^2 + (m-t_2)^2 + \dots + (m-t_n)^2}$$

Table 28 FACTORS FOR COMPUTING PROBABLE ERRORS (continued)

n	1	1	.6745	.6745	.8453	.8453
	\sqrt{n}	$\sqrt{n(n-1)}$	$\sqrt{n-1}$	$\sqrt{n(n-1)}$	$n\sqrt{n-1}$	$\sqrt{n(n-1)}$
50	.141 421	.020 202	.0984	.0136	.0024	.0171
51	.140 028	.019 803	.0954	.0134	.0023	.0167
52	.139 075	.019 418	.0945	.0131	.0023	.0164
53	.137 361	.019 048	.0935	.0129	.0022	.0161
54	.136 083	.018 692	.0927	.0126	.0022	.0158
55	.134 840	.018 349	.0918	.0124	.0021	.0155
56	.133 631	.018 019	.0910	.0122	.0020	.0152
57	.132 453	.017 700	.0901	.0119	.0020	.0150
58	.131 306	.017 392	.0893	.0117	.0019	.0147
59	.130 189	.017 095	.0886	.0115	.0019	.0145
60	.129 099	.016 807	.0878	.0113	.0018	.0142
61	.128 037	.016 529	.0871	.0112	.0018	.0140
62	.127 000	.016 261	.0864	.0110	.0018	.0138
63	.125 988	.016 001	.0857	.0108	.0017	.0135
64	.125 000	.015 749	.0850	.0106	.0017	.0133
65	.124 035	.015 504	.0843	.0105	.0016	.0131
66	.123 091	.015 268	.0837	.0103	.0016	.0129
67	.122 169	.015 038	.0830	.0101	.0016	.0127
68	.121 268	.014 815	.0824	.0100	.0015	.0125
69	.120 386	.014 599	.0818	.0099	.0015	.0123
70	.119 523	.014 389	.0812	.0097	.0015	.0122
71	.118 678	.014 185	.0806	.0096	.0014	.0120
72	.117 851	.013 986	.0801	.0094	.0014	.0118
73	.117 041	.013 793	.0795	.0093	.0014	.0117
74	.116 249	.013 605	.0789	.0092	.0013	.0115
75	.115 470	.013 423	.0784	.0091	.0013	.0113
76	.114 708	.013 245	.0779	.0089	.0013	.0112
77	.113 961	.013 072	.0773	.0088	.0013	.0111
78	.113 228	.012 904	.0769	.0087	.0012	.0109
79	.112 509	.012 739	.0764	.0086	.0012	.0108
80	.111 803	.012 579	.0759	.0085	.0012	.0106
81	.111 111	.012 423	.0754	.0084	.0012	.0105
82	.110 432	.012 270	.0749	.0083	.0012	.0104
83	.109 764	.012 121	.0745	.0082	.0011	.0103
84	.109 109	.011 976	.0740	.0081	.0011	.0101
85	.108 465	.011 835	.0736	.0080	.0011	.0100
86	.107 833	.011 698	.0732	.0079	.0011	.0099
87	.107 211	.011 561	.0727	.0078	.0011	.0098
88	.106 600	.011 429	.0723	.0077	.0010	.0097
89	.106 000	.011 300	.0719	.0076	.0010	.0096
90	.105 409	.011 173	.0715	.0075	.0010	.0094
91	.104 828	.011 050	.0711	.0075	.0010	.0093
92	.104 257	.010 929	.0707	.0074	.0010	.0092
93	.103 696	.010 811	.0703	.0073	.0010	.0091
94	.103 142	.010 696	.0699	.0072	.0009	.0090
95	.102 598	.010 582	.0696	.0071	.0009	.0089
96	.102 062	.010 471	.0692	.0071	.0009	.0089
97	.101 535	.010 363	.0688	.0070	.0009	.0088
98	.101 015	.010 257	.0685	.0069	.0009	.0087
99	.100 504	.010 152	.0681	.0069	.0009	.0086
100	.100 000	.010 050	.0678	.0068	.0008	.0085

Approximate values of e and E are

$$e = \frac{0.8453}{\sqrt{n(n-1)}} D \quad E = \frac{0.8453}{n\sqrt{n-1}} D$$

where

$$D = |m - t_1| + |m - t_2| + \dots + |m - t_n|$$

Table 24 χ^2 DISTRIBUTION

Entries in this table give values of χ_0^2 for the indicated values of m and ϵ such that

$$\int_{\omega=\chi_0^2}^{\omega=\infty} f_m(\omega) d\omega = \epsilon$$

where $f_m(\omega) = \left[\frac{\omega^{(m/2)-1}}{2^{m/2}\Gamma(m/2)} \right] \exp\left(-\frac{\omega}{2}\right)$ $\omega > 0$

Degrees of Freedom, m	ϵ						
	.99	.98	.95	.90	.80	.70	.50
	χ_0^2						
1	0.000	0.001	0.004	0.016	0.064	0.148	0.455
2	0.020	0.040	0.103	0.211	0.446	0.713	1.386
3	0.115	0.185	0.352	0.584	1.005	1.422	2.366
4	0.297	0.429	0.711	1.064	1.649	2.185	3.357
5	0.554	0.752	1.145	1.610	2.343	3.000	4.351
6	0.872	1.134	1.635	2.204	3.070	3.828	5.348
7	1.239	1.564	2.167	2.833	3.822	4.671	6.346
8	1.646	2.032	2.733	3.420	4.584	5.527	7.344
9	2.088	2.532	3.325	4.169	5.380	6.393	8.343
10	2.558	3.059	3.940	4.865	6.179	7.207	9.342
11	3.053	3.609	4.575	5.618	6.989	8.148	10.341
12	3.571	4.178	5.226	6.324	7.807	9.034	11.340
13	4.107	4.765	5.892	7.042	8.634	9.925	12.340
14	4.660	5.368	6.571	7.790	9.457	10.821	13.339
15	5.229	5.985	7.261	8.547	10.307	11.721	14.339
16	5.812	6.614	7.962	9.312	11.152	12.624	15.338
17	6.408	7.255	8.672	10.085	12.002	13.531	16.338
18	7.018	7.906	9.390	10.865	12.857	14.440	17.338
19	7.633	8.567	10.117	11.651	13.716	15.352	18.338
20	8.260	9.237	10.851	12.443	14.578	16.266	19.337
21	8.897	9.915	11.591	13.240	15.445	17.182	20.337
22	9.542	10.600	12.338	14.041	16.314	18.101	21.337
23	10.196	11.293	13.091	14.848	17.187	19.021	22.337
24	10.856	11.992	13.848	15.659	18.062	19.943	23.337
25	11.524	12.697	14.611	16.473	18.940	20.867	24.337
26	12.198	13.409	15.379	17.292	19.820	21.792	25.336
27	12.879	14.125	16.151	18.114	20.703	22.719	26.336
28	13.565	14.847	16.928	18.930	21.588	23.647	27.336
29	14.256	15.574	17.708	19.768	22.475	24.577	28.336
30	14.953	16.306	18.493	20.599	23.364	25.508	29.336

Table 24 is abridged from Table III of Fisher: *Statistical Methods for Research Workers*, published by Oliver and Boyd Ltd., Edinburgh, by permission of the author and publishers.

Table 24 χ^2 DISTRIBUTION (continued)

Degrees of Freedom, m	ϵ						
	.30	.20	.10	.05	.02	.01	.001
	χ^2						
1	1.074	1.642	2.706	3.841	5.412	6.635	10.827
2	2.408	3.219	4.605	5.991	7.824	9.310	13.815
3	3.665	4.642	6.251	7.815	9.837	11.341	16.268
4	4.878	5.989	7.779	9.488	11.669	13.277	18.465
5	6.064	7.289	9.236	11.070	13.388	15.086	20.517
6	7.231	8.558	10.645	12.592	15.033	16.812	22.457
7	8.383	9.803	12.017	14.067	16.622	18.475	24.322
8	9.524	11.030	13.362	15.507	18.168	20.090	26.126
9	10.655	12.242	14.684	16.919	19.679	21.666	27.877
10	11.781	13.442	15.987	18.307	21.161	23.209	29.588
11	12.899	14.631	17.275	19.675	22.618	24.725	31.204
12	14.011	15.812	18.549	21.026	24.054	26.217	32.909
13	15.119	16.986	19.812	22.362	25.472	27.688	34.568
14	16.222	18.151	21.064	23.685	26.873	29.141	36.183
15	17.322	19.311	22.307	24.998	28.259	30.578	37.697
16	18.418	20.466	23.542	26.296	29.637	32.000	39.252
17	19.511	21.615	24.769	27.587	30.995	33.409	40.790
18	20.601	22.760	25.989	28.869	32.346	34.805	42.312
19	21.689	23.900	27.204	30.144	33.687	36.191	43.820
20	22.775	25.038	28.412	31.410	35.020	37.566	45.315
21	23.858	26.171	29.615	32.671	36.343	38.932	46.797
22	24.939	27.301	30.812	33.924	37.659	40.289	48.268
23	26.018	28.429	32.007	35.172	38.968	41.638	49.728
24	27.096	29.553	33.196	36.416	40.270	42.980	51.179
25	28.172	30.675	34.382	37.652	41.566	44.314	52.620
26	29.246	31.795	35.568	38.885	42.856	45.642	54.052
27	30.319	32.912	36.741	40.113	44.140	46.965	55.475
28	31.391	34.027	37.916	41.337	45.419	48.278	56.893
29	32.461	35.139	39.087	42.557	46.692	49.588	58.302
30	33.530	36.250	40.266	43.773	47.952	50.892	59.703

INTEREST AND ACTUARIAL EXPERIENCE

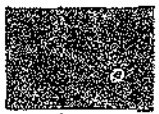
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Table 25 AMOUNT OF 1 AT COMPOUND INTEREST*

<i>n</i>	1%	2%	2½%	3%	3½%	4%	4½%	5%	6%
1	1.0100	1.0200	1.0250	1.0300	1.0350	1.0400	1.0450	1.0500	1.0600
2	1.0201	1.0404	1.0506	1.0609	1.0712	1.0816	1.0920	1.1025	1.1236
3	1.0303	1.0612	1.0769	1.0927	1.1087	1.1249	1.1412	1.1576	1.1910
4	1.0406	1.0824	1.1038	1.1256	1.1475	1.1699	1.1925	1.2155	1.2625
5	1.0510	1.1041	1.1314	1.1593	1.1877	1.2167	1.2462	1.2763	1.3382
6	1.0615	1.1262	1.1597	1.1941	1.2293	1.2653	1.3023	1.3401	1.4185
7	1.0721	1.1487	1.1867	1.2299	1.2723	1.3159	1.3609	1.4071	1.5036
8	1.0829	1.1717	1.2184	1.2668	1.3168	1.3686	1.4221	1.4775	1.5938
9	1.0937	1.1951	1.2489	1.3048	1.3629	1.4233	1.4851	1.5513	1.6895
10	1.1046	1.2180	1.2801	1.3439	1.4108	1.4802	1.5530	1.6299	1.7908
11	1.1157	1.2434	1.3121	1.3842	1.4600	1.5395	1.6229	1.7103	1.8983
12	1.1268	1.2682	1.3449	1.4258	1.5111	1.6010	1.6959	1.7969	2.0122
13	1.1381	1.2936	1.3785	1.4685	1.5640	1.6651	1.7722	1.8856	2.1329
14	1.1495	1.3195	1.4130	1.5126	1.6187	1.7317	1.8519	1.9799	2.2609
15	1.1610	1.3459	1.4483	1.5630	1.6753	1.8009	1.9353	2.0789	2.3966
16	1.1725	1.3728	1.4845	1.6047	1.7340	1.8730	2.0224	2.1829	2.5404
17	1.1843	1.4002	1.5216	1.6528	1.7947	1.9479	2.1134	2.2920	2.6928
18	1.1961	1.4282	1.5597	1.7024	1.8575	2.0256	2.2085	2.4086	2.8543
19	1.2081	1.4568	1.5987	1.7535	1.9225	2.1088	2.3079	2.5270	3.0265
20	1.2202	1.4869	1.6388	1.8061	1.9936	2.1911	2.4117	2.6533	3.2071
21	1.2324	1.5177	1.6798	1.8603	2.0694	2.2738	2.5202	2.7860	3.3995
22	1.2447	1.5490	1.7216	1.9161	2.1513	2.3699	2.6337	2.9253	3.6035
23	1.2572	1.5769	1.7646	1.9736	2.2061	2.4647	2.7522	3.0715	3.8197
24	1.2697	1.6044	1.8087	2.0328	2.2833	2.5633	2.8760	3.2251	4.0489
25	1.2824	1.6406	1.8539	2.0938	2.3632	2.6658	3.0054	3.3884	4.2919
26	1.2953	1.6734	1.9003	2.1566	2.4460	2.7725	3.1407	3.5557	4.5494
27	1.3082	1.7069	1.9478	2.2213	2.5318	2.8834	3.2820	3.7335	4.8223
28	1.3212	1.7410	1.9965	2.2879	2.6202	2.9987	3.4297	3.9201	5.1117
29	1.3345	1.7758	2.0454	2.3566	2.7119	3.1167	3.5940	4.1161	5.4184
30	1.3478	1.8114	2.0976	2.4273	2.8068	3.2434	3.7453	4.3219	5.7435
31	1.3613	1.8476	2.1500	2.5001	2.9053	3.3731	3.9139	4.5380	6.0881
32	1.3749	1.8845	2.2038	2.5751	3.0067	3.5081	4.0900	4.7649	6.4534
33	1.3887	1.9222	2.2580	2.6523	3.1119	3.6484	4.2740	5.0032	6.8406
34	1.4026	1.9607	2.3153	2.7319	3.2209	3.7943	4.4664	5.2533	7.2510
35	1.4166	1.9999	2.3732	2.8139	3.3330	3.9481	4.6673	5.5160	7.6881
36	1.4308	2.0399	2.4325	2.8983	3.4503	4.1039	4.8774	5.7918	8.1473
37	1.4451	2.0807	2.4933	2.9852	3.5710	4.2631	5.0969	6.0814	8.6361
38	1.4595	2.1223	2.5557	3.0748	3.6960	4.4388	5.3262	6.3855	9.1543
39	1.4741	2.1647	2.6196	3.1670	3.8254	4.6164	5.5659	6.7048	9.7035
40	1.4889	2.2080	2.6851	3.2620	3.9593	4.8010	5.8164	7.0400	10.2857
41	1.5038	2.2522	2.7522	3.3599	4.0978	4.9931	6.0781	7.3920	10.9029
42	1.5188	2.2972	2.8210	3.4607	4.2413	5.1928	6.3516	7.7616	11.5570
43	1.5340	2.3432	2.8915	3.5645	4.3897	5.4005	6.6374	8.1497	12.2505
44	1.5493	2.3901	2.9638	3.6715	4.5433	5.6165	6.9361	8.5572	12.9855
45	1.5648	2.4379	3.0379	3.7816	4.7024	5.8412	7.2482	8.9850	13.7646
46	1.5805	2.4866	3.1139	3.8950	4.8669	6.0748	7.5744	9.4343	14.5905
47	1.5963	2.5363	3.1917	4.0119	5.0373	6.3178	7.9153	9.9060	15.4659
48	1.6122	2.5871	3.2715	4.1323	5.2136	6.5705	8.2715	10.4013	16.3939
49	1.6283	2.6389	3.3533	4.2562	5.3961	6.8333	8.6437	10.9213	17.3775
50	1.6446	2.6916	3.4371	4.3839	5.5849	7.1067	9.0326	11.4674	18.4202

$$s = (1 + r)^n$$

* See Sec. 14. A principal of 1 placed at a rate of interest *r* (expressed as decimal), compounded annually, accumulates to an amount $(1 + r)^n$ at end of *n* years. In this table the rate *r* is expressed in per cent.

Table 26 PRESENT VALUE OF 1 AT COMPOUND INTEREST*

n	1%	2%	2½%	3%	3½%	4%	4½%	5%	6%
1	.99010	.98039	.97561	.97087	.96618	.96154	.95694	.95238	.94340
2	.98030	.96117	.95181	.94260	.93351	.92456	.91573	.90703	.89000
3	.97098	.94232	.92960	.91714	.90494	.89290	.88103	.86934	.84962
4	.96098	.92385	.90595	.88849	.87144	.85480	.83856	.82270	.79209
5	.95147	.90572	.88385	.86261	.84197	.82193	.80246	.78353	.74726
6	.94205	.88797	.86230	.83748	.81350	.79031	.76790	.74622	.70485
7	.93272	.87056	.84127	.81399	.78859	.76492	.74283	.72136	.67508
8	.92348	.85349	.82075	.78941	.76341	.73869	.71519	.69284	.64171
9	.91434	.83676	.80073	.76642	.73733	.71259	.68909	.66681	.61160
10	.90529	.82035	.78120	.74499	.71292	.68556	.66093	.63801	.57839
11	.89632	.80428	.76214	.72242	.68955	.66193	.63620	.61438	.55079
12	.88745	.78849	.74355	.70138	.66789	.64009	.61326	.59134	.52371
13	.87866	.77303	.72542	.68095	.64694	.61897	.59204	.57002	.50000
14	.86996	.75786	.70773	.66112	.62678	.59878	.57285	.55083	.47833
15	.86135	.74301	.69047	.64186	.60729	.57926	.55433	.53231	.45833
16	.85282	.72845	.67362	.62317	.58851	.56046	.53553	.51351	.43833
17	.84436	.71416	.65720	.60572	.57103	.54296	.51803	.49601	.42000
18	.83602	.70016	.64117	.58977	.55503	.52696	.50203	.48001	.40333
19	.82774	.68643	.62553	.57422	.53953	.51146	.48653	.46451	.38733
20	.81954	.67297	.61027	.55896	.52426	.49619	.47126	.44924	.37200
21	.81143	.65978	.59599	.54468	.51003	.48196	.45703	.43501	.35833
22	.80340	.64684	.58185	.53054	.49589	.46782	.44289	.42087	.34500
23	.79544	.63416	.56817	.51686	.48221	.45414	.42921	.40719	.33200
24	.78757	.62172	.55573	.50442	.46977	.44170	.41677	.39475	.32000
25	.77977	.60953	.54354	.49223	.45758	.42951	.40458	.38256	.30833
26	.77205	.59758	.53159	.48028	.44563	.41756	.39263	.37061	.29666
27	.76440	.58586	.51987	.46856	.43391	.40584	.38091	.35889	.28500
28	.75684	.57437	.50838	.45725	.42260	.39453	.36960	.34758	.27400
29	.74934	.56311	.49712	.44581	.41116	.38309	.35816	.33614	.26333
30	.74192	.55207	.48608	.43477	.39912	.37105	.34612	.32410	.25333
31	.73458	.54125	.47527	.42396	.38831	.36024	.33531	.31329	.24333
32	.72730	.53063	.46465	.41334	.37760	.34953	.32460	.30258	.23333
33	.72010	.52023	.45427	.40296	.37219	.34412	.31919	.29717	.22500
34	.71297	.51003	.44410	.39275	.36678	.33871	.31378	.29176	.22000
35	.70591	.50003	.43413	.38275	.35678	.32871	.30378	.28176	.21000
36	.69892	.49022	.42437	.37296	.34678	.31871	.29378	.27176	.20000
37	.69200	.48061	.41481	.36337	.33719	.30912	.28419	.26217	.19000
38	.68515	.47119	.40549	.35406	.32791	.30084	.27591	.25389	.18000
39	.67837	.46195	.39637	.34496	.31881	.29174	.26681	.24479	.17000
40	.67165	.45289	.38743	.33606	.31003	.28296	.25803	.23601	.16000
41	.66500	.44401	.37865	.32737	.30134	.27427	.24934	.22732	.15000
42	.65842	.43530	.37003	.31888	.29331	.26624	.24131	.21929	.14000
43	.65190	.42677	.36157	.31059	.28492	.25785	.23292	.21090	.13000
44	.64545	.41840	.35337	.30246	.27679	.24972	.22479	.20277	.12000
45	.63905	.41020	.34541	.29449	.26882	.24175	.21682	.19480	.11000
46	.63273	.40218	.33769	.28667	.26100	.23393	.20900	.18708	.10000
47	.62646	.39427	.33021	.27908	.25331	.22624	.20131	.17929	.09000
48	.62026	.38654	.32296	.27173	.24584	.21877	.19384	.17182	.08000
49	.61412	.37896	.31593	.26461	.23857	.21150	.18657	.16455	.07000
50	.60804	.37153	.30912	.25760	.23156	.20449	.17956	.15754	.06000

$$v^n = (1 + r)^{-n}$$

* See Sec. 20. The present quantity which in n years will accumulate to 1 at the rate of interest r (expressed as decimal), compounded annually, is $(1 + r)^{-n}$. In this table the rate r is expressed in per cent.

Table 27 AMOUNT OF AN ANNUITY* OF 1

n	1%	2%	2½%	3%	3½%	4%	4½%	5%	6%
1	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
2	2.0100	2.0200	2.0250	2.0300	2.0350	2.0400	2.0450	2.0500	2.0600
3	3.0301	3.0604	3.0755	3.0909	3.1062	3.1216	3.1370	3.1525	3.1836
4	4.0604	4.1216	4.1525	4.1836	4.2149	4.2465	4.2782	4.3101	4.3746
5	5.1010	5.2040	5.2563	5.3091	5.3625	5.4163	5.4707	5.5256	5.6371
6	6.1520	6.3081	6.3877	6.4684	6.5502	6.6330	6.7169	6.8019	6.9763
7	7.2135	7.4343	7.5474	7.6625	7.7794	7.8983	8.0192	8.1420	8.3938
8	8.2857	8.5330	8.7361	8.8523	8.9717	9.0937	9.2182	9.3450	9.6975
9	9.3685	9.7546	9.9545	10.1591	10.3685	10.5828	10.8021	11.0266	11.4913
10	10.4622	10.9497	11.2034	11.4639	11.7314	12.0061	12.2882	12.5779	13.1808
11	11.5668	12.1887	12.4835	12.8076	13.1420	13.4864	13.8412	14.2068	14.9716
12	12.6825	13.4121	13.7956	14.1920	14.6020	15.0258	15.4640	15.9171	16.8699
13	13.8093	14.6903	15.1404	15.6178	16.1130	16.6268	17.1599	17.7130	18.8821
14	14.9474	15.9739	16.5190	17.0883	17.6770	18.2819	18.9321	19.5965	21.0151
15	16.0969	17.2934	17.9319	18.5989	19.2957	20.0238	20.7841	21.5786	23.2760
16	17.2579	18.6393	19.3802	20.1569	20.9710	21.8245	22.7193	23.6576	25.6725
17	18.4304	20.0121	20.8547	21.7616	22.7050	23.6975	24.7417	25.8404	28.2129
18	19.6147	21.4123	22.3883	23.4144	24.4997	25.6454	26.8551	28.1324	30.9067
19	20.8109	22.8405	23.9480	25.1169	26.3572	27.6712	29.0636	30.5390	33.7600
20	22.0180	24.2974	25.5447	26.8704	28.2797	29.7781	31.3714	33.0660	36.7856
21	23.2392	25.7833	27.1833	28.6755	30.2695	31.9992	33.7831	35.7193	39.9927
22	24.4716	27.2990	28.8629	30.5368	32.3289	34.2480	36.3034	38.5052	43.3923
23	25.7163	28.8450	30.5844	32.4529	34.4604	36.6179	38.9370	41.7130	46.9958
24	26.9735	30.4219	32.3490	34.4265	36.6865	39.0826	41.6892	44.5020	50.8156
25	28.2432	32.0303	34.1578	36.4593	38.9499	41.8459	44.5692	47.7271	54.8645
26	29.5256	33.6709	36.0117	38.5530	41.3131	44.2117	47.5706	51.1135	59.1564
27	30.8209	35.3443	37.9120	40.7095	43.7591	47.0842	50.7113	54.8691	63.7058
28	32.1291	37.0512	39.8598	42.9309	46.2906	49.9576	53.9333	58.4026	68.5281
29	33.4504	38.7922	41.8563	45.2189	48.9108	52.9663	57.4230	62.3227	73.6398
30	34.7849	40.5681	43.9027	47.5754	51.6227	56.0849	61.0071	66.4388	79.0592
31	36.1327	42.3794	46.0003	50.0027	54.4295	59.3283	64.7524	70.7608	84.8017
32	37.4941	44.2270	48.1503	52.5028	57.3345	62.7015	68.6882	75.2988	90.8698
33	38.8690	46.1118	50.3540	55.0773	60.3412	66.2095	72.7582	80.0838	97.3432
34	40.2577	48.0338	52.6129	57.7302	63.4532	69.3579	77.0303	85.0670	104.1838
35	41.6603	49.9945	54.9282	60.4621	66.6740	73.0522	81.4266	90.3203	111.4348
36	43.0769	51.9944	57.3014	63.2789	70.0076	77.5983	86.1540	95.8353	119.1209
37	44.5076	54.0343	59.7339	66.1742	73.4879	81.7022	91.0413	101.6281	127.2691
38	45.9527	56.1149	62.2273	69.1594	77.0280	85.9703	95.1382	107.7095	135.9042
39	47.4123	58.2372	64.7830	72.2342	80.7249	90.4091	101.4644	114.0950	145.0595
40	48.8864	60.4020	67.4026	75.4013	84.5503	95.0255	107.0303	120.7998	154.7820
41	50.3752	62.6100	70.0876	78.6633	88.5095	99.8265	112.8467	127.8398	165.0477
42	51.8790	64.8622	72.8398	82.0232	92.6074	104.8196	118.9249	135.2318	175.9505
43	53.3978	67.1595	75.6608	85.4839	96.8486	110.0124	125.2764	142.9933	187.5076
44	54.9318	69.5027	78.5523	89.0484	101.2393	115.4129	131.9138	151.1430	199.7580
45	56.4811	71.8927	81.5161	92.7199	105.7817	121.0294	138.8500	159.7002	212.7435
46	58.0459	74.3305	84.5540	95.5015	110.4840	126.8706	145.9982	168.6852	226.5031
47	59.6263	76.8172	87.6579	100.3965	115.3810	132.9454	153.6726	178.1194	241.0936
48	61.2226	79.3535	90.8596	104.4084	120.3883	139.2632	161.5870	188.0254	256.5645
49	62.8348	81.9406	94.1311	108.5406	125.5018	145.8337	169.8594	198.4287	272.9584
50	64.4632	84.5794	97.4843	112.7969	130.9979	152.6671	178.5030	209.3480	290.3359

$$s_{\overline{n}|i} = \frac{(1+i)^n - 1}{i}$$

* See Sec. 23. If 1 is deposited at the end of each successive year (beginning one year hence), and interest at rate r (expressed as decimal), compounded annually, is paid on the accumulated deposit at the end of each year, the total amount accumulated at the end of n years is $[(1+r)^n - 1]/r$. In this table the rate r is expressed in per cent.

Table 28 PRESENT VALUE OF AN ANNUITY* OF 1 372

n	1%	2%	2½%	3%	3½%	4%	4½%	5%	6%
1	0.9901	.9804	.9755	.9709	.9662	.9615	.9569	.9524	.9434
2	1.9704	1.9416	1.9274	1.9135	1.8997	1.8861	1.8727	1.8594	1.8334
3	2.9410	2.8839	2.8560	2.8286	2.8018	2.7751	2.7480	2.7232	2.6730
4	3.9020	3.8077	3.7520	3.7171	3.6731	3.6299	3.5875	3.5460	3.4651
5	4.8534	4.7135	4.6458	4.5797	4.5151	4.4513	4.3900	4.3295	4.2124
6	5.7956	5.6014	5.5081	5.4172	5.3286	5.2421	5.1579	5.0757	4.9173
7	6.7282	6.4720	6.3494	6.2303	6.1145	6.0021	5.8927	5.7864	5.5824
8	7.6517	7.3255	7.1701	7.0197	6.8740	6.7327	6.5959	6.4632	6.2098
9	8.5660	8.1622	7.9709	7.7861	7.6077	7.4353	7.2688	7.1078	6.8017
10	9.4713	8.9826	8.7521	8.5302	8.3166	8.1109	7.9127	7.7217	7.3601
11	10.3676	9.7868	9.5142	9.2526	9.0016	8.7605	8.5289	8.3064	7.8869
12	11.2551	10.5753	10.2578	9.9540	9.6633	9.3851	9.1166	8.8538	8.3838
13	12.1337	11.3484	10.9832	10.6350	10.3027	9.9856	9.6829	9.3936	8.8527
14	13.0037	12.1062	11.6909	11.2961	10.9205	10.5631	10.2228	9.8966	9.2950
15	13.8651	12.8493	12.3814	11.9379	11.5174	11.1184	10.7395	10.3797	9.7122
16	14.7179	13.5777	13.0550	12.5811	12.0941	11.6323	11.2340	10.8378	10.1059
17	15.5623	14.2919	13.7122	13.1861	12.6513	12.1657	11.7072	11.2741	10.4773
18	16.3983	14.9920	14.3534	13.7535	13.1897	12.6593	12.1600	11.6896	10.8276
19	17.2250	15.6785	14.9795	14.3033	13.7028	13.1329	12.5933	12.0853	11.1561
20	18.0456	16.3514	15.5892	14.8775	14.2124	13.5903	13.0079	12.4622	11.4699
21	18.8570	17.0112	16.1846	15.4150	14.6980	14.0292	13.4047	12.8212	11.7641
22	19.6604	17.6580	16.7654	15.9369	15.1571	14.4511	13.7844	13.1630	12.0416
23	20.4558	18.2922	17.3321	16.4436	15.6204	14.8598	14.1476	13.4886	12.3034
24	21.2434	18.9139	17.8850	16.9355	16.0584	15.2470	14.4956	13.7986	12.5504
25	22.0232	19.5235	18.4244	17.4131	16.4815	15.6221	14.8282	14.0939	12.7834
26	22.7952	20.1210	18.9508	17.8763	16.8904	15.9828	15.1466	14.3752	13.0032
27	23.5596	20.7059	19.4640	18.3270	17.2854	16.3296	15.4513	14.6430	13.2105
28	24.3164	21.2813	19.9649	18.7641	17.6670	16.6631	15.7429	14.8961	13.4052
29	25.0658	21.8444	20.4535	19.1885	18.0356	16.9837	16.0219	15.1411	13.5907
30	25.8077	22.3955	20.9303	19.6004	18.3920	17.2920	16.2899	15.3725	13.7648
31	26.5423	22.9377	21.3954	20.0004	18.7363	17.5885	16.5444	15.5928	13.9291
32	27.2696	23.4693	21.8492	20.3888	19.0688	17.8736	16.7888	15.8027	14.0840
33	27.9897	23.9886	22.2919	20.7658	19.3902	18.1476	17.0229	16.0026	14.2302
34	28.7027	24.4986	22.7238	21.1318	19.7007	18.4112	17.2468	16.1929	14.3681
35	29.4086	24.9986	23.1452	21.4872	20.0007	18.6646	17.4610	16.3742	14.4982
36	30.1075	25.4888	23.5563	21.8323	20.2905	18.9083	17.6650	16.5459	14.6210
37	30.7995	25.9695	23.9573	22.1672	20.5705	19.1426	17.8622	16.7113	14.7388
38	31.4847	26.4406	24.3486	22.4925	20.8411	19.3679	18.0500	16.8679	14.8460
39	32.1630	26.9026	24.7303	22.8082	21.1025	19.5845	18.2297	17.0170	14.9491
40	32.8347	27.3555	25.1028	23.1148	21.3551	19.7928	18.4016	17.1591	15.0483
41	33.4997	27.7995	25.4661	23.4124	21.5991	19.9931	18.5661	17.2944	15.1380
42	34.1581	28.2348	25.8206	23.7014	21.8349	20.1856	18.7235	17.4232	15.2245
43	34.8100	28.6616	26.1664	23.9819	22.0627	20.3708	18.8742	17.5459	15.3082
44	35.4555	29.0800	26.5038	24.2543	22.2828	20.5488	19.0184	17.6628	15.3882
45	36.0945	29.4902	26.8330	24.5187	22.4955	20.7200	19.1553	17.7741	15.4558
46	36.7272	29.8923	27.1542	24.7754	22.7009	20.8847	19.2894	17.8801	15.5244
47	37.3537	30.2866	27.4675	25.0247	22.8994	21.0429	19.4147	17.9810	15.5890
48	37.9740	30.6731	27.7732	25.2667	23.0912	21.1951	19.5356	18.0772	15.6500
49	38.5881	31.0521	28.0714	25.5017	23.2768	21.3415	19.6513	18.1687	15.7076
50	39.1961	31.4236	28.3623	25.7298	23.4556	21.4822	19.7620	18.2559	15.7619

$$a_{\overline{n}|r} = \frac{1 - (1 + r)^{-n}}{r}$$

* See Sec. 24. The total present amount which will supply an annuity of 1 at the end of each year for n years, beginning one year hence (assuming that in successive years the amount not yet paid out earns interest at rate r , compounded annually), is $[1 - (1 + r)^{-n}]/r$. In this table the rate r is expressed in per cent.

Table 29 THE ANNUITY* THAT I WILL PURCHASE 373

n	1%	2%	2½%	3%	3½%	4%	4½%	5%	6%
1	1.01000	1.02000	1.02500	1.03000	1.03500	1.04000	1.04500	1.05000	1.06000
2	0.50751	0.51505	0.51893	0.52261	0.52640	0.53020	0.53400	0.53780	0.54544
3	.34002	.34675	.35014	.35353	.35693	.36035	.36377	.36721	.37411
4	.25628	.26262	.26582	.26903	.27225	.27649	.27874	.28201	.28859
5	.20604	.21216	.21526	.21826	.22148	.22483	.22779	.23097	.23740
6	.17255	.17853	.18155	.18460	.18767	.19076	.19388	.19702	.20336
7	.14863	.15451	.15750	.16051	.16354	.16661	.16970	.17282	.17914
8	.13088	.13661	.13947	.14246	.14548	.14853	.15161	.15472	.16104
9	.11674	.12252	.12546	.12843	.13145	.13449	.13757	.14069	.14702
10	.10558	.11133	.11426	.11723	.12024	.12329	.12638	.12950	.13587
11	.09645	.10218	.10511	.10808	.11109	.11415	.11725	.12039	.12679
12	.08885	.09456	.09749	.10046	.10348	.10655	.10967	.11283	.11928
13	.08247	.08812	.09105	.09403	.09706	.10014	.10328	.10646	.11296
14	.07690	.08260	.08554	.08853	.09157	.09467	.09782	.10102	.10758
15	.07212	.07783	.08077	.08377	.08683	.08994	.09311	.09634	.10296
16	.06794	.07365	.07659	.07961	.08268	.08582	.08902	.09227	.09895
17	.06426	.06997	.07293	.07595	.07904	.08220	.08542	.08870	.09544
18	.06098	.06670	.06967	.07271	.07582	.07899	.08224	.08555	.09236
19	.05806	.06378	.06678	.06981	.07294	.07614	.07941	.08275	.08962
20	.05542	.06115	.06415	.06722	.07036	.07358	.07688	.08024	.08718
21	.05303	.05878	.06179	.06487	.06804	.07128	.07460	.07800	.08500
22	.05085	.05663	.05965	.06275	.06593	.06920	.07256	.07600	.08305
23	.04889	.05467	.05770	.06081	.06402	.06731	.07068	.07414	.08128
24	.04707	.05287	.05591	.05905	.06227	.06559	.06899	.07247	.07968
25	.04541	.05122	.05428	.05743	.06067	.06401	.06744	.07095	.07823
26	.04387	.04970	.05277	.05594	.05921	.06257	.06602	.06956	.07690
27	.04245	.04829	.05138	.05465	.05805	.06144	.06492	.06850	.07590
28	.04112	.04699	.05009	.05329	.05660	.06001	.06352	.06712	.07459
29	.03990	.04578	.04889	.05211	.05543	.05888	.06241	.06608	.07358
30	.03875	.04465	.04778	.05102	.05437	.05783	.06139	.06505	.07255
31	.03768	.04360	.04674	.05000	.05337	.05686	.06044	.06413	.07169
32	.03667	.04261	.04577	.04905	.05244	.05595	.05956	.06328	.07088
33	.03573	.04169	.04486	.04816	.05157	.05510	.05874	.06249	.07012
34	.03484	.04082	.04401	.04732	.05076	.05431	.05798	.06176	.06940
35	.03400	.04000	.04321	.04654	.05000	.05358	.05727	.06107	.06877
36	.03321	.03923	.04245	.04580	.04928	.05289	.05661	.06043	.06819
37	.03247	.03851	.04174	.04511	.04861	.05224	.05598	.05984	.06766
38	.03176	.03782	.04107	.04446	.04798	.05163	.05540	.05928	.06716
39	.03109	.03717	.04044	.04384	.04739	.05106	.05486	.05876	.06689
40	.03046	.03656	.03984	.04326	.04683	.05052	.05434	.05828	.06646
41	.02986	.03597	.03927	.04271	.04630	.05002	.05386	.05782	.06606
42	.02928	.03542	.03873	.04219	.04580	.04954	.05341	.05739	.06568
43	.02873	.03489	.03822	.04170	.04533	.04909	.05298	.05699	.06533
44	.02820	.03439	.03773	.04123	.04488	.04866	.05258	.05662	.06501
45	.02771	.03391	.03727	.04079	.04445	.04826	.05220	.05628	.06470
46	.02723	.03345	.03683	.04036	.04405	.04788	.05184	.05593	.06441
47	.02677	.03300	.03641	.03996	.04367	.04752	.05151	.05561	.06415
48	.02633	.03256	.03601	.03958	.04331	.04718	.05119	.05532	.06390
49	.02591	.03212	.03559	.03917	.04292	.04682	.05089	.05504	.06366
50	.02551	.03172	.03520	.03877	.04253	.04655	.05060	.05478	.06344

$$a_{\overline{n}|}^{-1} \text{ at } r = \frac{r}{1 - (1 + r)^{-n}}$$

* See Sec. 24. One (1) now will purchase an annuity $r/[1 - (1 + r)^{-n}]$ at the end of each year for n years, beginning one year hence (assuming that in successive years the amount not yet paid out earns interest at rate r , expressed as decimal, compounded annually). In this table the rate r is expressed in per cent.

The total amount accumulated at the end of n years is 1 if an annuity $s_{\overline{n}|}^{-1}$ is deposited at the end of each successive year (beginning one year hence), and interest at rate r , compounded annually, is paid on the accumulated deposit at the end of each year.

$$s_{\overline{n}|}^{-1} = a_{\overline{n}|}^{-1} - r$$

Table 30 LOGARITHMS FOR INTEREST COMPUTATIONS

r	1+r	log(1+r)	r	1+r	log(1+r)
1/2%	1.005	00216 60617 55508	5 1/4%	1.055	02325 24596 33711
1%	1.010	00432 13737 82643	6%	1.060	02530 58552 64770
1 1/4%	1.015	00646 60422 49232	6 1/4%	1.065	02734 96077 74757
1 1/2%	1.020	00860 01717 81918	7%	1.070	02938 87776 85210
2%	1.025	01072 38863 91773	7 1/4%	1.075	03140 84664 51624
2 1/2%	1.030	01283 72247 05172	8%	1.080	03342 37554 85950
3%	1.035	01494 03497 92937	8 1/4%	1.085	03542 97381 84548
3 1/2%	1.040	01703 33392 98780	9%	1.090	03742 64979 40624
4%	1.045	01911 62904 47073	9 1/4%	1.095	03941 41191 78137
5%	1.050	02118 92990 69938	10%	1.100	04139 26851 58226

The amount A of principal P at compound interest after n years is $A = P(1+r)^n$.

Table 31 AMERICAN EXPERIENCE MORTALITY TABLE

x	l_x	d_x	q_x	p_x	x	l_x	d_x	q_x	p_x
10	100 000	749	0.007 490	0.992 510	53	66 797	1 091	0.016 333	0.983 667
11	99 281	746	0.007 516	0.992 484	54	65 706	1 143	0.017 396	0.982 604
12	98 503	743	0.007 542	0.992 459	55	64 588	1 199	0.018 571	0.981 429
13	97 762	740	0.007 568	0.992 431	56	63 364	1 260	0.019 885	0.980 115
14	97 022	737	0.007 596	0.992 404	57	62 104	1 325	0.021 336	0.978 663
15	96 285	735	0.007 634	0.992 366	58	60 779	1 394	0.022 936	0.977 064
16	95 559	732	0.007 681	0.992 339	59	59 385	1 468	0.024 720	0.975 280
17	94 818	729	0.007 688	0.992 312	60	57 917	1 546	0.026 693	0.973 307
18	94 089	727	0.007 727	0.992 273	61	56 371	1 628	0.028 880	0.971 120
19	93 362	725	0.007 765	0.992 235	62	54 743	1 713	0.031 292	0.968 708
20	92 637	723	0.007 805	0.992 195	63	53 030	1 800	0.033 943	0.966 057
21	91 914	722	0.007 855	0.992 145	64	51 230	1 889	0.036 873	0.963 127
22	91 192	721	0.007 906	0.992 094	65	49 341	1 980	0.040 129	0.959 871
23	90 471	720	0.007 958	0.992 042	66	47 361	2 070	0.043 707	0.956 293
24	89 751	719	0.008 011	0.991 989	67	45 291	2 158	0.047 647	0.952 353
25	89 032	718	0.008 065	0.991 935	68	43 133	2 243	0.052 002	0.947 998
26	88 314	718	0.008 130	0.991 870	69	40 890	2 321	0.056 762	0.943 238
27	87 566	718	0.008 197	0.991 803	70	38 589	2 391	0.061 993	0.938 007
28	86 878	718	0.008 264	0.991 735	71	36 178	2 448	0.067 658	0.932 335
29	86 160	719	0.008 345	0.991 655	72	33 730	2 487	0.073 733	0.926 267
30	85 441	720	0.008 427	0.991 573	73	31 243	2 505	0.080 176	0.919 822
31	84 721	721	0.008 510	0.991 490	74	28 738	2 501	0.087 028	0.912 972
32	84 000	723	0.008 607	0.991 393	75	26 237	2 476	0.094 371	0.905 629
33	83 277	726	0.008 718	0.991 282	76	23 761	2 431	0.102 311	0.897 689
34	82 551	729	0.008 831	0.991 169	77	21 330	2 369	0.111 084	0.888 936
35	81 822	732	0.008 946	0.991 054	78	18 961	2 291	0.120 827	0.879 173
36	81 090	737	0.009 089	0.990 911	79	16 670	2 196	0.131 734	0.868 266
37	80 353	742	0.009 234	0.990 766	80	14 474	2 091	0.144 486	0.855 534
38	79 611	749	0.009 408	0.990 592	81	12 363	1 964	0.158 605	0.841 395
39	78 862	756	0.009 586	0.990 414	82	10 419	1 816	0.174 297	0.825 703
40	78 106	765	0.009 794	0.990 206	83	8 603	1 648	0.191 561	0.808 439
41	77 341	774	0.010 008	0.989 992	84	6 965	1 470	0.211 359	0.788 641
42	76 567	785	0.010 252	0.989 748	85	5 485	1 292	0.235 562	0.764 448
43	75 782	797	0.010 517	0.989 483	86	4 193	1 114	0.265 681	0.734 319
44	74 985	812	0.010 829	0.989 171	87	3 079	933	0.303 020	0.696 980
45	74 173	828	0.011 163	0.988 837	88	2 146	744	0.346 692	0.653 308
46	73 345	848	0.011 532	0.988 438	89	1 402	555	0.395 863	0.604 137
47	72 497	870	0.012 000	0.988 000	90	847	385	0.454 545	0.545 455
48	71 627	896	0.012 509	0.987 491	91	462	246	0.532 468	0.467 532
49	70 731	927	0.013 106	0.986 894	92	216	137	0.634 259	0.365 741
50	69 804	962	0.013 781	0.986 219	93	79	58	0.734 177	0.265 823
51	68 842	1 001	0.014 541	0.985 459	94	21	18	0.857 143	0.142 857
52	67 841	1 044	0.015 389	0.984 611	95	3	3	1.000 000	0.000 000

Based on 100,000 living at age 10 years (x = age in years):

- l_x = number of original 100,000 who live to reach age x
- d_x = number of original 100,000 who live to reach age x but die before age $(x + 1)$
- q_x = probability of dying before age $(x + 1)$ if alive at age x . $q_x = d_x/l_x$
- p_x = probability of living to age $(x + 1)$ if alive at age x . $p_x = 1 - q_x$

CONSTANTS, EQUIVALENTS, AND CONVERSION FACTORS

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**Table 32 DECIMAL EQUIVALENTS OF
COMMON FRACTIONS**

$1/64 = 0.015625$	$1/2 = 16/32 = 32/64 = 0.50$
$1/32 = 2/64 = .03125$	$33/64 = .515625$
$3/64 = .046875$	$17/32 = 34/64 = .53125$
$1/16 = 2/32 = 4/64 = .0625$	$35/64 = .546875$
$5/64 = .078125$	$9/16 = 18/32 = 36/64 = .5625$
$3/32 = 6/64 = .09375$	$37/64 = .578125$
$7/84 = .109375$	$19/32 = 38/64 = .59375$
$1/8 = 4/32 = 8/64 = 0.125$	$39/64 = .609375$
$9/64 = .140625$	$5/8 = 20/32 = 40/64 = 0.625$
$5/32 = 10/64 = .15625$	$41/64 = .640625$
$11/64 = .171875$	$21/32 = 42/64 = .65625$
$3/16 = 6/32 = 12/64 = .1875$	$43/64 = .671875$
$13/64 = .203125$	$11/16 = 22/32 = 44/64 = .6875$
$7/32 = 14/64 = .21875$	$45/64 = .703125$
$15/64 = .234375$	$23/32 = 46/64 = .71875$
$1/4 = 8/32 = 16/64 = 0.25$	$47/64 = .734375$
$17/64 = .265625$	$3/4 = 24/32 = 48/64 = 0.75$
$9/32 = 18/64 = .28125$	$49/64 = .765625$
$19/64 = .296875$	$25/32 = 50/64 = .78125$
$5/16 = 10/32 = 20/64 = .3125$	$51/64 = .796875$
$21/64 = .328125$	$13/16 = 26/32 = 52/64 = .8125$
$11/32 = 22/64 = .34375$	$53/64 = .828125$
$23/64 = .359375$	$27/32 = 54/64 = .84375$
$3/8 = 12/32 = 24/64 = 0.375$	$55/64 = .859375$
$25/64 = .390625$	$7/8 = 28/32 = 56/64 = 0.875$
$13/32 = 26/64 = .40625$	$57/64 = .890625$
$27/64 = .421875$	$29/32 = 58/64 = .90625$
$7/16 = 14/32 = 28/64 = .4375$	$59/64 = .921875$
$29/64 = .453125$	$15/16 = 30/32 = 60/64 = .9375$
$15/32 = 30/64 = .46875$	$61/64 = .953125$
$31/64 = .484375$	$31/32 = 62/64 = .96875$
	$63/64 = .984375$

SQUARE ROOTS OF CERTAIN COMMON FRACTIONS

N	√N	N	√N	N	√N
1/2	0.7071	1/8	0.3536	1/16	0.2500
1/3	0.5774	3/8	0.6124	3/16	0.4330
2/3	0.8165	5/8	0.7906	5/16	0.5590
1/4	0.5000	7/8	0.9354	7/16	0.6614
3/4	0.8660	1/9	0.3333	9/16	0.7500
1/5	0.4472	2/9	0.4714	11/16	0.8292
2/5	0.6325	4/9	0.6667	13/16	0.9014
3/5	0.7746	5/9	0.7454	15/16	0.9682
4/5	0.8944	7/9	0.8819	1/32	0.1768
1/6	0.4082	8/9	0.9428	1/64	0.1250
5/6	0.9129	1/12	0.2887	1/50	0.1414
1/7	0.3780	5/12	0.6456		
2/7	0.5345	7/12	0.7638		
3/7	0.6547	11/12	0.9574		
4/7	0.7559				
5/7	0.8452				
6/7	0.9258				

Table 33 IMPORTANT CONSTANTS

N	Log N	N	Log N
$\pi = 3.14159265$	0.4971499	$\pi^2 = 9.86960440$	0.9942997
$2\pi = 6.28318531$	0.7981799	$\frac{1}{\pi^2} = 0.10132118$	9.0057003-10
$4\pi = 12.56637061$	1.0992099	$\sqrt{\pi} = 1.77245385$	0.2485749
$\frac{\pi}{2} = 1.57079633$	0.1961199	$\frac{1}{\sqrt{\pi}} = 0.56418958$	9.7514251-10
$\frac{\pi}{3} = 1.04719755$	0.0200206	$\sqrt[3]{\frac{3}{\pi}} = 0.97720502$	9.9899867-10
$\frac{4\pi}{3} = 4.18879020$	0.6220886	$\sqrt[4]{\frac{4}{\pi}} = 1.12837917$	0.0524551
$\frac{\pi}{4} = 0.78539816$	9.8959899-10	$\sqrt[3]{\frac{3}{\pi}} = 1.46459189$	0.1657166
$\frac{\pi}{6} = 0.52359878$	9.7189086-10	$\frac{1}{\sqrt[5]{\pi}} = 0.68278406$	9.8342834-10
$\frac{1}{\pi} = 0.31830969$	9.5028501-10	$\sqrt[3]{\pi^2} = 2.14502940$	0.2314332
$\frac{1}{2\pi} = 0.15915494$	9.2018201-10	$\sqrt[3]{\frac{3}{4\pi}} = 0.62035049$	9.7926371-10
$\frac{3}{\pi} = 0.95492966$	9.9799714-10	$\sqrt[3]{\frac{\pi}{6}} = 0.80599598$	9.9063329-10
$\frac{4}{\pi} = 1.27323954$	0.1049101		
<p>e = Napierian base = 2.71828183</p> <p>M = log₁₀ e = 0.43429448</p> <p>1/M = log_e 10 = 2.30258509</p> <p>180/π = degrees in 1 radian = 57.2957795</p> <p>π/180 = radians in 1° = 0.01745329</p> <p>π/10800 = radians in 1' = 0.0002908882</p> <p>π/648000 = radians in 1" = 0.00004848136811095</p> <p>sin 1" = 0.00004848136811133</p> <p>tan 1" = 0.00004848136811133</p> <p>centimeters in 1 (U.S.) ft = 30.48006096 (legal)</p> <p>feet in 1 cm = 0.03280833</p> <p>inches in 1 m = 39.37 (exact legal value)</p> <p>pounds in 1 kg = 2.204622621</p> <p>kilograms in 1 lb = 0.4535924277</p> <p>cubic inches in 1 (U.S.) gal = 231 (exact legal value)</p> <p>g (average value) = 32.16 ft/sec/sec</p> <p>g (legal) = 980.665 cm/sec²</p> <p>weight of 1 cu ft of water = 62.426 lb (max. density)</p> <p>weight of 1 cu ft of air = 0.0807 lb (at 32°F)</p> <p>ft.lb. per sec in 1 hp = 550 (exact legal value)</p> <p>kg.m per sec in 1 hp = 76.0404</p> <p>watts in 1 hp (legal) = 745.70</p>		<p>0.43429448</p> <p>9.63778431-10</p> <p>0.36221569</p> <p>1.76812268</p> <p>8.24187737-10</p> <p>6.46372612-10</p> <p>4.68557487-10</p> <p>4.68557487-10</p> <p>4.68557487-10</p> <p>1.4840158</p> <p>8.5159842-10</p> <p>1.5931664</p> <p>0.3433342</p> <p>9.3566660-10</p> <p>2.3636120</p> <p>1.5073</p> <p>2.9915207</p> <p>1.7953586</p> <p>8.907-10</p> <p>2.7403627</p> <p>1.8810445</p> <p>2.8725648</p>	
<p>π = 3.14159 26535 89793 23846</p> <p>e = 2.71828 18284 59045 23836</p> <p>M = 0.43429 44819 03251 82765</p> <p>1/M = 2.30258 50929 94045 63402</p> <p>log₁₀ π = 0.49714 98726 94133 85435</p> <p>log_e M = 9.63778 43113 09536 78912</p>			

Table 34 COMPLETE ELLIPTIC INTEGRALS, K AND E ,
FOR DIFFERENT VALUES OF THE MODULUS k

$\sin^{-1}k$	K	E	$\sin^{-1}k$	K	E	$\sin^{-1}k$	K	E
0°	1.5708	1.5708	50°	1.9356	1.3055	81°0	3.2553	1.0338
1	1.5709	1.5707	51	1.9539	1.2963	81.2	3.2771	1.0326
2	1.5713	1.5703	52	1.9729	1.2870	81.4	3.2995	1.0314
3	1.5719	1.5697	53	1.9927	1.2776	81.6	3.3223	1.0302
4	1.5727	1.5689	54	2.0133	1.2681	81.8	3.3458	1.0290
5	1.5738	1.5678	55	2.0347	1.2587	82.0	3.3699	1.0278
6	1.5751	1.5665	56	2.0571	1.2492	82.2	3.3946	1.0267
7	1.5767	1.5649	57	2.0804	1.2397	82.4	3.4199	1.0256
8	1.5785	1.5632	58	2.1047	1.2301	82.6	3.4460	1.0245
9	1.5805	1.5611	59	2.1300	1.2206	82.8	3.4728	1.0234
10	1.5828	1.5589	60	2.1565	1.2111	83.0	3.5004	1.0223
11	1.5854	1.5564	61	2.1842	1.2015	83.2	3.5288	1.0213
12	1.5882	1.5537	62	2.2132	1.1920	83.4	3.5581	1.0202
13	1.5913	1.5507	63	2.2435	1.1826	83.6	3.5884	1.0192
14	1.5946	1.5476	64	2.2754	1.1732	83.8	3.6196	1.0182
15	1.5981	1.5442	65	2.3088	1.1638	84.0	3.6519	1.0172
16	1.6020	1.5406	65.5	2.3261	1.1592	84.2	3.6852	1.0163
17	1.6061	1.5367	66.0	2.3433	1.1545	84.4	3.7193	1.0153
18	1.6105	1.5326	66.5	2.3622	1.1499	84.6	3.7557	1.0144
19	1.6151	1.5283	67.0	2.3809	1.1453	84.8	3.7930	1.0135
20	1.6200	1.5238	67.5	2.4001	1.1408	85.0	3.8317	1.0127
21	1.6252	1.5191	68.0	2.4198	1.1362	85.2	3.8721	1.0118
22	1.6307	1.5141	68.5	2.4401	1.1317	85.4	3.9142	1.0110
23	1.6365	1.5090	69.0	2.4610	1.1272	85.6	3.9583	1.0102
24	1.6425	1.5037	69.5	2.4825	1.1228	85.8	4.0044	1.0094
25	1.6490	1.4981	70.0	2.5046	1.1184	86.0	4.0526	1.0086
26	1.6557	1.4924	70.6	2.5273	1.1140	86.2	4.1037	1.0079
27	1.6627	1.4864	71.0	2.5507	1.1096	86.4	4.1574	1.0072
28	1.6701	1.4803	71.5	2.5749	1.1053	86.6	4.2142	1.0065
29	1.6777	1.4740	72.0	2.5998	1.1011	86.8	4.2744	1.0059
30	1.6858	1.4675	72.5	2.6256	1.0968	87.0	4.3387	1.0053
31	1.6941	1.4608	73.0	2.6521	1.0927	87.2	4.4073	1.0047
32	1.7028	1.4539	73.5	2.6796	1.0885	87.4	4.4811	1.0041
33	1.7119	1.4469	74.0	2.7081	1.0844	87.6	4.5609	1.0036
34	1.7214	1.4397	74.5	2.7375	1.0804	87.8	4.6477	1.0031
35	1.7312	1.4323	75.0	2.7681	1.0764	88.0	4.7427	1.0026
36	1.7415	1.4248	75.5	2.7998	1.0726	88.2	4.8476	1.0021
37	1.7522	1.4171	76.0	2.8327	1.0686	88.4	4.9654	1.0017
38	1.7633	1.4092	76.5	2.8669	1.0648	88.6	5.0988	1.0014
39	1.7748	1.4013	77.0	2.9025	1.0611	88.8	5.2527	1.0010
40	1.7868	1.3931	77.5	2.9397	1.0574	89.0	5.4349	1.0008
41	1.7992	1.3849	78.0	2.9786	1.0538	89.1	5.5402	1.0006
42	1.8122	1.3765	78.5	3.0192	1.0502	89.2	5.6579	1.0005
43	1.8256	1.3680	79.0	3.0617	1.0468	89.3	5.7914	1.0004
44	1.8396	1.3594	79.5	3.1064	1.0434	89.4	5.9455	1.0003
45	1.8541	1.3506	80.0	3.1534	1.0401	89.5	6.1278	1.0002
46	1.8691	1.3418	80.2	3.1729	1.0388	89.6	6.3509	1.0001
47	1.8846	1.3329	80.4	3.1928	1.0375	89.7	6.6285	1.0001
48	1.9011	1.3238	80.6	3.2132	1.0363	89.8	7.0440	1.0000
49	1.9186	1.3147	80.8	3.2340	1.0350	89.9	7.7371	1.0000

$$K = \int_0^{\pi/2} \frac{dx}{\sqrt{1 - k^2 \sin^2 x}} \quad E = \int_0^{\pi/2} \sqrt{1 - k^2 \sin^2 x} \, dx$$

Table 35 A TABLE OF CONVERSION FACTORS 379
(WEIGHTS AND MEASURES)

To convert from	To	Multiply by
Acres (British)	sq. meters	4046.849
Acres (U. S.)	sq. miles	0.0015625
Acres (U. S.)	sq. yards	4840
Ares	sq. meters	100
Ares	sq. yards	119.60
Barrels, oil	gallons (U. S.)	42
Barrels (U. S., dry)	cu. inches	7056
Barrels (U. S., dry)	quarts (dry)	105.0
Barrels (U. S., liquid)	gallons	31.5
Bars	dynes/sq. cm.	1.000×10^6
Bars	pounds/sq. inch	14.504
Board feet	cu. feet	1/12
B. T. U. (mean)	calories, gram (mean)	251.98
B. T. U. (mean)	foot pounds	777.98
B. T. U. (mean)	horse power hours	3.9292×10^{-4}
B. T. U. (mean)	joules (Abs.)	1054.8
B. T. U. (mean)	kilowatt hours	2.930×10^{-4}
B. T. U. (mean)	kg. meters	107.56
Bushels (British, dry)	liters	36.3677048
Bushels (U. S., dry), (level)	cu. inches	2150.42
Bushels (U. S., dry)	liters	35.238329
Bushels (U. S., dry)	pecks	4
Calories, gram (mean)	B. T. U. (mean)	3.9585×10^{-3}
Centimeters	inches (U. S.)	0.393700
Centimeters	meters	10^{-2}
Centimeters	yards (British)	0.01093614
Centimeters	yards (U. S.)	0.01093611
Chains (surveyors' or Gunter's)	yards	22
Circular inches	sq. inches	0.78540
Circular mils	sq. inches	7.854×10^{-7}
Cords	cord feet	8
Cord feet	cu. feet	18
Cubic cm.	cubic inches	0.061023
Cubic ft. (U. S.)	cubic inches	1728
Cubic ft. (U. S.)	cubic yards	0.037037
Cu. inches (British)	bushels (British)	4.5081×10^{-4}
Cu. inches (British)	cubic cm.	16.3870253
Cu. inches (U. S.)	cubic cm.	16.387162
Cu. inches (U. S.)	cu. ft. (U. S.)	5.78704×10^{-4}
Cu. inches (British)	gallons (British)	0.003606
Cu. inches (U. S.)	gallons (U. S.)	0.0043290
Cu. inches (U. S.)	liters	0.0163868
Cu. inches (U. S.)	quarts (U. S., dry)	0.0148808
Cubic yds. (British)	cubic ft.	27
Cubic yds. (U. S.)	cubic ft.	27
Cubic yds. (British)	cubic meters	0.76455285
Cubic yds. (U. S.)	cubic meters	0.76455945
Decameter	meters	10
Decigrams	grams	0.1
Decimeter	meter	10^{-1}
Degrees	minutes	60
Degrees (See Table 12)	radians	0.0174533
Dekagrams	grams	10
Drams (apothecaries' or troy)	ounces (avoirdupois)	0.1371429

Table 35 A TABLE OF CONVERSION FACTORS (WEIGHTS AND MEASURES) 330
(continued)

To convert from	To	Multiply by
Drams (apothecaries' or troy)	ounces (troy)	0.125
Drams (avoirdupois)	ounces (avoirdupois)	0.0825
Drams (avoirdupois)	ounces (troy)	0.055966146
Drams (U. S., fluid or apoth.)	cubic cm.	3.6967
Dynes	pounds	2.2481×10^{-6}
Ergs	B. T. U. (mean)	9.4806×10^{-11}
Ergs	foot pounds	7.3756×10^{-8}
Ergs	joules	1×10^{-7}
Ergs	kg. meters	1.0197×10^{-8}
Ergs/sec.	horse power	1.3410×10^{-10}
Ergs/sec.	K. W.	1×10^{-10}
Fathoms	feet	6
Feet (U. S.)	inches (U. S.)	12
Feet (U. S.)	meters	0.3048006096
Furlongs	mites (U. S.)	0.125
Gallons (British)	cubic inches	277.41
Gallons (British)	liters	4.5459631
Gallons (British)	quarts (British, liquid)	4
Gallons (U. S.)	cubic inches	231
Gallons (U. S.)	liters	3.78533
Gallons (U. S.)	quarts (U. S., liquid)	4
Gills (British)	cubic cm.	142.07
Gills (British)	pints (British, liquid)	0.25
Gills (U. S.)	cubic cm.	118.294
Gills (U. S.)	pints (U. S., liquid)	0.25
Grains	drams (avoirdupois)	0.03657143
Grains	drams (troy)	0.016667
Grains	milligrams	64.798918
Grains	ounces (avoirdupois)	0.0022857
Grains	ounces (troy)	0.0020833
Grams	ounces (avoirdupois)	0.0352739
Grams	ounces (troy)	0.0321507
Hectometers	meters	100
Hogsheads (British)	cubic ft.	10.114
Hogsheads (U. S.)	cubic ft.	8.42184
Hogsheads (U. S.)	gallons (U. S.)	63
Horse power	B. T. U. (mean)/min.	42.418
Horse power	calories, kg. (mean)/min.	10.588
Horse power	foot pounds/min.	33000
Horse power	foot pounds/sec.	550
Horse power	horse power (metric)	1.0139
Horse power	K. W. ($g = 980.665$)	0.74570
Horse power, electrical (U.S. or British)	watts (Abs.)	746.00
Horse power (metric)	horse power (U. S.)	0.98632
Hours (mean solar)	minutes (mean solar)	60
Hundredweights (long)	pounds	112
Hundredweights (long)	tons (long)	0.05
Hundredweights (short)	ounces (avoirdupois)	1600
Hundredweights (short)	pounds	100
Hundredweights (short)	tons (metric)	0.0453592
Hundredweights (short)	tons (long)	0.045429
Inches (British)	centimeters	2.53998
Inches (U. S.)	centimeters	2.54006
Inches (U. S.)	feet (U. S.)	1/12
Joules (Abs.)	B. T. U. (mean)	9.480×10^{-8}
Joules (Abs.)	ergs	1×10^7
Joules (Abs.)	foot pounds	0.73756
Kilograms	grams	1000

Table 35 A TABLE OF CONVERSION FACTORS (WEIGHTS AND MEASURES) 381
(continued)

To convert from	To	Multiply by
Kilograms	tons (long)	9.84207×10^{-4}
Kilograms	tons (metric)	0.001
Kilograms	tons (short)	0.0011023112
Kilometers	meters	1000
Kilometers	miles (nautical)	0.539953
Kilometers	miles (U. S.)	0.6213699495
Kilowatt hours	B. T. U. (mean)	3413.0
Kilowatt hours	foot pounds	2.6552×10^6
Kilowatt hours	joules (Abs.)	3.6000×10^6
Kilowatts	B. T. U. (mean)/min.	56.884
Kilowatts	foot pounds/min.	44254
Kilowatts	horse power	1.3410
Kilowatts	watts	1000
Knots (per hour)	feet/hour	6080.20
Knots (per hour)	miles/hour	1.15155
Leagues (nautical)	nautical miles	3
Leagues (statute)	statute miles	3
Links (surveyors' or Gunter's)	inches	7.92
Liters	cubic inches	61.025
Liters	gallons (British)	0.219976
Liters	gallons (U. S.)	0.26417782
Liters	quarts (British, liquid)	0.87990
Liters	quarts (U. S., dry)	0.908096
Liters	quarts (U. S., liquid)	1.056681369
Meters	feet (U. S.)	3.280833333
Meters	inches (British)	39.370113
Meters	inches (U. S.)	39.3700
Meters	yards (U. S.)	1.093611
Microns	meters	1×10^{-6}
Miles (nautical)	feet	6080.20
Miles (nautical)	kilometers	1.85325
Miles (nautical)	miles (U. S., statute)	1.1516
Miles (U. S., statute)	feet	5280
Miles (U. S., statute)	kilometers	1.609347219
Miles/hour	feet/min.	88
Miles/hour	knots (per hour)	0.8684
Milligrams	grains	0.01543236
Milligrams	grams	0.001
Millimeters	meters	0.001
Millimicrons	meters	1×10^{-9}
Minis	inches	0.001
Minims (British)	cubic cm.	0.059192
Minims (U. S., fluid)	cubic cm.	0.061612
Minutes (angle) (See Table 12)	radians	2.90888×10^{-4}
Minutes (angle)	seconds	60
Minutes (time)	seconds	60
Myriagrams	grams	10000
Myriameters	meters	10000
Ounces (avoirdupois)	grams	28.349527
Ounces (avoirdupois)	pounds (avoirdupois)	1/16
Ounces (avoirdupois)	pounds (troy)	0.075964861
Ounces (British, fluid)	cubic cm.	28.4130
Ounces (U. S., fluid)	cubic cm.	29.5737
Ounces (U. S., fluid)	liters	0.0295729
Ounces (U. S., fluid)	pints (U. S., liquid)	1/16
Pecks (British)	cubic inches	554.6
Pecks (British)	liters	9.091901
Pecks (U. S.)	bushels	0.25
Pecks (U. S.)	cubic inches	537.605
Pecks (U. S.)	liters	8.809582
Pecks (U. S.)	quarts (dry)	8

Table 35 A TABLE OF CONVERSION FACTORS (WEIGHTS AND MEASURES) 332
(continued)

To convert from	To	Multiply by
Pennyweights	grains	24
Pennyweights	grams	1.55517
Pennyweights	ounces (avoirdupois)	0.0548571
Pennyweights	ounces (troy)	0.05
Pints (British, liquid)	cubic cm.	568.26
Pints (British, liquid)	quarts (British)	0.5
Pints (U. S., dry)	cubic cm.	550.61
Pints (U. S., dry)	quarts (U. S., dry)	0.5
Pints (U. S., liquid)	cubic cm.	473.179
Pints (U. S., liquid)	gallons (U. S.)	0.125
Pints (U. S., liquid)	quarts (U. S., liquid)	0.5
Poundals	pounds	0.031061
Pounds (avoirdupois)	kilograms	0.4535924277
Pounds (avoirdupois)	ounces (avoirdupois)	16
Pounds (avoirdupois)	ounces (troy)	14.5833
Pounds	poundals	32.174
Pounds (troy)	kilograms	0.3732418
Pounds (troy)	ounces (avoirdupois)	13.160714
Pounds (troy)	ounces (troy)	12
Quarts (British, liquid)	cubic cm.	1136.521
Quarts (U. S., dry)	cubic cm.	1101.23
Quarts (U. S., dry)	cubic inches	67.2006
Quarts (U. S., dry)	pecks (U. S.)	0.125
Quarts (U. S., dry)	pints (dry)	2
Quarts (U. S., liquid)	cubic cm.	946.358
Quarts (U. S., liquid)	cubic inches	57.749
Quarts (U. S., liquid)	gallons (U. S.)	0.25
Quarts (U. S., liquid)	liters	0.946333
Radians (See Table 14)	degrees	57.29578
Rods (Surveyors' measure)	yards	5.5
Scruples	grains	20.
Seconds	minutes	1/60
Sq. cm.	sq. inches	0.15500
Sq. feet (British)	sq. meters	0.09290289
Sq. feet (U. S.)	sq. inches	144
Sq. feet (U. S.)	sq. meters	0.09290341
Sq. inches (British)	sq. cm.	6.4515898
Sq. inches (U. S.)	sq. cm.	6.4516258
Sq. inches (U. S.)	sq. feet (U. S.)	1/144
Sq. meters	sq. inches	1550.0
Sq. meters	sq. yards (British)	1.195992
Sq. meters	sq. yards (U. S.)	1.195985
Sq. miles	acres	640
Sq. yards (British)	sq. meters	0.836126
Sq. yards (U. S.)	sq. feet	9
Sq. yards (U. S.)	sq. meters	0.83613
Tons (long)	hundredweights (short)	22.400
Tons (long)	pounds (avoirdupois)	2240
Tons (long)	pounds (troy)	2722.22
Tons (long)	tons (metric)	1.0160470
Tons (long)	tons (short)	1.12000
Tons (metric)	tons (long)	0.984207
Tons (metric)	tons (short)	1.10231
Tons (short)	hundredweights (short)	20
Tons (short)	kilograms	907.1846
Tons (short)	tons (long)	0.892857
Tons (short)	tons (metric)	0.907185
Watts (Abs.)	B. T. U. (mean)/min.	0.066884
Watts (Abs.)	ergs/sec.	1×10 ⁷
Watts (Abs.)	joules/sec.	1
Yards (British)	meters	0.9143992
Yards (U. S.)	feet	3
Yards (U. S.)	meters	0.91440183

FOUR-PLACE TABLES

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Table 36 COMMON LOGARITHMS OF TRIGONOMETRIC FUNCTIONS

Deg.	Log Rad	Log Sin	Log Cos	Log Tan	Log Ctn	Log Sec	Log Csc			
0			10.0000			10.0000			10.1961	90
1	8.2418	8.2419	9.9999	8.2418	11.7581	10.0001	11.7581		10.1913	89
2	8.5429	8.5428	9.9997	8.5431	11.4569	10.0003	11.4572		10.1864	88
3	8.7190	8.7189	9.9994	8.7194	11.2606	10.0006	11.2612		10.1814	87
4	8.8439	8.8436	9.9989	8.8446	11.1564	10.0011	11.1564		10.1764	86
5	8.9408	8.9403	9.9983	8.9420	11.0580	10.0017	11.0597		10.1713	85
6	9.0200	9.0192	9.9976	9.0216	10.9784	10.0024	10.9808		10.1662	84
7	9.0870	9.0859	9.9968	9.0891	10.9109	10.0032	10.9141		10.1610	83
8	9.1450	9.1435	9.9958	9.1478	10.8522	10.0042	10.8564		10.1557	82
9	9.1961	9.1943	9.9946	9.1997	10.8003	10.0054	10.8057		10.1504	81
10	9.2419	9.2397	9.9934	9.2468	10.7537	10.0066	10.7603		10.1450	80
11	9.2833	9.2808	9.9919	9.2887	10.7113	10.0081	10.7194		10.1395	79
12	9.3211	9.3179	9.9904	9.3275	10.6725	10.0096	10.6821		10.1340	78
13	9.3552	9.3521	9.9887	9.3634	10.6366	10.0113	10.6479		10.1284	77
14	9.3880	9.3837	9.9869	9.3968	10.6032	10.0131	10.6163		10.1227	76
15	9.4180	9.4130	9.9849	9.4281	10.5719	10.0151	10.5870		10.1169	75
16	9.4460	9.4403	9.9828	9.4575	10.5425	10.0172	10.5597		10.1111	74
17	9.4725	9.4659	9.9806	9.4865	10.5147	10.0194	10.5311		10.1052	73
18	9.4971	9.4900	9.9782	9.5118	10.4882	10.0218	10.5100		10.0992	72
19	9.5206	9.5126	9.9757	9.5370	10.4630	10.0243	10.4874		10.0931	71
20	9.5429	9.5341	9.9730	9.5611	10.4389	10.0270	10.4659		10.0870	70
21	9.5641	9.5543	9.9702	9.5842	10.4158	10.0298	10.4457		10.0807	69
22	9.5843	9.5736	9.9672	9.6064	10.3936	10.0328	10.4264		10.0744	68
23	9.6036	9.5919	9.9640	9.6279	10.3721	10.0360	10.4081		10.0680	67
24	9.6221	9.6093	9.9607	9.6486	10.3514	10.0393	10.3907		10.0614	66
25	9.6398	9.6259	9.9573	9.6687	10.3313	10.0427	10.3741		10.0548	65
26	9.6569	9.6418	9.9537	9.6882	10.3118	10.0463	10.3592		10.0481	64
27	9.6732	9.6570	9.9499	9.7072	10.2928	10.0501	10.3430		10.0412	63
28	9.6890	9.6716	9.9459	9.7257	10.2743	10.0541	10.3284		10.0343	62
29	9.7042	9.6858	9.9418	9.7438	10.2562	10.0582	10.3144		10.0272	61
30	9.7190	9.6990	9.9375	9.7614	10.2386	10.0625	10.3010		10.0200	60
31	9.7332	9.7118	9.9331	9.7786	10.2212	10.0669	10.2882		10.0127	59
32	9.7470	9.7242	9.9284	9.7958	10.2042	10.0716	10.2758		10.0053	58
33	9.7604	9.7361	9.9236	9.8125	10.1875	10.0764	10.2639		9.9978	57
34	9.7734	9.7478	9.9186	9.8290	10.1710	10.0814	10.2524		9.9901	56
35	9.7859	9.7585	9.9134	9.8452	10.1548	10.0866	10.2414	9.9822		55
36	9.7982	9.7692	9.9080	9.8613	10.1387	10.0920	10.2308	9.9743		54
37	9.8101	9.7795	9.9023	9.8771	10.1229	10.0977	10.2205	9.9662		53
38	9.8217	9.7893	9.8965	9.8928	10.1072	10.1035	10.2107	9.9579		52
39	9.8329	9.7989	9.8905	9.9084	10.0916	10.1095	10.2011	9.9494		51
40	9.8439	9.8081	9.8843	9.9238	10.0762	10.1157	10.1919	9.9408		50
41	9.8547	9.8169	9.8778	9.9392	10.0608	10.1222	10.1831	9.9321		49
42	9.8651	9.8255	9.8711	9.9544	10.0456	10.1289	10.1745	9.9231		48
43	9.8753	9.8338	9.8641	9.9697	10.0303	10.1359	10.1662	9.9140		47
44	9.8853	9.8418	9.8569	9.9848	10.0152	10.1431	10.1582	9.9048		46
45	9.8951	9.8495	9.8495	10.0000	10.0000	10.1505	10.1505	9.8951		45
		Log Cos	Log Sin	Log Ctn	Log Tan	Log Sec	Log Csc	Log Rad	Deg.	

For degrees indicated in the left-hand column use the column headings at the top. For degrees indicated in the right-hand column use the column headings at the bottom.

The -10 portion of the characteristic of the logarithm is not printed but must be written down whenever such a logarithm is used.

Table 37 NATURAL TRIGONOMETRIC FUNCTIONS

Deg.	Rad	Sin	Cos	Tan	Ctn	Sec	Csc		
0	0.0000	0.0000	1.0000	0.0000	-----	1.0000	-----	1.5708	90
1	0.0175	0.0175	0.9998	0.0175	57.290	1.0002	57.299	1.5533	89
2	0.0349	0.0349	0.9994	0.0349	28.636	1.0006	28.664	1.5369	88
3	0.0524	0.0523	0.9986	0.0524	19.081	1.0014	19.107	1.5184	87
4	0.0698	0.0698	0.9976	0.0699	14.301	1.0024	14.336	1.5010	86
5	0.0873	0.0872	0.9962	0.0875	11.430	1.0038	11.474	1.4835	85
6	0.1047	0.1045	0.9945	0.1051	9.5144	1.0055	9.5668	1.4661	84
7	0.1222	0.1219	0.9925	0.1229	8.1443	1.0075	8.2055	1.4486	83
8	0.1396	0.1392	0.9903	0.1405	7.1154	1.0098	7.1853	1.4312	82
9	0.1571	0.1564	0.9877	0.1584	6.3138	1.0126	6.3925	1.4137	81
10	0.1745	0.1736	0.9848	0.1763	5.6713	1.0154	5.7588	1.3963	80
11	0.1920	0.1908	0.9816	0.1944	5.1446	1.0187	5.2406	1.3789	79
12	0.2094	0.2079	0.9781	0.2126	4.7046	1.0223	4.8097	1.3614	78
13	0.2269	0.2250	0.9744	0.2309	4.3315	1.0263	4.4454	1.3439	77
14	0.2443	0.2419	0.9703	0.2493	4.0108	1.0306	4.1336	1.3265	76
15	0.2618	0.2588	0.9659	0.2679	3.7321	1.0353	3.8837	1.3090	75
16	0.2793	0.2756	0.9613	0.2867	3.4874	1.0403	3.6280	1.2915	74
17	0.2967	0.2924	0.9563	0.3057	3.2709	1.0457	3.4203	1.2741	73
18	0.3142	0.3090	0.9511	0.3249	3.0777	1.0515	3.2361	1.2566	72
19	0.3316	0.3256	0.9455	0.3443	2.9042	1.0576	3.0716	1.2392	71
20	0.3491	0.3420	0.9397	0.3640	2.7478	1.0642	2.9238	1.2217	70
21	0.3665	0.3584	0.9336	0.3839	2.6051	1.0711	2.7904	1.2043	69
22	0.3840	0.3746	0.9272	0.4040	2.4751	1.0785	2.6695	1.1868	68
23	0.4014	0.3907	0.9205	0.4245	2.3559	1.0864	2.5593	1.1694	67
24	0.4189	0.4067	0.9135	0.4452	2.2460	1.0946	2.4586	1.1519	66
25	0.4363	0.4225	0.9063	0.4663	2.1446	1.1034	2.3662	1.1345	65
26	0.4538	0.4384	0.8988	0.4877	2.0503	1.1126	2.2812	1.1170	64
27	0.4712	0.4540	0.8910	0.5095	1.9626	1.1223	2.2027	1.0996	63
28	0.4887	0.4695	0.8829	0.5317	1.8807	1.1326	2.1301	1.0821	62
29	0.5061	0.4848	0.8745	0.5543	1.8046	1.1434	2.0627	1.0647	61
30	0.5236	0.5000	0.8660	0.5774	1.7321	1.1547	2.0000	1.0472	60
31	0.5411	0.5150	0.8572	0.6009	1.6643	1.1666	1.9416	1.0297	59
32	0.5585	0.5299	0.8480	0.6249	1.6003	1.1792	1.8871	1.0123	58
33	0.5760	0.5446	0.8387	0.6494	1.5399	1.1924	1.8361	0.9948	57
34	0.5934	0.5592	0.8290	0.6745	1.4826	1.2062	1.7883	0.9774	56
35	0.6109	0.5735	0.8192	0.7002	1.4281	1.2206	1.7434	0.9599	55
36	0.6283	0.5878	0.8090	0.7265	1.3764	1.2351	1.7013	0.9425	54
37	0.6458	0.6018	0.7986	0.7536	1.3270	1.2501	1.6616	0.9250	53
38	0.6632	0.6157	0.7880	0.7813	1.2798	1.2659	1.6243	0.9076	52
39	0.6807	0.6293	0.7771	0.8098	1.2349	1.2828	1.5890	0.8901	51
40	0.6981	0.6428	0.7660	0.8391	1.1918	1.3004	1.5557	0.8727	50
41	0.7155	0.6561	0.7547	0.8693	1.1504	1.3250	1.5243	0.8552	49
42	0.7330	0.6691	0.7431	0.9004	1.1106	1.3456	1.4945	0.8378	48
43	0.7505	0.6820	0.7314	0.9325	1.0724	1.3673	1.4663	0.8203	47
44	0.7679	0.6947	0.7193	0.9657	1.0355	1.3902	1.4396	0.8029	46
45	0.7854	0.7071	0.7071	1.0000	1.0000	1.4142	1.4142	0.7854	45
		Cos	Sin	Ctn	Tan	Csc	Sec	Rad	Deg.

For degrees indicated in the left-hand column use the column headings at the top. For degrees indicated in the right-hand column use the column headings at the bottom.

Table 38 COMMON LOGARITHMS OF NUMBERS

N											Proportional parts				
	0	1	2	3	4	5	6	7	8	9	1	2	3	4	5
10	0000	0043	0086	0128	0170	0212	0253	0294	0334	0374	4	8	12	17	21
11	0414	0453	0492	0531	0569	0607	0645	0682	0719	0755	4	8	11	15	19
12	0792	0828	0864	0899	0934	0969	1004	1038	1072	1106	3	7	10	14	17
13	1139	1173	1206	1239	1271	1303	1335	1367	1399	1430	3	6	9	13	16
14	1461	1492	1523	1553	1584	1614	1644	1673	1703	1732	3	6	9	12	15
15	1761	1790	1818	1847	1875	1903	1931	1959	1987	2014	3	6	8	11	14
16	2041	2068	2095	2122	2148	2175	2201	2227	2253	2279	3	5	8	11	13
17	2304	2330	2355	2380	2406	2430	2455	2480	2504	2529	2	5	7	10	12
18	2553	2577	2601	2625	2648	2672	2695	2718	2742	2765	2	5	7	9	11
19	2788	2810	2833	2856	2878	2900	2923	2945	2967	2989	2	4	7	9	11
20	3010	3032	3054	3075	3096	3118	3139	3160	3181	3201	2	4	6	8	11
21	3222	3243	3263	3284	3304	3324	3345	3365	3385	3404	2	4	6	8	10
22	3424	3444	3464	3483	3502	3522	3541	3560	3579	3598	2	4	6	8	10
23	3617	3636	3655	3674	3692	3711	3729	3747	3766	3784	2	4	6	7	9
24	3802	3820	3838	3856	3874	3892	3909	3927	3945	3962	2	4	5	7	9
25	3979	3997	4014	4031	4048	4065	4082	4099	4116	4133	2	4	5	7	9
26	4150	4166	4183	4200	4216	4232	4248	4265	4281	4298	2	3	5	6	8
27	4314	4330	4346	4362	4378	4393	4409	4425	4440	4456	2	3	5	6	7
28	4472	4487	4502	4518	4533	4548	4564	4579	4594	4609	2	3	5	6	7
29	4624	4639	4654	4669	4683	4698	4713	4728	4742	4757	1	3	4	6	7
30	4771	4786	4800	4814	4829	4843	4857	4871	4886	4900	1	3	4	6	7
31	4914	4928	4942	4955	4969	4983	4997	5011	5024	5038	1	3	4	5	7
32	5051	5065	5079	5092	5105	5119	5132	5145	5159	5172	1	3	4	5	7
33	5185	5198	5211	5224	5237	5250	5263	5276	5289	5302	1	3	4	5	6
34	5315	5328	5340	5352	5365	5378	5391	5403	5416	5428	1	2	4	5	6
35	5441	5453	5465	5478	5490	5502	5514	5527	5539	5551	1	2	4	5	6
36	5563	5575	5587	5599	5611	5623	5635	5647	5658	5670	1	2	4	5	6
37	5682	5694	5705	5717	5729	5740	5752	5763	5775	5786	1	2	4	5	6
38	5798	5809	5821	5832	5843	5855	5866	5877	5888	5899	1	2	3	5	6
39	5911	5922	5933	5944	5955	5966	5977	5988	5999	6010	1	2	3	4	5
40	6021	6031	6042	6053	6064	6075	6085	6096	6107	6117	1	2	3	4	5
41	6128	6138	6149	6160	6170	6180	6191	6201	6212	6222	1	2	3	4	5
42	6232	6243	6253	6263	6274	6284	6294	6304	6314	6325	1	2	3	4	5
43	6335	6345	6355	6365	6375	6385	6395	6405	6415	6425	1	2	3	4	5
44	6435	6444	6454	6464	6474	6484	6493	6503	6513	6522	1	2	3	4	5
45	6532	6542	6551	6561	6571	6580	6590	6599	6609	6618	1	2	3	4	5
46	6628	6637	6646	6656	6665	6675	6684	6693	6702	6712	1	2	3	4	5
47	6721	6730	6739	6749	6758	6767	6776	6785	6794	6803	1	2	3	4	4
48	6812	6821	6830	6839	6848	6857	6866	6875	6884	6893	1	2	3	4	4
49	6902	6911	6920	6928	6937	6946	6955	6964	6972	6981	1	2	3	4	4
50	6990	6998	7007	7016	7024	7033	7042	7050	7059	7067	1	2	3	3	4
51	7076	7084	7093	7101	7110	7118	7126	7135	7143	7152	1	2	3	3	4
52	7160	7168	7177	7185	7193	7202	7210	7218	7226	7235	1	2	3	3	4
53	7243	7251	7259	7267	7275	7284	7292	7300	7308	7316	1	2	2	3	4
54	7324	7332	7340	7348	7356	7364	7372	7380	7388	7396	1	2	2	3	4
N	0	1	2	3	4	5	6	7	8	9	1	2	3	4	5

Table 38 COMMON LOGARITHMS OF NUMBERS (continued)

N											Proportional parts				
	0	1	2	3	4	5	6	7	8	9	1	2	3	4	5
55	7404	7412	7419	7427	7435	7443	7451	7459	7466	7474	1	2	2	3	4
56	7482	7490	7497	7505	7513	7520	7528	7536	7543	7551	1	2	2	3	4
57	7559	7566	7574	7582	7589	7597	7604	7612	7619	7627	1	1	2	3	4
58	7634	7642	7649	7657	7664	7672	7679	7686	7694	7701	1	1	2	3	4
59	7709	7716	7723	7731	7738	7745	7752	7760	7767	7774	1	1	2	3	4
60	7782	7789	7796	7803	7810	7818	7825	7832	7839	7846	1	1	2	3	4
61	7853	7860	7868	7875	7882	7889	7896	7903	7910	7917	1	1	2	3	3
62	7924	7931	7938	7945	7952	7959	7966	7973	7980	7987	1	1	2	3	3
63	7993	8000	8007	8014	8021	8028	8035	8041	8048	8055	1	1	2	3	3
64	8062	8069	8075	8082	8089	8096	8102	8109	8116	8122	1	1	2	3	3
65	8129	8136	8142	8149	8156	8162	8169	8176	8182	8189	1	1	2	3	3
66	8195	8202	8209	8215	8222	8228	8235	8241	8248	8254	1	1	2	3	3
67	8261	8267	8274	8280	8287	8293	8299	8305	8312	8319	1	1	2	3	3
68	8325	8331	8338	8344	8351	8357	8363	8370	8376	8382	1	1	2	3	3
69	8388	8395	8401	8407	8414	8420	8426	8432	8439	8445	1	1	2	3	3
70	8451	8457	8463	8470	8475	8482	8488	8494	8500	8506	1	1	2	3	3
71	8513	8519	8525	8531	8537	8543	8549	8555	8561	8567	1	1	2	3	3
72	8573	8579	8585	8591	8597	8603	8609	8615	8621	8627	1	1	2	3	3
73	8633	8639	8645	8651	8657	8663	8669	8675	8681	8686	1	1	2	3	3
74	8692	8698	8704	8710	8716	8722	8727	8733	8739	8745	1	1	2	3	3
75	8751	8756	8762	8768	8774	8779	8785	8791	8797	8802	1	1	2	3	3
76	8808	8814	8820	8825	8831	8837	8842	8848	8854	8859	1	1	2	3	3
77	8865	8871	8876	8882	8887	8893	8899	8904	8910	8915	1	1	2	3	3
78	8921	8927	8932	8938	8943	8949	8954	8960	8965	8971	1	1	2	3	3
79	8976	8982	8987	8993	8998	9004	9009	9015	9020	9025	1	1	2	3	3
80	9031	9036	9042	9047	9053	9058	9063	9069	9074	9079	1	1	2	3	3
81	9085	9090	9096	9101	9106	9112	9117	9122	9128	9133	1	1	2	3	3
82	9138	9143	9149	9154	9159	9165	9170	9175	9180	9186	1	1	2	3	3
83	9191	9196	9201	9206	9212	9217	9222	9227	9232	9238	1	1	2	3	3
84	9243	9248	9253	9258	9263	9269	9274	9279	9284	9289	1	1	2	3	3
85	9294	9299	9304	9309	9315	9320	9325	9330	9335	9340	1	1	2	3	3
86	9345	9350	9355	9360	9365	9370	9375	9380	9385	9390	1	1	2	3	3
87	9395	9400	9405	9410	9415	9420	9425	9430	9435	9440	1	1	2	3	3
88	9445	9450	9455	9460	9465	9469	9474	9479	9484	9489	0	1	1	2	2
89	9494	9499	9504	9509	9513	9518	9523	9528	9533	9538	0	1	1	2	2
90	9542	9547	9552	9557	9562	9566	9571	9576	9581	9586	0	1	1	2	2
91	9590	9595	9600	9605	9609	9614	9619	9624	9628	9633	0	1	1	2	2
92	9638	9643	9647	9652	9657	9661	9666	9671	9675	9680	0	1	1	2	2
93	9685	9689	9694	9699	9703	9708	9713	9717	9722	9727	0	1	1	2	2
94	9731	9735	9741	9745	9750	9754	9759	9763	9768	9773	0	1	1	2	2
95	9777	9782	9786	9791	9795	9800	9805	9809	9814	9818	0	1	1	2	2
96	9823	9827	9832	9836	9841	9845	9850	9854	9859	9863	0	1	1	2	2
97	9868	9872	9877	9881	9886	9890	9894	9899	9903	9908	0	1	1	2	2
98	9912	9917	9921	9926	9930	9934	9939	9943	9948	9952	0	1	1	2	2
99	9956	9961	9965	9969	9974	9978	9983	9987	9991	9995	0	1	1	2	2

Table 39 COMMON ANTILOGARITHMS

L											Proportional parts				
	0	1	2	3	4	5	6	7	8	9	1	2	3	4	5
.00	1000	1002	1005	1007	1009	1012	1014	1016	1019	1021	0	0	1	1	1
.01	1023	1026	1028	1030	1033	1035	1038	1040	1042	1045	0	0	1	1	1
.02	1047	1050	1052	1054	1057	1059	1062	1064	1067	1069	0	0	1	1	1
.03	1072	1074	1075	1079	1081	1084	1086	1089	1091	1094	0	0	1	1	1
.04	1096	1099	1102	1104	1107	1109	1112	1114	1117	1119	0	1	1	1	1
.05	1122	1125	1127	1130	1132	1135	1138	1140	1143	1146	0	1	1	1	1
.06	1148	1151	1153	1156	1159	1161	1164	1167	1169	1172	0	1	1	1	1
.07	1175	1178	1180	1183	1186	1189	1191	1194	1197	1199	0	1	1	1	1
.08	1202	1205	1208	1211	1213	1216	1219	1222	1225	1227	0	1	1	1	1
.09	1230	1233	1236	1239	1242	1245	1247	1250	1253	1256	0	1	1	1	1
.10	1259	1262	1265	1268	1271	1274	1276	1279	1282	1285	0	1	1	1	1
.11	1288	1291	1294	1297	1300	1303	1306	1309	1312	1315	0	1	1	1	2
.12	1318	1321	1324	1327	1330	1334	1337	1340	1343	1346	0	1	1	1	2
.13	1349	1352	1355	1358	1361	1365	1368	1371	1374	1377	0	1	1	1	2
.14	1380	1384	1387	1390	1393	1396	1400	1403	1406	1409	0	1	1	1	2
.15	1413	1416	1419	1422	1426	1429	1432	1435	1439	1442	0	1	1	1	2
.16	1445	1449	1452	1455	1459	1462	1466	1469	1472	1476	0	1	1	1	2
.17	1479	1483	1486	1489	1493	1496	1500	1503	1507	1510	0	1	1	1	2
.18	1514	1517	1521	1524	1528	1531	1535	1538	1542	1545	0	1	1	1	2
.19	1549	1552	1556	1560	1563	1567	1570	1574	1578	1581	0	1	1	1	2
.20	1585	1589	1592	1596	1600	1603	1607	1611	1614	1618	0	1	1	1	2
.21	1622	1626	1629	1633	1637	1641	1644	1648	1652	1656	0	1	1	2	2
.22	1660	1663	1667	1671	1675	1679	1683	1687	1690	1694	0	1	1	2	2
.23	1698	1702	1706	1710	1714	1718	1722	1726	1730	1734	0	1	1	2	2
.24	1738	1742	1746	1750	1754	1758	1762	1766	1770	1774	0	1	1	2	2
.25	1778	1782	1786	1791	1795	1799	1803	1807	1811	1816	0	1	1	2	2
.26	1820	1824	1828	1832	1837	1841	1845	1849	1854	1858	0	1	1	2	2
.27	1862	1866	1871	1875	1879	1884	1888	1892	1897	1901	0	1	1	2	2
.28	1905	1910	1914	1919	1923	1928	1932	1936	1941	1945	0	1	1	2	2
.29	1950	1954	1959	1963	1968	1972	1977	1982	1986	1991	0	1	1	2	2
.30	1995	2000	2004	2009	2014	2018	2023	2028	2032	2037	0	1	1	2	2
.31	2042	2046	2051	2056	2061	2065	2070	2075	2080	2084	0	1	1	2	2
.32	2089	2094	2099	2104	2109	2113	2118	2123	2128	2133	0	1	1	2	2
.33	2138	2143	2148	2153	2158	2163	2168	2173	2178	2183	0	1	1	2	2
.34	2188	2193	2198	2203	2208	2213	2218	2223	2228	2234	1	1	2	2	3
.35	2239	2244	2249	2254	2259	2265	2270	2275	2280	2286	1	1	2	2	3
.36	2291	2296	2301	2307	2312	2317	2323	2328	2333	2339	1	1	2	2	3
.37	2344	2350	2355	2360	2365	2371	2377	2382	2388	2393	1	1	2	2	3
.38	2399	2404	2410	2415	2421	2427	2432	2438	2443	2449	1	1	2	2	3
.39	2455	2460	2466	2472	2477	2483	2489	2495	2500	2506	1	1	2	2	3
.40	2512	2518	2523	2529	2535	2541	2547	2553	2559	2564	1	1	2	2	3
.41	2570	2576	2582	2588	2594	2600	2606	2612	2618	2624	1	1	2	2	3
.42	2630	2636	2642	2649	2655	2661	2667	2673	2679	2685	1	1	2	2	3
.43	2692	2698	2704	2710	2716	2723	2729	2735	2742	2748	1	1	2	2	3
.44	2754	2761	2767	2773	2780	2786	2793	2799	2805	2812	1	1	2	2	3
.45	2818	2825	2831	2838	2844	2851	2858	2864	2871	2877	1	1	2	2	3
.46	2884	2891	2897	2904	2911	2917	2924	2931	2938	2944	1	1	2	2	3
.47	2951	2958	2965	2972	2979	2985	2992	2999	3006	3013	1	1	2	2	3
.48	3020	3027	3034	3041	3048	3055	3062	3069	3076	3083	1	1	2	2	3
.49	3090	3097	3105	3112	3119	3126	3133	3141	3148	3155	1	1	2	2	4
L	0	1	2	3	4	5	6	7	8	9	1	2	3	4	5

Enter from the margins with the given (common) logarithm, the corresponding number is given in the body of the table.

Table 39 COMMON ANTILOGARITHMS (continued)

L											Proportional parts				
	0	1	2	3	4	5	6	7	8	9	1	2	3	4	5
.60	3162	3170	3177	3184	3192	3199	3206	3214	3221	3228	1	1	2	3	4
.61	3236	3243	3251	3258	3266	3273	3281	3289	3296	3304	1	1	2	3	4
.62	3311	3319	3327	3334	3342	3350	3357	3365	3373	3381	1	1	2	3	4
.63	3388	3396	3404	3412	3420	3428	3436	3443	3451	3459	1	2	3	4	
.64	3467	3475	3483	3491	3499	3508	3516	3524	3532	3540	1	2	3	4	
.65	3548	3556	3565	3573	3581	3589	3597	3606	3614	3622	1	2	2	3	4
.66	3631	3639	3648	3656	3664	3673	3681	3689	3698	3707	1	2	2	3	4
.67	3715	3724	3733	3741	3750	3758	3767	3776	3784	3793	1	2	3	3	4
.68	3802	3811	3819	3828	3837	3846	3855	3864	3873	3882	1	2	3	3	4
.69	3890	3899	3908	3917	3926	3936	3945	3954	3963	3972	1	2	3	4	5
.70	3981	3990	3999	4009	4018	4027	4036	4046	4055	4064	1	2	3	4	5
.71	4074	4083	4093	4102	4111	4121	4130	4140	4150	4159	1	2	3	4	5
.72	4169	4178	4188	4198	4207	4217	4227	4236	4246	4256	1	2	3	4	5
.73	4266	4275	4285	4295	4305	4315	4325	4335	4345	4355	1	2	3	4	5
.74	4365	4375	4385	4395	4406	4416	4426	4436	4446	4457	1	2	3	4	5
.75	4467	4477	4487	4498	4508	4519	4529	4539	4550	4560	1	2	3	4	5
.76	4571	4581	4592	4603	4613	4624	4634	4645	4656	4667	1	2	3	4	5
.77	4677	4688	4699	4710	4721	4732	4742	4753	4764	4775	1	2	3	4	5
.78	4786	4797	4808	4819	4831	4842	4853	4864	4875	4887	1	2	3	4	5
.79	4898	4909	4920	4932	4943	4955	4966	4977	4989	5000	1	2	3	5	6
.80	5012	5023	5035	5047	5058	5070	5082	5093	5105	5117	1	2	3	5	6
.81	5129	5140	5152	5164	5175	5188	5200	5212	5224	5236	1	2	4	5	6
.82	5248	5260	5272	5284	5297	5309	5321	5333	5346	5358	1	2	4	5	6
.83	5370	5383	5395	5408	5420	5433	5445	5458	5470	5483	1	3	4	5	6
.84	5495	5508	5521	5534	5546	5559	5572	5585	5598	5610	1	3	4	5	6
.85	5623	5636	5649	5662	5675	5689	5702	5715	5728	5741	1	3	4	5	7
.86	5764	5778	5791	5804	5818	5831	5844	5858	5871	5885	1	3	4	5	7
.87	5898	5912	5925	5939	5952	5967	5980	5994	6008	6021	1	3	4	5	7
.88	6026	6039	6053	6067	6081	6095	6109	6124	6138	6152	1	3	4	6	7
.89	6166	6180	6194	6209	6223	6237	6252	6266	6281	6295	1	3	4	6	7
.90	6310	6324	6339	6353	6368	6383	6397	6412	6427	6442	1	3	4	6	7
.91	6457	6471	6486	6501	6516	6531	6546	6561	6577	6592	2	3	5	6	8
.92	6607	6622	6637	6653	6668	6683	6698	6714	6730	6745	2	3	5	6	8
.93	6761	6776	6792	6808	6823	6839	6855	6871	6887	6902	2	3	5	6	8
.94	6918	6934	6950	6966	6982	6998	7015	7031	7047	7063	2	3	5	7	8
.95	7079	7096	7112	7129	7145	7161	7178	7194	7211	7228	2	3	5	7	8
.96	7244	7261	7278	7295	7311	7328	7345	7362	7379	7396	2	3	5	7	8
.97	7413	7430	7447	7464	7482	7499	7516	7534	7551	7569	2	4	5	7	9
.98	7588	7603	7621	7638	7656	7674	7691	7709	7727	7745	2	4	5	7	9
.99	7762	7780	7798	7816	7834	7852	7870	7889	7907	7925	2	4	5	7	9
.90	7943	7962	7980	7998	8017	8035	8054	8072	8091	8110	2	4	6	7	9
.91	8128	8147	8166	8185	8204	8222	8241	8260	8279	8299	2	4	6	8	9
.92	8318	8337	8356	8375	8395	8414	8433	8453	8472	8492	2	4	6	8	10
.93	8511	8531	8551	8570	8590	8610	8630	8650	8670	8690	2	4	6	8	10
.94	8710	8730	8750	8770	8790	8810	8831	8851	8872	8892	2	4	6	8	10
.95	8913	8933	8954	8974	8995	9016	9036	9057	9078	9099	2	4	6	8	10
.96	9120	9141	9162	9183	9204	9226	9247	9268	9290	9311	2	4	6	9	11
.97	9333	9354	9375	9397	9419	9441	9462	9484	9506	9528	2	4	6	9	11
.98	9550	9572	9594	9616	9638	9661	9683	9705	9727	9750	2	4	7	9	11
.99	9772	9795	9817	9840	9863	9886	9908	9931	9954	9977	2	5	7	9	11
L	0	1	2	3	4	5	6	7	8	9	1	2	3	4	5

GLOSSARY OF SYMBOLS

<i>Symbol</i>	<i>Name or meaning</i>
<i>Algebra</i>	
$ a $	absolute value of a
$n!$	n factorial
$0!$	zero factorial, $0! = 1$
$\binom{n}{r} = \frac{n!}{(n-r)!r!}$	binomial coefficient; $\binom{n}{r}$ is coefficient of $a^{n-r}b^r$ in expansion of $(a + b)^n$
$\sqrt[n]{a} = a^{1/n}$	the positive n th root of a , $a > 0$
$\sqrt[q]{a^p} = a^{p/q}$	the positive q th root of a^p , $a > 0$
$\sum_{i=1}^n a_i$	$\sum_{i=1}^n a_i = a_1 + a_2 + \dots + a_n$
<i>Sets</i>	
\in	belongs to; is an element of; is a member of
\notin	does not belong to; is not an element of; is not a member of
\supset	contains
\subseteq	is included in; is a subset of
\subset	is a proper subset of; is properly included in; <i>inclusion</i>
$\not\subset$	is not a proper subset of; is not included in
\cup	union; cup; join; logical sum
\cap	intersection; cap; logical product
A' or $C_U A$	complement of A relative to universe U
\emptyset	the null set; the empty set
$=$	is equal to; equals; is identical with; is equivalent to
\neq	is not equal to; not equal
\approx or \leftrightarrow	is equivalent to
$\not\approx$	is not equivalent to
$\{ \}$	used to represent a set
$\{x\}$	the set of objects x
$\{ \text{ or } :$	such that; for which
$\{x \mid x \text{ has property } p\}$	the set of objects x having the property p
$\{x \mid \}$	a set builder, that is, a set consisting of all x such that
$[a, b]$	the set $a \leq x \leq b$
(a, b)	the set $a < x < b$

$[a, b]$	the set $a < x \leq b$
(a, b)	the set $a < x < b$
$\langle x, y \rangle$	ordered pair
$A \times B$	cartesian product of A and B , A cross B
$\cup A_r$ or $\bigcup_{r=1}^n A_r = A_1 \cup A_2 \cup \dots \cup A_n$	
$\cap A_r$ or $\bigcap_{r=1}^n A_r = A_1 \cap A_2 \cap \dots \cap A_n$	
\aleph_0	cardinal number of set of all natural numbers 1, 2, 3, ...; aleph null
\aleph	cardinal number of set of all real numbers; aleph

Logic

\wedge , &	and; <i>conjunction</i>
\vee	inclusive or; <i>inclusive disjunction</i>
$\underline{\vee}$	exclusive or; <i>exclusive disjunction</i>
\rightarrow or \Rightarrow	implies; <i>implication</i>
\leftrightarrow or \Leftrightarrow	is equivalent to; <i>equivalence</i>
\sim or $'$	not; <i>negation</i>
\forall	for all, for every; <i>universal quantifier</i>
\forall_x	for all x
\exists	there exists at least one; for some; <i>existential quantifier</i>
\exists_x	there exists an x such that; for some x
$\exists!$	there exists uniquely
$\exists!v$	there exists exactly one v such that; <i>uniqueness quantifier</i>
\vdash	it is asserted that

Relations

R	has the <i>relation</i> R to
\bar{R} or R'	does not have the relation R to
\bar{R}	the <i>converse</i> of relation R
\approx or $=$	is identical (or equivalent) to; <i>identity</i> or <i>equivalence</i> relation
$\not\approx$ or \neq	is not identical (or not equivalent) to; <i>diversity relation</i>
\subset	is included in
R^{-1}	the <i>inverse</i> of relation R
$x R y$	x stands in the (binary) relation R to y

$x \not R y$	x does not stand in the (binary) relation R to y
\leftrightarrow	one-to-one reciprocal correspondence
\rightarrow or \curvearrowright	corresponds to; gives; is transformed into; has for an image
\underline{T} or \underline{T}	passage by means of T ; is transformed by T into
$F: S \rightarrow T$	F maps S into T

Functions

$f(x)$	value of function f at x
f^{-1}	inverse of function f
$f \circ g$	composite of function f and function g
$f^{-1} \circ f = f \circ f^{-1} = E$	
$f: (x, y)$	the function f whose ordered pairs are (x, y)

Probabilities

$P(E)$	probability of E
$P(E \cap F)$ or $P(E \wedge F)$	probability of both E and F
$P(E F)$	probability of E , given F
$P(E \cup F)$ or $P(E \vee F)$	probability of E or F or both
$P(E \setminus F)$	probability of E or F , but not both E and F

Algebraic Structures

$\circ, \odot, \oplus, \otimes, *$	
$\cup, \cap, +, -$	symbols for operations
z or 0	zero
u or 1	unity
a^{-1}	inverse of a
\otimes	symbol used in ordering
\ominus	symbol used in ordering

Vectors

$V_1 \cdot V_2$	scalar product
$V_1 \times V_2$	vector product
$\nabla S, \text{grad } S$	gradient of S
$\text{div } V, \nabla \cdot V$	divergence of V

$\text{curl } V, \text{ rot } V,$
 $\nabla \times V$ curl of V
 $\nabla^2 S, \text{ div grad } S$ divergence of grad S

Matrices

$\begin{pmatrix} a_{11} & \cdots & a_{1n} \\ \cdots & \cdots & \cdots \\ a_{n1} & \cdots & a_{nn} \end{pmatrix}$ or $[a_{rs}]$ or (a_{ij}) or A matrix

(x_1, x_2, \dots, x_n) or \vec{x}_p or $\|x_1, x_2, \dots, x_n\|$ row matrix, row vector

$\begin{Bmatrix} y_1 \\ \cdots \\ y_n \end{Bmatrix}$ or $y_p \uparrow$ or $\begin{Bmatrix} y_1 \\ \cdots \\ y_n \end{Bmatrix}$ column matrix, column vector

$\det A = |A| = \begin{vmatrix} a_{11} & \cdots & a_{1n} \\ \cdots & \cdots & \cdots \\ a_{n1} & \cdots & a_{nn} \end{vmatrix}$ determinate of matrix A

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